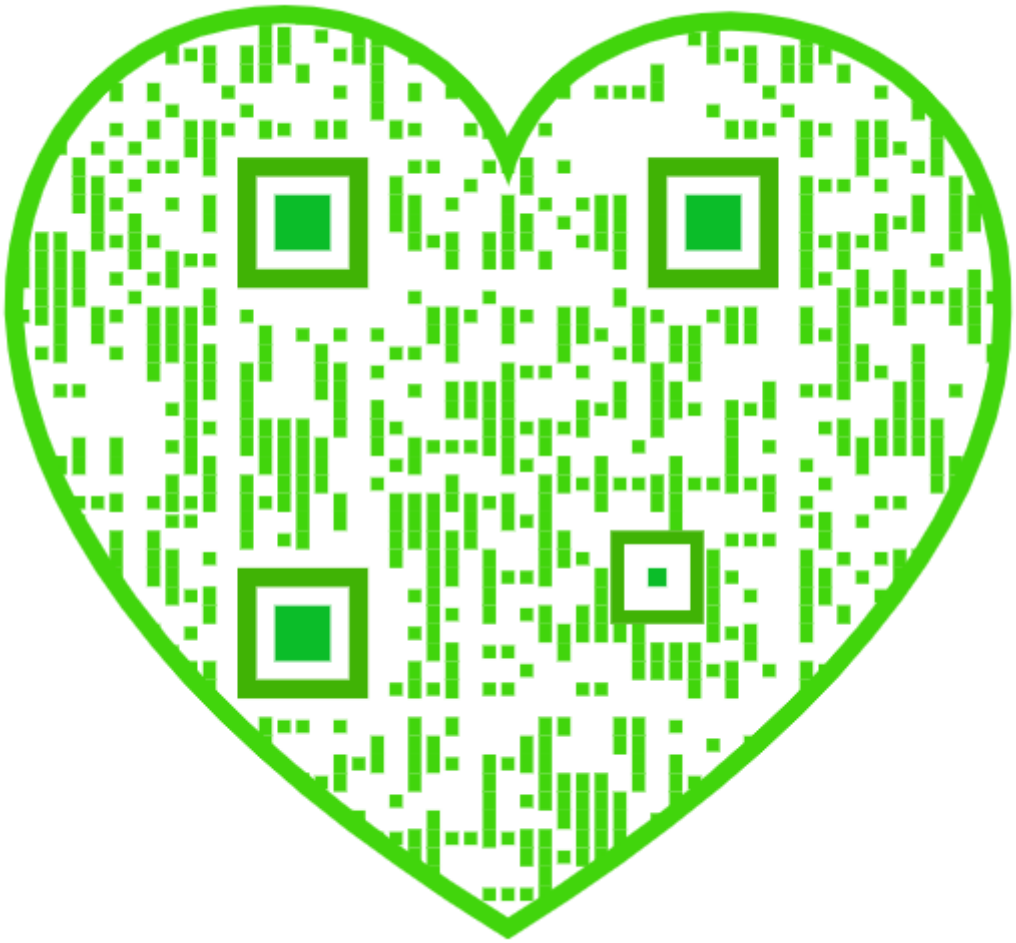

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*SHARPLESS & BROWN'S MATHEMATICAL SERIES.*

# GUMMERE'S

# SURVEYING.

REVISED AND ADAPTED TO MODERN METHODS,

By ISAAC SHARPLESS,
PROFESSOR OF MATHEMATICS IN HAVERFORD COLLEGE, PA.

PHILADELPHIA:
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# PREFACE.

GUMMERE'S SURVEYING was published in 1814. It was the first book which placed the subject in a systematic shape for the use of students, and the public fully appreciated its merits. The author made frequent revisions during his lifetime, as the numerous editions were called for, and these have enabled it to retain for so long a time the confidence of educators and practical surveyors.

The present edition contains, in addition to the prominent features of the old book, the following:

1. New tables of six-place logarithms and logarithmic sines and tangents, and the adaptation of all examples to them. These tables have been carefully compared with the latest German edition of Schrön's seven-place tables, and are undoubtedly reliable.

2. Surveying with Transit and Theodolite, with examples showing the methods employed in practice.

3. New rules and demonstrations for the calculation of areas and for laying out and dividing land.

4. A large number of minor revisions necessary to secure harmony through the book and to adapt the descriptions to modern methods and usages.

W. & L. E. Gurley, instrument-makers, Troy, N. Y., have furnished electrotypes, thus permitting the insertion of figures of the most approved and modern forms of surveying instruments.

I. S.

HAVERFORD COLLEGE, 1880.

# PREFACE.

The following compilation originated in the belief that our schools are in want of a Treatise on Surveying, adapted to the methods practised in this country, and freed from the defects of the systems now in use. Notwithstanding the importance of the science, and the large number that make it an object of study, it is believed we are not in possession of a treatise on this subject, suited to the wants of the student. The works of Gibson and Jess are the only ones at present in general use; the former, though much the better of the two is deficient in many respects. It may be sufficient here, merely to advert to its want of examples, which renders it entirely unsuitable for a school book. From the latter, the student would in vain expect to become acquainted with the principles of the science, or the *rationale* of any of the rules, necessary in performing the various calculations.*

In order to understand the principles of surveying, a previous knowledge of Geometry is absolutely necessary; and this knowledge will be best acquired from a regular treatise on the subject. In the demonstrations, therefore, throughout this work, the student is supposed to be acquainted with the elements of that science. The references are adapted to Playfair's Geometry but they will in general apply equally well to Simson's translation of Euclid's Elements.

As there are many who wish to obtain a practical knowledge of Surveying, whose leisure may be too limited to admit of their

* Each of these works has lately gone through a new edition, in which considerable additions are stated to have been made. On examination, however, it does not appear, that those additions are such as to supply the deficiencies.

The additions made to Gibson, consist principally of some nautical problems quite foreign to a treatise on Surveying. Those made to Jess, consist of a few extracts from Gibson, in one of which the Pennsylvania method of calculation is introduced, as being quite different from that given by Jess; whereas it is well known to be the method given by that author, and used, as well in the preceding as in the subsequent part of his work.

going through a course of Geometry, the author has adapted his work to this class, by introducing the necessary geometrical definitions and problems, and by giving plain and concise rules, entirely detached from the demonstrations; the latter being placed in the form of notes at the bottom of the page. Each rule is exemplified by one wrought example; and the most of them by several unwrought examples, with the answers annexed.

In the laying out and dividing of land, which forms the most difficult part of surveying, a variety of problems is introduced, adapted to the cases most likely to occur in practice. This part of the subject, however, presents such a great variety of cases, that we should in vain attempt to give rules that would apply to all of them. It cannot therefore be too strongly recommended to every one, who has the opportunity, to make himself well acquainted with Geometry, and also with Algebra, previous to entering on the study of Surveying. Furnished with these useful auxiliaries, and acquainted with the principles of the science, the practitioner will be able to perform, with ease, any thing likely to occur in his practice.

The compiler thinks proper to acknowledge, that in the arrangement of the work, he availed himself of the advice of his learned preceptor and friend, E. Lewis of New-Garden; and that several of the demonstrations were furnished by him.

J. G.

# CONTENTS.

## INTRODUCTION.


PAGE

Logarithms ..... 9
Geometrical Definitions ..... 25
Geometrical Problems ..... 30
Plane Trigonometry ..... 37


## SURVEYING.

### CHAPTER I.


Dimensions of a Survey ..... 80


### CHAPTER II.


Supplying Omissions in the Dimensions of a Survey ..... 116


### CHAPTER III.


Content of Land ..... 127


### CHAPTER IV.


Laying Out and Dividing Land ..... 156


### CHAPTER V.


Miscellaneous Examples ..... 180


### CHAPTER VI.


Variation of the Compass ..... 184


### CHAPTER VII.


The Theodolite and Transit ..... 197

## CHAPTER VIII.

PAGE
Levelling..... 213

## CHAPTER IX.

Topography..... 230

## CHAPTER X.

Coast Survey..... 244
Surveying the Public Lands of the U. S..... 247

## TABLES.

Logarithms..... 1
Logarithmic Sines and Tangents..... 18
Proportional Parts..... 63
Traverse Table..... 66
Natural Sines and Cosines..... 79

A

# TREATISE ON SURVEYING

## OF LOGARITHMS.

LOGARITHMS are a series of numbers so contrived, that by them the work of multiplication is performed by addition, and that of division by subtraction.

If a series of numbers in arithmetical progression be placed as indices, or exponents, to a series of numbers in geometrical progression, the sum or difference of any two of the former, will answer to the product or quotient of the two corresponding terms of the latter. Thus,

0. 1. 2. 3. 4. 5. 6. 7. &c., arith. series, or indices.
1. 2. 4. 8. 16. 32. 64. 128. &c., geom. series.

Now $2+3=5$. also $7-3=4$.
And $4\times8=32$. and $128\div8=16$.

Therefore the arithmetical series, or indices, have the same properties as logarithms; and these properties hold true, whatever may be the ratio of the geometrical series.

There may, therefore, be as many different systems of logarithms, as there can be taken different geometrical series having unity for the first term. But the most

convenient system is that in which the ratio of the geometrical series is 10, and this is the one in common use. Thus,

0, 1, 2, 3, 4, 5, &c. indices or logar.
1, 10, 100, 1000, 10000, 100000, &c. natural numbers.

In this system the log. of 1 is 0*, the log. of 10 is 1, the log. of 100 is 2, &c. Hence it is plain that the log. of any number between 1 and 10 will be expressed by a decimal, the log. of any number between 10 and 100 by 1 and a decimal, the log. of any number between 100 and 1000 by 2 and a decimal, &c.

The numbers, 0, 1, 2, 3, &c., that stand before the decimal part of logarithms are called *indices*, and are always less by unity than the number of figures in the integral part of the corresponding natural number.

The index of the logarithm of a number, consisting in whole, or in part, of integers, is positive; but if the number be a decimal the index is negative, and is marked by a negative sign (—), placed either before or above it. If the first significant figure of the decimal be adjacent to the decimal point, the index is $-1$, or $\bar{1}$; if there be one cipher between them, the index is $-2$, or $\bar{2}$; if there be two ciphers between them, the index is $-3$, or $\bar{3}$, &c.

The decimal parts of the logarithms of numbers, consisting of the same figures and in the same order, are the same, whether the number be integral, fractional, or mixed. This is illustrated as follows:

* In every system the logarithm of 1 is 0.

| | |
|---|---|
| Number 18960 | Logarithm 4.277838 |
| 1896 | 3.277838 |
| 189.6 | 2.277838 |
| 18.96 | 1.277838 |
| 1.896 | 0.277838 |
| .1896 | —1.277838 |
| .01896 | —2.277838 |
| .001896 | —3.277838 |
| .0001896 | —4.277838 |

The method of finding logarithms in the tables, and of multiplying, dividing, &c. by them, is contained in the following problems:

## PROBLEM I.

*To find the logarithm of a given number.*

If the given number consists of *one* or *two* figures only, find it in the column marked *N.* in the first page of the table, and against it, in the next column, marked *log.*, is the logarithm. Thus the log. of 7 will be found 0.845098, and the log. of 85 will be found 1.929419.

But if the given number be either wholly or in part decimal, the index must be changed accordingly. Observing that the index must always be one less than the number of figures in the integral part of the given number; also, when the given number is wholly a decimal, the index is negative, and must be one more than the number of the ciphers between the decimal point and first significant figure on the left hand. Thus the log. of .7 is —1.845098, and the log. of .0085 is —3.929419.

If the given number consists of *three* figures, find it in one of the other pages of the table, in the column marked *N.*, and against it, in the next column, is the decimal

part of the logarithm. The index must be placed before it agreeably to the above observation. Thus the log. of 421 is 2.624282, the log. of 4.21 is 0.624282, and the log. of .0421 is —2.624282.

If the given number consists of *four* figures, find the three left hand figures in the column marked *N.* as before, and the remaining, or right hand figure at the top of the table; in the column under this figure, and against the other three, is the decimal part of the logarithm. Thus the log. of 5163 is 3.712902, and the log. of .6387 is —1.805297.

If the given number consists of *five* or *six* figures, find the logarithm of the four left hand figures as before; then take the difference between this logarithm and the next greater in the table. Multiply this difference by the remaining figure or figures of the given number, and cut off one or two figures to the right hand of the product, according as the multiplier consists of one or two figures; then add the remaining figure or figures of the product to the logarithm first taken out of the table, and the sum will be the logarithm required. Thus, let it be required to find the logarithm of 59686; then,

| | |
|---|---|
| Logarithm of 5968 is - - - | 775829 |
| The next greater log. is - - | 775902 |
| Difference - - - - - | 73 |
| Remaining figure - - - | 6 |
| Product - - - - - | 43,8 |
| To - - - - - - | 775829 |
| Add - - - - - - | 44* |
| Decimal part of the log. - - | 775873 |

* Because 43.8 is nearer 44 than 43.

The natural number consisting of five integers, the index must be 4; therefore the log. of 59686 is 4.775873.

Again, let it be required to find the log. of .0131755; then,

| | |
|---|---|
| Logarithm of 1317 is - - - | 119586 |
| The next greater log. is - - | 119915 |
| * Difference - - - - - | 329 |
| Remaining figures - - - | 55 |
| Product - - - - - | 180,95 |
| To - - - - - - | 119586 |
| Add - - - - - - | 181 |
| Decimal part of the log. - - | 119767 |

As the given number is a decimal, and has one cipher between the decimal point and first significant figure, the index must be $-2$; therefore the log. of .0131755 is $-2.119767$.

EXAMPLES.

| | | | |
|---|---|---|---|
| 1. Required the log. of | 4.3 | *Ans.* | 0.633468 |
| 2. Required the log. of | 7986 | *Ans.* | 3.902329 |
| 3. Required the log. of | .3754 | *Ans.* | $-1.574494$ |
| 4. Required the log. of | 596.87 | *Ans.* | 2.775879 |
| 5. Required the log. of | 785925 | *Ans.* | 5.895381 |
| 6. Required the log. of | 6543900 | *Ans.* | 6.815836 |
| 7. Required the log. of | .0027863 | *Ans.* | $-3.445028$ |

* This difference is given in column D. In the first pages of the table the logarithms vary so rapidly that it is safer in accurate work to make the subtraction as above.

## PROBLEM II.

*To find the natural number corresponding to a given logarithm.*

If *four* figures only be required in the answer, look in the table for the decimal part of the given logarithm, and if it cannot be found exactly, take the one nearest to it, whether greater or less; then the three figures in the first column, marked *N*, which are in a line with the logarithm found, together with the figure at the top of the table directly above it, will form the number required. Observing that, when the index of the given logarithm is positive, the integers in the number found must be one more than the number expressed by the index; but when the index of the given logarithm is negative, the number found will be wholly a decimal, and must have one cipher less—placed between the decimal point and first significant figure on the left hand—than the number expressed by the index. Thus the natural number corresponding to the logarithm 2.902329 is 798.6, the natural number corresponding to the logarithm 3.770557 is 5896, and the natural number corresponding to the logarithm $-3.363612$ is .00231.

If the exact logarithm be found in the table, and the figures in the number corresponding do not exceed the index by one, annex ciphers to the right hand till they do. Thus the natural number corresponding to the logarithm 6.640680 is 4372000.

If *five* or *six* figures be required in the answer, find in the table the logarithm next less than the given one, and take out the four figures answering to it as before. Subtract this logarithm from the next greater in the table, and also from the given logarithm; to the latter difference annex one or two ciphers, according as five

or six figures are required, and divide the number thus produced by the former difference; annex the quotient to the right hand of the four figures already found, and it will give the natural number required.

Thus let it be required to find the natural number corresponding to the logarithm 2.538990 true to *five* figures; then,

Given logarithm - .538990
Next less - - .538951, the natural number corresponding is 3459.
Diff. with one cipher annexed 390

| | | |
|---|---|---|
| Next less log. | - - | .538951 |
| Next greater | - - | .539076 |
| Difference | - - | 125 |

Divide 390 by 125 and the quotient will be 3, which, annexed to the right hand of 3459, the four figures already found, makes 34593; therefore, as the index is 2, the required natural number is 345.93.

Again, let it be required to find the natural number corresponding to the logarithm 4.598590, true to *six* figures; then,

Given logarithm - .598590
Next less - - .598572, the natural number answering to it is 3968.
Diff. with two ciphers annexed - - 1800

| | | |
|---|---|---|
| Next less log. | - - | .598572 |
| Next greater | - - | .598681 |
| Difference | - - | 109 |

Divide 1800 by 109, and the quotient will be 17, which annexed to the right hand of 3968, the four

figures already found, makes 396817; therefore, as the index is 4, the required natural number is 39681.7.

EXAMPLES.

1. Required the natural number answering to the logarithm 1.880299. *Ans.* 75.91.

2. Required the natural number answering to the logarithm 5.370817. *Ans.* 234861.

3. Required the natural number answering to the logarithm 3.119770. *Ans.* 1317.56.

4. Required the natural number answering to the logarithm $-2.974350$. *Ans.* .094265.

## PROBLEM III.

*To multiply numbers by means of logarithms.*

Case 1.—When all the factors are whole or mixed numbers.

RULE.

Add together the logarithms of the factors, and the sum will be the logarithm of the product.

EXAMPLES.

1. Required the product of 84 by 56.

| | | | |
|---|---|---|---|
| Logarithm of | 84 | is | 1.924279 |
| Do. | of 56 | is | 1.748188 |
| Product | 4704 | Sum | 3.672467 |

2. Required the continued product of 17.3, 1.907 and 34.

| | | | |
|---|---|---|---|
| Logarithm of | 17.3 | is | 1.238046 |
| Do. | 1.907 | is | 0.280351 |
| Do. | 34. | is | 1.531479 |
| Product | 1121.71 | Sum | 3.049876 |

3. Find by logarithms the product of 76.5 by 5.5. *Ans.* 420.75.

4. Find by logarithms the continued product of 42.35, 1.7364, and 1.76. *Ans.* 129.424.

CASE 2.—When some or all of the factors are decimal numbers.

RULE.

Add the decimal parts of the logarithms as before, and if there be any to carry from the decimal part, add it to the positive index or indices, or else subtract it from the negative.

Then add the indices together, when they are all of the same kind; that is, all positive or all negative; but when they are of different kinds, take the difference between the sums of the positive and negative ones, and prefix the sign of the greater.

*Note.*—When the index is positive, it is not necessary to place any sign before it; but when it is negative, the sign must not be omitted.

EXAMPLES.

1. Required the continued product of 349.17, 25.43, .93521 and .00576.

| | | | |
|---|---|---|---|
| Logarithm of | 349.17 | is | 2.543037 |
| Do. | 25.43 | is | 1.405346 |
| Do. | .93521 | is | −1.970909 |
| Do. | .00576 | is | −3.760422 |
| Product 47.83. | | Sum | 1.679714 |

In this example there is 2 to carry from the decimal part of the logarithms, which added to 3, the sum of the affirmative indices, makes 5; from this taking 4, the sum of the negative indices, the remainder is 1, which is the index of the sum of the logarithms, and is affirmative, because the sum of the affirmative indices, together with the number carried, exceed the sum of the negative indices.

2. Required the continued product of .0839, .7536, and .003179.

| | | | |
|---|---|---|---|
| Logarithm of | .0839 | is | −2.923762 |
| Do. | .7536 | is | −1.877141 |
| Do. | .003179 | is | −3.502291 |
| Product .000201. | | Sum | −4.303194 |

In this example there is 2 to carry from the decimal part of the logarithms, which subtracted from 6, the sum of the negative indices, leaves 4, which is the index of the sum of the logarithms, and is negative, because the sum of the negative indices is the greater.

3. Required the continued product of 13.19, .3765, and .00415. *Ans.* .02061.

4. Required the continued product of 343, 1.794, 5.41, and .019. *Ans.* 63.25.

## PROBLEM IV.

*To divide numbers by means of Logarithms.*

Case 1.—When the dividend and divisor are both whole or mixed numbers

RULE.

From the logarithm of the dividend subtract the logarithm of the divisor; the remainder will be the logarithm of the quotient.

*Note.*—When the divisor exceeds the dividend, the question must be wrought by the rule given in the next case.

EXAMPLES.

1. Required the quotient of 3450 divided by 23.

| | | | |
|---|---|---|---|
| Logarithm of | 3450 | is | 3.537819 |
| Do. | 23 | is | 1.361728 |
| Quotient 150. | | Remainder | 2.176091 |

2. Required the quotient of 420.75 divided by 76.5. *Ans.* 5.5.

3. Required the quotient of 37.1542 divided by 1.73958. *Ans.* 21.3585.

Case 2.—When the dividend or divisor, or both of them, are decimal numbers.

RULE.

Subtract the decimal parts of the logarithms as before, and if 1 be borrowed in the left-hand place of the decimal part, add it to the index of the divisor when that index is positive, but subtract it when negative.

Then conceive the sign of the index of the divisor changed from positive to negative, or from negative to positive; and if, when changed, it be of the same name with that of the dividend, add the indices together,

but if it be of a different name, take the difference of the indices, and prefix the sign of the greater.

EXAMPLES.

1. Required the quotient of .7591 divided by 32.147.

| | | | |
|---|---|---|---|
| Logarithm of | .7591 | is | −1.880299 |
| Do. | 32.147 | is | 1.507140 |
| Quotient .02361 | | Remain. | −2.373159 |

In this example the index of the divisor, with its sign changed, is −1, which added to −1, the index of the dividend, makes −2, for the index of the quotient.

2. Required the quotient of .63153 divided by .00917.

| | | | |
|---|---|---|---|
| Logarithm of | .63153 | is | −1.800394 |
| Do. | .00917 | is | −3.962369 |
| Quotient 68.8683. | | Remain. | 1.838025 |

In this example there is 1 to carry from the decimal part of the logarithm, which subtracted from −3, the index of the divisor, leaves −2; this, with its sign changed, is +2; from which subtracting 1, the index of the dividend, the remainder is 1, and is affirmative, because the affirmative index is the greater.

3. Required the quotient of 13.921 divided by 7965.13.

| | | | |
|---|---|---|---|
| Logarithm of | 13.921 | is | 1.143670 |
| Do. | 7965.13 | is | 3.901194 |
| Quotient .001748 | | Remain. | −3.242476 |

In this example there is 1 to carry from the decimal part of the logarithm, which added to 3, the index of the divisor, makes 4; this, with its sign changed, is $-4$; from which subtracting 1, the index of the dividend, the remainder is $-3$.

4. Required the quotient of 79.35 divided by .05178.
*Ans.* 1532.46.

5. Required the quotient of .5903 divided by .931.
*Ans.* .63404.

## PROBLEM V.

*To involve a number to any power; that is, to find the square, cube, &c. of a number logarithmically.*

### RULE.

Multiply the logarithm of the given number by the index of the power, viz. by 2 for the square, by 3 for the cube, &c., and the product will be the logarithm of the power.

*Note.*—When the index of the logarithm is negative, if there be any to carry from the decimal part, instead of adding it to the product of the index and multiplier, subtract it, and the remainder will be the index of the logarithm of the power, and will always be negative.

### EXAMPLES.

1. Required the square of 317.

| | | |
|---|---|---|
| Logarithm of 317 | is | 2.501059 |
| | | 2 |
| Square 100489 | | 5.002118 |

2. Required the 5th power of 1.735.

| | | |
|---|---|---|
| Logarithm of 1.735 | is | 0.239299 |
| | | 5 |
| 5th power 15.7218 | | 1.196495 |

3. Required the cube of .08761.

| | | |
|---|---|---|
| Logarithm of .08761 | is | $-2.942554$ |
| | | 3 |
| Cube .0006725 | | $-4.827662$ |

4. Required the cube of 7.503. *Ans.* 422.37.

5. Required the 7th power of .32513. *Ans.* .0003841.

## PROBLEM VI.

*To extract any root of a number logarithmically.*

RULE.

Divide the logarithm of the given number by the index of the root—that is, by 2 for the square root, by 3 for the cube root, &c.—and the quotient will be the logarithm of the required root.

*Note.*—When the index of the logarithm is negative, and does not exactly contain the divisor, increase the index by a number just sufficient to make it exactly divisible by it, and carry the units borrowed, as so many tens, to the left-hand figure of the decimal part; then proceed with the division in the usual manner.

EXAMPLES.

1. Required the cube root of 391.27.

| | | 3) |
|---|---|---|
| Logarithm of 391.27 | is | 2.592477 |
| Cube root 7.314 | | 0.864159 |

2. Required the square root of .08593.

| | | 2) |
|---|---|---|
| Logarithm of .08593 | is | $-2.934145$ |
| Square root .29314 | | $-1.467072$ |

3. Required the cube root of .7596.

| | | 3) |
|---|---|---|
| Logarithm of .7596 | is | $-1.880585$ |
| Cube root .9124 | | $-1.960192$ |

4. Required the cube root of .0000613.

| | | 3) |
|---|---|---|
| Logarithm of .0000613 | is | $-5.787460$ |
| Cube root .03943 | | $-2.595820$ |

5. Required the square root of 365. *Ans.* 19.105.
6. Required the 5th root of .9563. *Ans.* .9911.
7. Required the 4th root of .00079. *Ans.* .16765.

*Of the Arithmetical Complements of Logarithms.*

When it is required to subtract several logarithms from others, it will be more convenient to convert the subtraction into an addition, by writing down, instead of the logarithms to be subtracted, what each of them wants of 10.000000, which may readily be done, by writ-

ing down what the first figure, on the right hand, wants of 10, and what every other figure wants of 9; this remainder is called the *Arithmetical Complement.* Thus, if the logarithm be 2.53061, its arithmetical complement will be 7.46939. If one or more figures to the right hand be ciphers, write ciphers in their place, and take the first significant figure from 10, and the remaining figures from 9. Thus, if the logarithm be 4.61300, its arithmetical complement will be 5.38700.

In any operation, where the arithmetical complements of logarithms are added to other logarithms, there must be as many tens subtracted from the sum, as there are arithmetical complements used.

As an example, let it be required to divide the product of 76.4 and 35.84 by the product of 473.9 and 4.76.

| | |
|---|---|
| 473.9 - - - | Ar. Co. 7.324313 |
| 4.76 - - - | Ar. Co. 9.322393 |
| 35.84 - - - | - log. 1.554368 |
| 76.4 - - - | - log. 1.883093 |
| Quotient 1.214 | 0.084167 |

# GEOMETRY.

## DEFINITIONS.

1. GEOMETRY is that science wherein the properties of magnitude are considered.

2. A *point* is that which has position, but not magnitude.

3. A *line* has length but not breadth.

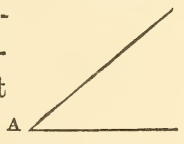
4. A *straight*, or *right line*, is the shortest line that can be drawn between any two points.

5. A *superficies* or *surface* is that which has length and breadth, but not thickness.

6. A *plane superficies* is that in which any two points being taken, the straight line which joins them lies wholly in that superficies.

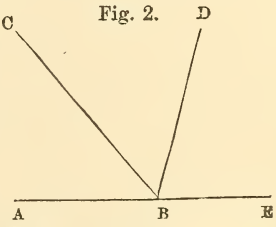
7. A *plane rectilineal angle* is the inclination of two straight lines to one another, which meet together, but are not in the same straight line, as A, Fig. 1.

Fig. 1.




*Note.*—When several angles are formed about the same point, as at B, Fig. 2, each particular angle is expressed by three letters, whereof the middle letter shows the angular point, and the other two the lines that form the angle; thus, CBD or DBC signifies the angle formed by the lines CB and DB.

Fig. 2.



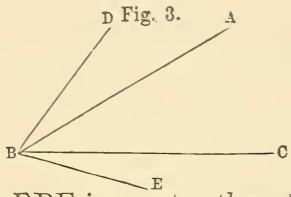


Fig. 3.

8. The *magnitude* of an *angle* depends on the inclination which the lines that form it have to each other, and not on the length of those lines. Thus the angle DBE is greater than the angle ABC, Fig. 3.

9. When a straight line stands on another straight line so as to incline to neither side, but makes the angles on each side equal, then each of those angles is called a *right angle,* and the line which stands on the other is said to be *perpendicular* to it. Thus ADC and BDC are right angles, and the line CD is perpendicular to AB, Fig. 4.

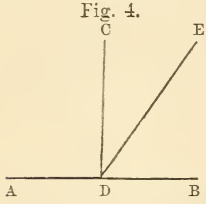


Fig. 4.

10. An *acute* angle is that which is less than a right angle, as BDE, Fig. 4.

11. An *obtuse* angle is that which is greater than a right angle, as ADE, Fig. 4.

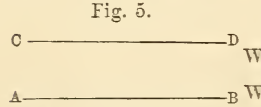


Fig. 5.

12. *Parallel straight lines* are those which are in the same plane, and which, being produced ever so far both ways, do not meet, as AB, CD, Fig. 5.

13. A *figure* is a space bounded by one or more lines.

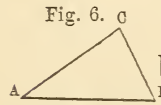


Fig. 6.

14. A *plane triangle* is a figure bounded by three straight lines, as ABC, Fig. 6.

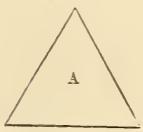


Fig. 7.

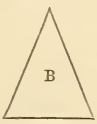


Fig. 8.

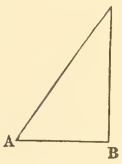
15. An *equilateral* triangle has its three sides equal to each other, as A, Fig. 7.

16. An *isosceles* triangle has only two of its sides equal, as B, Fig. 8.

17. A *scalene* triangle has three unequal sides, as ABC, Fig. 6.

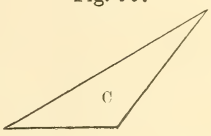
18. A *right angled* triangle has one right angle, as ABC, Fig. 9; in which the side AC, opposite to the right angle, is called the *hypothenuse.*

Fig. 9.




19. An *obtuse angled triangle* has one obtuse angle, as C, Fig. 10.

Fig. 10.



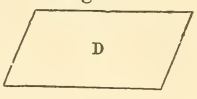

20. An *acute angled triangle* has all its angles acute, as ABC, Fig. 6.

21. Acute and obtuse angled triangles are called *oblique angled triangles.*

22. Any plane figure bounded by four right lines, is called a *quadrilateral.*

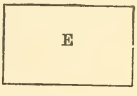
23. Any quadrilateral, whose opposite sides are parallel, is called a *parallelogram*, as D, Fig. 11.

Fig. 11.




24. A parallelogram, whose angles are all right angles, is called a *rectangle*, as E, Fig. 12.

Fig. 12.



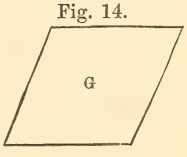

25. A parallelogram whose sides are all equal, and angles right, is called a *square*, as F, Fig. 13.

Fig. 13.

26. A *rhomboid* is a parallelogram, whose opposite sides are equal, and angles oblique, as D, Fig. 11.

27. A *rhombus* is a parallelogram, whose sides are all equal and angles oblique, as G, Fig. 14.

Fig. 14.



28. Any quadrilateral figure that is not a parallelogram, is called a *trapezium.*

29. A trapezium that has two parallel sides is called a *trapezoid.*

30. A right line joining any two opposite angles of a quadrilateral figure, is called a *diagonal.*

31. That side upon which any parallelogram, or triangle is supposed to stand, is called the *base;* and the perpendicular falling thereon from the opposite angle is called the *altitude* of the parallelogram, or triangle. Thus AD is the base of the parallelogram ABEC, or triangle ABC, and CD is the altitude, Fig. 15.

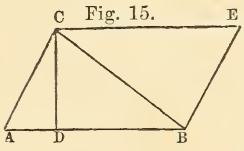


Fig. 15.

32. All plane figures contained by straight lines are called *polygons;* of which those having five sides are called *pentagons;* those having six sides, *hexagons,* and so on.

33. A *regular polygon* is one whose angles, as well as sides, are all equal.

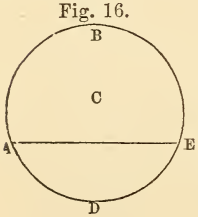


Fig. 16.

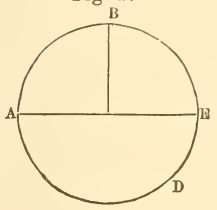
34. A *circle* is a plane figure, bounded by one curve line called the *circumference* or *periphery,* every part of which is equally distant from a certain point within the circle; and this point is called the *centre,* Fig. 16.

35. The *radius* of a circle is a straight line drawn from the centre to the circumference, as CB, Fig. 17.

36. The *diameter* of a circle is a straight line drawn through the centre, and terminated both ways by the

circumference, as AE, Fig. 17. It divides the circle into two equal parts, called *semicircles.*

Fig 17.




37. A *quadrant* is one quarter of a circle, as ACB, Fig. 17.

*Note.*—The fourth part of the circumference of a circle is also called a quadrant.

38. A *segment* of a circle is the figure contained by a right line, and the part of the circumference it cuts off: thus AEBA and AEDA are segments of the circle ABED, Fig. 16.

39. An *arc* of a circle is any part of the circumference; as AD or DE, Fig. 17.

40. *Ratio* is a mutual relation between two quantities of the same kind with respect to magnitude.

*Note.*—A ratio is generally expressed, either by two numbers or by two right lines.

41. When two quantities have the same ratio as two other quantities, the four quantities taken in order are called *proportionals;* and the last is said to be a *fourth proportional* to the other three.

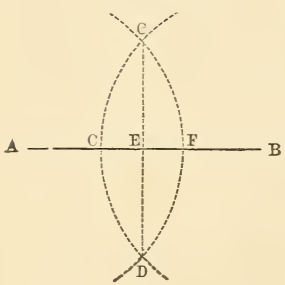
42. When three quantities of the same kind are such that the first has to the second the same ratio which the second has to the third, the third is called a *third proportional* to the first and second, and the second is called a *mean proportional* between the first and third.

# GEOMETRICAL PROBLEMS.

## PROBLEM I.

*To bisect a right line,* AB, Fig. 18.

Fig. 18.

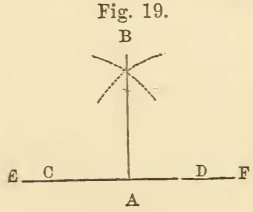



Open the dividers to any distance more than half the line AB, and with one foot in A, describe the arc CFD; with the same opening, and one foot in B, describe the arc CGD, meeting the first arc in C and D; from C to D draw the right line CD, cutting AB in E, which will be equally distant from A and B.

## PROBLEM II.

*At a given point* A, *in a right line* EF, *to erect a perpendicular,* Fig. 19.

Fig. 19.



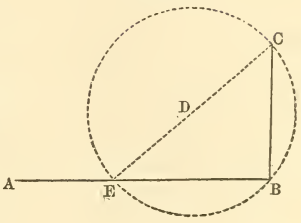

From the point A, lay off on each side, the equal distances AC, AD; from C and D, as centres, with any radius greater than AC or AD, describe two arcs intersecting each other in B; from A to B, draw the line AB, which will be the perpendicular required.

## PROBLEM III.

*To raise a perpendicular on the end* B *of a right line* AB, Fig. 20.

Take any point D not in the line AB, and with the distance from D to B, describe a circle cutting AB in E; from E through D draw the right line EDC, cutting the periphery in C, and join CB, which will be perpendicular to AB.

Fig. 20.

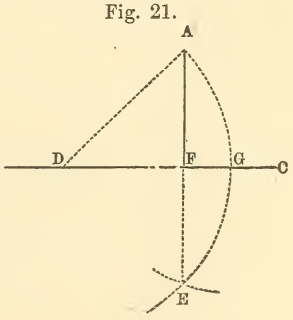



## PROBLEM IV.

*To let fall a perpendicular upon a given line* BC, *from a given point* A, *without it*, Fig. 21.

In the line BC take any point D, and with it as a centre and distance DA describe an arc AGE, cutting BC in G; with G as centre, and distance GA, describe an arc cutting AGE in E, and from A to E draw the line AFE; then AF will be perpendicular to AB.

Fig. 21.

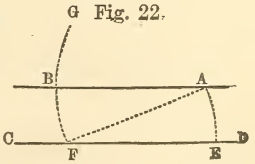



## PROBLEM V.

*Through a given point* A *to draw a right line* AB, *parallel to a given right line* CD, Fig. 22.

From the point A to any point F, in the line CD, draw the right

Fig. 22.

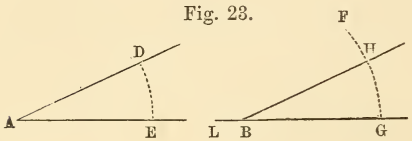


line AF; with F as a centre and distance FA, describe the arc AE, and with the same distance and centre A describe the arc FB; make FB equal to AE, and through A and B draw the line AB, and it will be parallel to CD.

## PROBLEM VI.

*At a given point* B, *in a given right line* LG, *to make an angle equal to a given angle* A, Fig. 23.

Fig. 23.

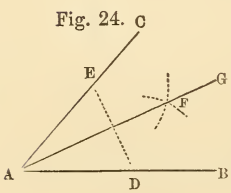



With the centre A and any distance AE, describe the arc DE, and with the same distance and centre B describe the arc FG; make HG equal to DE, and through B and H draw the line BH; then will the angle HBG be equal to the angle A.

## PROBLEM VII.

*To bisect any right lined angle* BAC, Fig. 24.

Fig. 24.

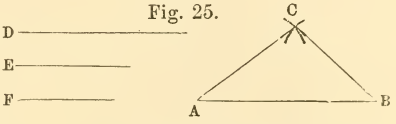



In the lines AB and AC, from the point A, set off equal distances, AD and AE; with the centres D and E and any distance more than half DE, describe two arcs cutting each other in F; from A through F draw the line AG, and it will bisect the angle BAC.

## PROBLEM VIII.

*To describe a triangle that shall have its sides respectively equal to three right lines,* D, E, *and* F, *of which any two must be together greater than the third,* Fig. 25.

Fig. 25.

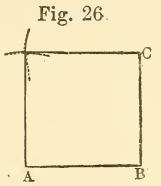



Make AB equal to D; with the centre A and distance equal to E, describe an arc, and with the centre B and distance equal to F describe another arc, cutting the former in C; draw AC and BC, and ABC is the triangle required.

## PROBLEM IX.

*Upon a given line* AB *to describe a square,* Fig. 26.

Fig. 26.

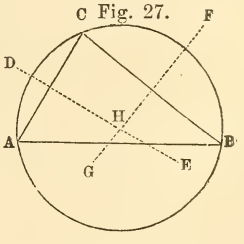



At the end B of the line AB, by Problem III. erect the perpendicular BC, and make it equal to AB; with A and C as centres, and distance AB or BC, describe two arcs cutting each other in D; draw AD, and CD, then will ABCD be the square required.

## PROBLEM X.

*To describe a circle that shall pass through the angular points* A, B, *and* C, *of a triangle* ABC, Fig. 27.

Fig. 27.



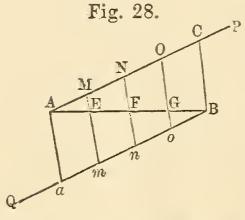

By Problem I. bisect any two of the sides, as AC, BC, by the perpendiculars DE and FG; the point H where they intersect each other will be the centre of the circle: with this centre, and the distance from

it to either of the points A, B, or C, describe the circle.

## PROBLEM XI.

*To divide a given right line* AB *into any number of equal parts,* Fig. 28.

Fig. 28.

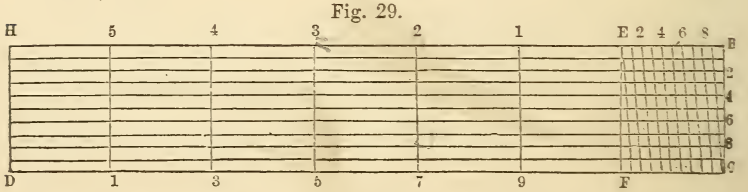



Draw the indefinite right line AP, making an angle with AB, also draw BQ, parallel to AP, in each of which, take as many equal parts AM, MN, &c. *Bo, on,* &c., as the line AB is to be divided into; then draw *Mm, Nn,* &c., intersecting AB in E, F, &c., which will divide the line as required.

## PROBLEM XII.

*To make a plane diagonal scale,* Fig. 29.

Draw eleven lines parallel to, and equidistant from each other; cut them at right angles by the equidistant lines BC; EF; 1, 9; 2, 7; &c. then will BC. &c. be divided into ten equal parts; divide the lines EB, and FC, each into ten equal parts; and from the points of division on the line EB, draw diagonals to the points of division on the line FC: thus join E and the first division on FC, the first division on EB, and the second on FC, &c.

Fig. 29.

*Note.*—Diagonal scales serve to take off dimensions or numbers of three figures. If the first large divisions be units, the second set of divisions along EB, will be

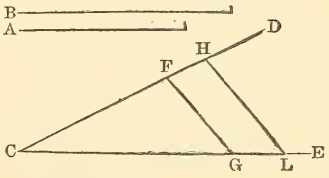
10th parts, and the divisions in the altitude, along BC, will be 100th parts. If HE be tens, EB will be units, and BC will be tenth parts. If HE be hundreds, BE will be tens, and BC units. And so on, each set of divisions being tenth parts of the former ones.

For example, suppose it were required to take off 242 from the scale. Extend the dividers from E to 2 towards H; and with one leg fixed in the point 2, extend the other till it reaches 4 in the line EB; move one leg of the dividers along the line 2, 7, and the other along the line 4, till they come to the line marked 2, in the line BC, and that will give the extent required.

## PROBLEM XIII.

*To find a third proportional to two given right lines,* A *and* B.

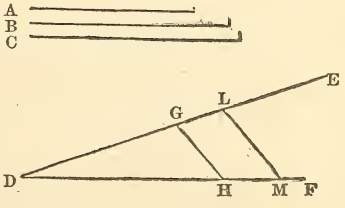
Draw two right lines, CD, CE, containing any angle; make CF equal A, and CG, CH each equal B; join FG, and draw HL parallel to it; then will CL be the third proportional required.




## PROBLEM XIV.

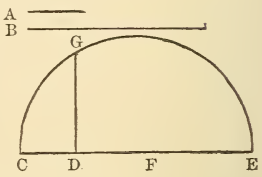
*To find a fourth proportional to three given right lines,* A, B *and* C.

Draw two right lines, DE, DF, containing any angle; make DG equal A, DH equal B, and DL equal C; join GH, and draw LM parallel to it; DM will be a fourth proportional to A, B and C.



## PROBLEM XV.

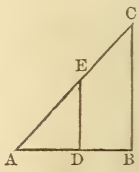
*To find a mean proportional between two given right lines,* A *and* B.




Draw any right line, CE, and in it take CD equal A, and DE equal B; bisect CE in F, and with the centre F and radius FC or FE describe the semicircle CGE; draw DG perpendicular to CE; then DG will be a mean proportional between A and B.

## PROBLEM XVI.

*To divide a given right line* AB *in two parts in the point* D, *so that* AD *may be to* DB *in the ratio of two given numbers m and n. For example, let* $m = 3$, *and* $n = 2$.




Draw AC, making any angle with AB; take the number *m* from any convenient scale of equal parts, and lay it on AC, from A to E, and take the number *n* from the same scale, and lay it from E to C; join CB and draw ED parallel to it; then AB will be divided as required.

# PLANE TRIGONOMETRY.

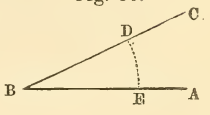
## DEFINITIONS.

1. Plane Trigonometry is the art by which, when any three parts of a plane triangle, except the three angles, are given, the others are determined.

2. The periphery of every circle is supposed to be divided into 360 equal parts, called degrees; each degree into 60 equal parts, called minutes; and each minute into 60 equal parts, called seconds, &c.

3. The measure of an angle is the arc of a circle, contained between the two lines that form the angle, the angular point being the centre; thus the angle ABC, Fig. 30, is measured by the arc DE, and contains the same number of degrees that the arc does. The measure of a right angle is therefore 90 degrees; for DH, Fig. 31, which measures the right angle DCH, is one-fourth part of the circumference, or 90 degrees.

Fig. 30.

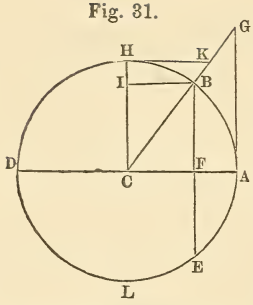



*Note.*—The degrees, minutes, seconds, &c., contained in any arc, or angle, are written in this manner, 50° 18′ 35″; which signifies that the given arc or angle contains 50 degrees, 18 minutes, and 35 seconds.

4. The *complement* of an arc, or of an angle, is what it wants of 90°; and the supplement of an arc, or of an angle, is what it wants of 180°.

5. The *chord* of an arc, is a right line drawn from one extremity of the arc to the other: thus the line BE is the chord of the arc BAE or BDE, Fig. 31.

Fig. 31.




6. The *sine* of an arc, is a right line drawn from one extremity of the arc perpendicular to the diameter which passes through the other extremity: thus BF is the sine of the arc AB or BD, Fig. 31.

7. The *cosine* of an arc, is that part of the diameter which is intercepted between the sine and the centre: thus CF is the cosine of the arc AB, and is equal to BI, the sine of its complement HB, Fig. 31

8. The *versed sine* of an arc, is that part of the diameter which is intercepted between the sine and the arc: thus AF is the versed sine of AB; and DF of DB, Fig. 31.

9. The *tangent* of an arc, is a right line touching the circle in one end of the arc, being perpendicular to the diameter which passes through that end, and is terminated by a right line drawn from the centre through the other end: thus AG is the tangent of the arc AB, Fig. 31.

10. The *secant* of an arc, is the right line drawn from the centre and terminating the tangent; thus CG is the secant of AB, Fig. 31.

11. The *cotangent* of an arc, is the tangent of the complement of that arc; thus HK is the cotangent of AB, Fig. 31.

12. The *cosecant* of an arc, is the secant of the complement of that arc; thus CK is the cosecant of AB, Fig. 31.

13. The sine, cosine, &c., of an *angle* is the same as the sine, cosine, &c., of the *arc* that measures the angle.

In measuring arcs it is customary to consider A, the right-hand extremity of the horizontal diameter, as the place of beginning, and measure around through 360° in the direction opposite to the motion of the hands of a watch. Thus, AH is the first quadrant, HD the second, DL the third, and LA the fourth.

An angle in the first quadrant is between 0° and 90°; in the second, between 90° and 180°; in the third, between 180° and 270°; in the fourth, between 270° and 360°.

The sine of 0° is 0; as the arc increases through the first quadrant, the sine also increases, till for 90° it is equal to the radius; through the second quadrant the sine diminishes as the arc increases. The cosine, on the other hand, decreases from radius at 0° to 0 at 90°, thence increases to radius at 180°. The tangent increases from 0 at 0° to infinity at 90°, and then decreases to 0 at 180°.

If we suppose radius to be unity, the length of the sine, tangent, &c. may be calculated by a method which it is out of place to explain here. It is evident that the sine and cosine, being never greater than radius, will be represented by a decimal. These decimals, radius being the unit, for every degree and minute of the first quadrant, are given in Table V. Such sines and cosines are known as natural sines and cosines, to distinguish them from the logarithmic sines and cosines. The sine of any angle being equal to the cosine of its complement, a table of natural sines will also be a table of natural cosines. Thus, if we desire the natural cosine of 80°, we may look for the natural sine of 10°. As far as 45° the degrees are given at the top of the page; above 45°, at the bottom.

To obtain from the table the natural sine or cosine

of a given angle, find the degrees at the top or bottom of the page, and the minutes in the left-hand or right-hand column; in the column of degrees, and opposite the given minute, is the required sine or cosine.

EXAMPLES.

1. Find the natural sine of 27° 42′. *Ans.* .46484.
2. Find the natural cosine of 46° 52′. *Ans.* .68370.
3. Find the natural cosine of 38°. *Ans.* .78801.
4. Find the natural sine of 30°. *Ans.* .50000.
5. Find the angle whose natural sine is .57742.
   *Ans.* 35° 16′.
6. Find the angle whose natural cosine is .61120.
   *Ans.* 52° 19′.

*Note.*—In the case of the last two examples, take the degrees and minutes corresponding to the sine and cosine nearest the given ones.

Should degrees, minutes and *seconds* be given, find the sine or cosine for the degrees and minutes. Take the difference between this and that corresponding to the next greater minute; divide this difference by 60, and multiply by the given number of seconds; add this to the number first taken out in the case of sines, and subtract in the case of cosines. The result will be the required number.

EXAMPLES.

1. Find the natural sine of 23° 42′ 17″.

| | |
|---|---|
| Natural sine of 23° 42′ - - - | .40195 |
| Natural sine of 23° 43′ - - - | .40221 |
| Difference - - - - | 26 |

This being the difference corresponding to 1′ or 60″, the difference corresponding to 17″ will be $\frac{17}{60}$ of 26, or 7.

.40195 + 7 = .40202. *Ans.*

2. Find the natural cosine of 23° 42′ 17″.

*Ans.* .91563.

*Note.*—We subtract in the case of the cosine, because the cosine decreases as the arc increases.

If it be required to find the degrees, minutes and seconds corresponding to a given sine or cosine, we proceed as in the following examples:

3. Find the degrees, minutes and seconds corresponding to the natural sine .47728.

| | |
|---|---|
| Next greater sine - - - - | .47741 |
| Next less sine - - - - | .47716 |
| Difference for 60″ - - - | 25 |
| Difference for 1″ - - - | .416 |
| Given sine - - - - - | .47728 |
| Next less sine - - - - | .47716 |
| Difference - - - - - | 12 |

.416 being the difference for 1″, 12 will correspond to 12 ÷ .416, or 29. Hence the answer is 28° 30′ 29″.

4. Find the angle corresponding to the natural cosine .40000.

By inspection of table, we find the given cosine falls between the cosines of 66° 25′ and 66° 26′.

| | |
|---|---|
| Cosine of 66° 25′ - - - - | 40008 |
| Cosine of 66° 26′ - - - - | 39982 |
| Difference for 60″ - - - | 26 |
| Difference for 1″ - - - | .433 |
| Cosine of next less angle - - - | 40008 |
| Given cosine - - - - - | 40000 |
| Difference - - - - - | 8 |

8 ÷ .433 = 18. *Ans.* 66° 25′ 18″.

## OF THE TABLE

OF

## LOGARITHMIC OR ARTIFICIAL SINES, TANGENTS, &c.

---

This table contains the logarithms of the sine, tangent, &c. to every degree and minute of the quadrant, the radius being 10000000000, and consequently its logarithm 10.

Let the radius be supposed to consist of 10000000000 equal parts as above, and let the quadrant be divided into 5400 equal arcs, each of these will therefore contain 1′; and if from the several points of division in the quadrant right lines be drawn perpendicular to the radius, the sine of every minute of the quadrant to the given radius will be exhibited. The lengths of these lines, being computed and arranged in a table, constitute what is usually termed a table of *natural* sines. The logarithms of those numbers taken from a table of logarithms, and properly arranged, form the table of *logarithmic* or *artificial sines*. In like manner the logarithmic tangents and secants are to be understood.

The method by which the sines are computed is too abstruse to be explained in this work, but a familiar exposition of this subject may be seen in *Sharpless's Trigonometry*.

### *To find, by the table, the sine, tangent, &c. of an arc or angle.*

If the degrees in the given angle be less than 45, look for them at the top of the table, and for the minutes, in

the left hand column; then in the column marked at the top of the table, sine, tangent, &c. and against the minutes, is the sine, tangent, &c. required. If the degrees are more than 45, look for them at the bottom of the table, and for the minutes, in the right hand column; then in the column marked at the bottom of the table, sine, tangent, &c. and against the minutes, is the sine, tangent, &c. required.

*Note.*—The sine of an angle and of its supplement being the same, if the given number of degrees be above 90, subtract them from 180°, and find the sine of the remainder.

EXAMPLES.

1. Required the sine of 32° 27′. *Ans.* 9.729621.
2. Required the tangent of 57° 39′. *Ans.* 10.198325.
3. What is the cosine of 89° 31′? *Ans.* 7.926119.
4. What is the sine of 157° 43′? *Ans.* 9.578853.

*To find the degrees and minutes corresponding to a given sine, tangent, &c.*

Find, in the table, the nearest logarithm to the given one, and the degrees answering to it will be found at the top of the table, if the name be there, and the minutes on the left hand; but if the name be at the bottom of the table, the degrees must be found at the bottom, and the minutes at the right hand.

EXAMPLES.

1. Required the degrees and minutes in the angle whose sine is 9.643908. *Ans.* 26° 8′.

2. Required the degrees and minutes in the angle whose tangent is 10.474641. *Ans.* 71° 28′.

*To find the sine, tangent, &c. of an arc or angle when seconds are required.*

Find the sine, tangent, &c. of the degrees and minutes as before. Multiply the corresponding number in column D by the given seconds, and add this to the number first taken out, for sines and tangents, and subtract for cosines and cotangents.

EXAMPLES.

1. Find the sine of 46° 25′ 32″.

Sine of 46° 25′ - - - - 9.859962
Corresponding number in column D 2.00

$$32 \times 2.00 = 64.$$

$$9.859962 + 64 = 9.860026.$$

*Note.*—The number in column D is the variation in the sine, tangent, &c. for a change of 1″ in the arc. Hence, to multiply by the number of seconds gives the corresponding variation.

2. Find the cotangent of 30° 27′ 48″.

*Ans.* 10.230488.

3. Find the tangent of 26° 51′ 32″.

*Ans.* 9.704517.

*To find the degrees, minutes and seconds corresponding to a given sine, &c.*

Find the two sines, &c. between which the given sine, &c. falls, and take the less of the corresponding minutes. Take the difference between the given sine,

&c. and that opposite to the minutes taken, and divide this difference by the corresponding number in column D for the seconds.

EXAMPLES.

1. Find the angle whose sine is 9.764236.

| | |
|---|---|
| The next less angle is - - - | 35° 31′ |
| Corresponding sine is - - - | 9.764131 |

Difference between this and given sine is 105 (millionths). Number in column D is 2.95. 2.95 being the number corresponding to 1″, 105 will correspond to $\frac{105}{2.95}$ seconds, or 35.6. Hence, the complete angle is 35° 31′ 36″.

2. Find the angle whose cosine is 9.620248.

The tables tell that the true angle is between 65° 20′ and 65° 21′. The cosine corresponding to 65° 20′ is 9.620488. Difference between this and given logarithm is 240.

$$240 \div 4.58 = 52.4.$$

Hence the angle is 65° 20′ 52″.

3. Find the angle whose cosine is 9.764227.

*Ans.* 54° 28′ 26″.

4. Find the angle whose tangent is 10.876429.

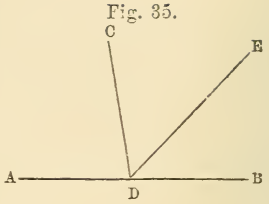
*Ans.* 82° 25′ 44″.

*Remarks on Angles, Triangles, &c.*

1. If from a point D in a right line AB, one or more right lines be drawn on the same side of it, the angles thus formed at the point D will be together equal to two

right angles, or 180°; thus ADE +EDB = two right angles, or 180°: also ADC + CDE + EDB = two right angles, or 180°. Fig. 35.

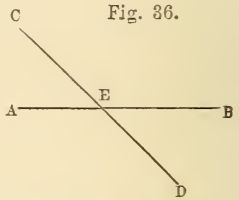
Fig. 35.




2. Since the angles thus formed at the point D, on the other side of AB, would also be equal to two right angles, the sum of all the angles formed about a point is equal to four right angles, or 360°.

3. If two right lines cut one another, the opposite angles will be equal: thus AEC = BED, and AED = CEB, Fig. 36.

Fig. 36.




4. The sum of the three angles of a plane triangle is equal to two rightangles, or 180°.

5. If the sum of two angles of a triangle be subtracted from 180°, the remainder will be the third angle.

6. If one angle of a triangle be subtracted from 180°, the remainder will be the sum of the other two angles.

7. In right-angled triangles, if one of the acute angles be subtracted from 90°, the remainder will be the other acute angle.

8. The angles at the base of an isosceles triangle are equal to one another.

9. If one side of a triangle be produced, the external angle will be equal to the sum of the two internal and opposite angles: thus the external angle CBD, of the triangle ABC, is equal to the sum of the internal and opposite angles A and C, Fig. 37.

Fig. 37

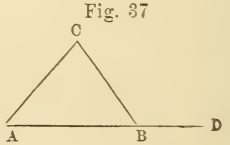
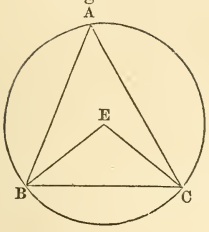
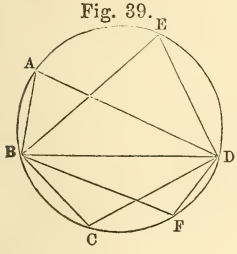


Fig. 38.




10. The angle at the centre of a circle is double of the angle at the circumference, upon the same base, that is upon the same part of the circumference: thus the angle BEC is double of the angle BAC, Fig. 38.

Fig. 39.




11. The angles in the same segment of a circle are equal to one another: thus the angle BAD is equal to the angle BED; also the angle BCD is equal to the angle BFD, Fig. 39.

12. The angle in a semicircle is a right angle: thus the angle ECF, Fig. 45, is a right angle.

13. This mark ′ placed on the sides or in the angles of a triangle, indicates that they are given; and this mark ° placed in the same way, indicates that they are required.

## PRACTICAL RULES FOR SOLVING ALL THE CASES OF PLANE TRIGONOMETRY.

### CASE 1.

*The angles and one side of any plane triangle being given, to find the other sides.*

#### RULE.

As the sine of the angle opposite the given side,
Is to the sine of the angle opposite the required side,
So is the given side,
To the required side.*

---

* Demonstration. Let ABC, Fig 40, be any plane triangle; take BF= AC, and upon AB let fall the perpendiculars CD and FE, which will be

*Note* 1.—The proportions in trigonometry are worked by logarithms: thus, from the sum of the logarithm of the second and third terms, subtract the logarithms of the first term, and the remainder will be the logarithm of the fourth term.

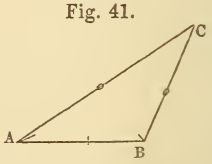
2. The logarithmic sine of a right angle or 90° is 10.00000, being the same as the logarithm of the radius.

EXAMPLES.

1. In the triangle ABC, there are given the angle A $= 32° 15'$, the angle B $= 114° 24'$, and consequently the angle C $= 33° 21'$, and the sides AB $= 98$;* required the sides AC and BC.

*By Construction*, Fig. 41.

Fig. 41.




Make AB equal to 98 by a scale of equal parts, and draw AC, making the angle A $= 32° 15'$; also make the angle B $= 114° 24'$, and produce BC, AC, till they meet in C, then is ABC the triangle required; and AC measured by the same scale of equal parts, is 162, and BC is 95.

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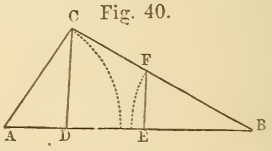
Fig. 40.




the sines of the angles A and B to the equal radii AC and BF. Now the triangles BDC and BEF being similar, we have CD : FE : : BC : BF : or AC; that is sin. A : sin. B : : BC : AC. In like manner it is proved, that sin. A : sin. C : : BC : AB. When one of the angles is obtuse, the demonstration is the same. Hence it appears, that in any plane triangle, the sides are to one another as the sines of their opposite angles.

* This 98 may express so many feet, or yards, &c., and the other sides will be of the same denomination as the given.

*By Calculation.*

| | |
|---|---|
| As sine of the angle C 33° 21′ - - | 9.740167 |
| Is to sine of the angle B 114° 24′ - | 9.959368 |
| So is AB 98 - - - - - | 1.991226 |
| | 11.950594 |
| | 9.740167 |
| To AC 162.3 - - - - - | 2.210427 |
| As sine of C 33° 21′ - - - - | 9.740167 |
| Is to sine of A 32° 15′ - - - | 9.727228 |
| So is AB 98 - - - - - | 1.991226 |
| | 11.718454 |
| | 9.740167 |
| To BC 95.12 - - - - - | 1.978287 |

This calculation could also be performed by adding the arithmetical complement of the logarithm of the first term to the other logarithms, and subtracting 10 from the result. This is usually the simplest method of working a proportion.

The first proportion would now become—

| | |
|---|---|
| As sine of C 33° 21′ - - | Ar. Co. 0.259833 |
| Is to sine of B 114° 24′ - - - | 9.959368 |
| So is AB 98 - - - - - | .991226 |
| To AC 162.3 - - - - | 1.210427 |

2. In the right-angled triangle ABC are given the hypothenuse AC = 480, and the angle A = 53° 8′. To find the base AB and perpendicular BC.

Say, As sine B : sine A :: AC : BC.
90°, 53° 8′, 480, 384.

As sine B : sine C :: AC : AB.
90°, 36° 52′, 480, 288.

The sine of 90° being equal to radius, its logarithm is 10.

3. In the triangle ABC, are given the angle A=79° 23′, the angle B=54° 22′, and the side BC=125; required AC and AB. *Ans.* AC=103.4, and AB=91.87.

4. In a right-angled triangle, there are given the angle A=56° 48′, and the base AB=53.66, to find the perpendicular BC and hypothenuse AC. *Ans.* BC=82 and AC=98.

5. In the right-angled triangle ABC, are given the angle A=39° 10′, and the perpendicular BC=407.37, to find the base AB and hypothenuse AC. *Ans.* AB=500.1, and AC=645.

### CASE 2.

*Two sides and an angle opposite one of them being given, to find the other angles and side.*

#### RULE.

As the side opposite the given angle,
Is to the other given side,
So is the sine of the angle opposite the former,
To the sine of the angle opposite the latter.*

* This is evident from the demonstration of the rule in the preceding case.

Add the angle thus found to the given angle, and subtract their sum from 180°, the remainder will be the third angle.

After finding the angles, the other side may be found by Case 1.

*Note.*—The angle found by this rule is sometimes ambiguous; for the operation only gives the sine of the angle, not the angle itself; and the sine of every angle is also the sine of its supplement.

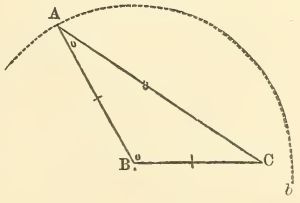
When the side opposite the given angle is equal to, or greater than the other given side, then the angle opposite that other given side is always acute; but when this is not the case, that angle may be either acute or obtuse, and is consequently ambiguous.

EXAMPLES.

1. In the triangle ABC, are given the angle C = 33° 21′, the side AB = .98 and the side BC = .7912; required the angles A and B, and the side BC.

*By Construction*, Fig. 43.

Fig. 43.




Make BC = .7912 by a scale of equal parts, and draw CA, making the angle C = 33° 21′; with the side AB = .98, in the compasses, taken from the same scale of equal parts, and B as a centre, describe the arc *ab*, cutting AC in the point A, and join BA; then is ABC the triangle required: the side AC, measured by the scale of equal parts, will be 1.54, and the angles A and B, measured by a protractor, will be 26° 21′ and 120° 18′.

Here the arc *ab* cuts AC in one point only, because AB is greater than BC; therefore the angle A is acute, and not ambiguous.

*By Calculation.*

| | |
|---|---|
| As AB, .98 - - - - - | — 1.991226 |
| Is to BC, .7912 - - - - | — 1.898286 |
| So is sine of C, 33° 21′ - - - | 9.740167 |
| | 9.638453 |
| To sine of A, 26° 21′ - - - | 9.647227 |

To the angle C = 33° 21′ add the angle A = 26° 21′, and the sum is 59° 42′, which, subtracted from 180°, leaves the angle B = 120° 18′.

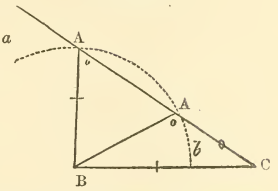
| | |
|---|---|
| As sine of C, 33° 21′ - - - | 9.740167 |
| Is to sine of B, 120° 18′ - - - | 9.936210 |
| So is AB, .98 - - - - | — 1.991226 |
| | 9.927436 |
| To AC, 1.539 - - - - | 0.187269 |

If the side BA had been given of such a length that, when the compass was swung around with B as a centre, it would not cut AC, the problem would have been impossible. No triangle could answer the conditions of the problem. If the arc had touched AC, the triangle would have been right-angled. If it had cut AC in two points, as in next example, either one of the two triangles formed would satisfy the given values. The problem would be said to be ambiguous.

2. In the triangle ABC are given the angle C = 33° 21′, the side BC = 95.12, and the side AB = 60, to find the angles A and B and the side AC.

*By Construction*, Fig. 44.

Fig. 44.




This is constructed in the same manner as the preceding example; only, AB being shorter than BC, the arc *ab* cuts AC in two points on the same side of BC; hence the angle A may be either acute or obtuse. The side required has also two values, as AC and AC.

*By Calculation.*

| | |
|---|---|
| As AB, 60 - - - - - | 1.778151 |
| Is to BC, 95.12 - - - - | 1.978272 |
| So is sine C, 33° 21′ - - - | 9.740167 |
| | 11.718440 |
| To sine of A { 60° 38′ acute / 119 22 obtuse } | 9.940289 |

The sum of the angles C and A subtracted from 180° leaves the angle B = 86° 1′ if A be acute, or 27° 17′ if A be obtuse.

To find the side AC answering to the acute value of the angle A.

| | |
|---|---|
| As sine of C, 33° 21′ - - - | 9.740167 |
| Is to sine of B, 86° 1′ - - - | 9.998950 |
| So is AB, 60 - - - - - | 1.778151 |
| | 11.777101 |
| To AC, 108.9 - - - - - | 2.036934 |

To find the side AC, answering to the obtuse value of the angle A.

| | |
|---|---|
| As sine of C, 33° 21′ - - - | 9.740167 |
| Is to sine of B, 27° 17′ - - - | 9.661236 |
| So is AB, 60 - - - - - | 1.778151 |
| | 11.439387 |
| To AC, 50.03 - - - - - | 1.699220 |

3. In a triangle ABC, the side AB is 274, AC 306. and the angle B 78° 13′; required the angles A and C. and the side BC.

*Ans.* A=40° 33′, C=61° 14′, and BC=203.2.

4. In a right-angled triangle, there are given the hypothenuse AC=272, and the base AB=232; to find the angles A and C, and the perpendicular BC.

*Ans.* A=31° 28′ C=58° 32′ and BC=142.

5. In a right-angled triangle ABC, the hypothenuse AC is 150, and one side BC 69; required the angles and other side.

*Ans.* C=62° 37′, A=27° 23′, and AB 133.2.

### CASE 3.

*Two sides and the included angle being given, to find the other angles and side.*

#### RULE.

Subtract the given angle from 180°, and the remainder will be the sum of the two unknown angles. Then,

As the sum of the two given sides,

Is to their difference;

So is the tangent of half the sum of the two unknown angles,

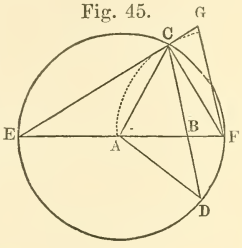
To the tangent of half their difference.*

This half difference of the two unknown angles, added to their half sum, will give the angle opposite the greater of the two given sides, and being subtracted

---

* Demonstration. Let ABC, Fig. 45, be the proposed triangle, having the two given sides AB, AC, including the given angle A. About A as a centre, with AC the greater of the given sides for a radius, describe a circle meeting AB produced in E and F, and BC in D; join DA, EC, and FC, and draw FG parallel to BC, meeting EC produced in G.

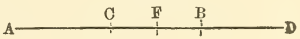
Fig. 45.




The angle EAC (I. 30)† is equal to the sum of the unknown angles ABC, ACB; and the angle EFC at the circumference, is equal to the half of EAC at the centre (III. 16;) therefore EFC is half the sum of the unknown angles; but (I. 30) the angle ABC is equal to the sum of the angles BAD and ADB, or BAD and ACB; therefore FAD is the difference of the unknown angles ABC, ACB; and FCD, at the circumference is the half of that difference; but because of the parallels DC, FG, the angles GFC, FCD are equal; therefore GFC is equal to half the difference of the unknown angles ABC, ACB; but since the angle ECF in a semicircle, is a right angle, EG is perpendicular to CF, and therefore CF being radius, EC, CG are the tangents of the angles EFC, CFG; it is also evident that EB is the sum of the sides BA, AC, and that BF is the difference; therefore since BC, FG are parallel, EB : BF : : EC : CG (V. 2:) that is, the sum of the sides AC, AB, is to their difference, as the tangent of half the sum of the angles ABC, ACB, is to the tangent of half their difference.

To demonstrate the latter part of the rule, let AC and AB, Fig. 46, represent any two magnitudes whatever; in AB produced, take BD equal to AC the less, and bisect AD in E.

Fig. 46.




Then because AE is equal to ED, and AC to BD, CE is equal to EB, therefore AE or ED is half the sum of the given magnitudes AB, AC, and CE, or EB is half their difference; but AB the greater is equal to AE, EB, that is to half the sum added to half the difference; and AC the less, is equal to the excess of AE, half the sum, above CE, half the difference.

† These references are to Sharpless's "Geometry."

from the half sum will give the angle opposite the less given side.

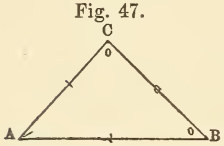
After finding the angles, the other side may be found by Case 1.

EXAMPLES.

1. In the triangle ABC, there are given AB=128, AC=90, and the angle A=48° 12′, to find the angles B and C, and the side BC.

*By Construction*, Fig. 47.

Fig. 47.




Draw AB=128, and make the angle A=48° 12′; draw AC=90, and join BC. The angle B will measure 44° 37′, the angle C 87° 11′, and the side BC 95.5.

*By Calculation.*

| | | | | |
|---|---|---|---|---|
| AB | 128 | | 180° | 0′ |
| AC | 90 | Angle A - - - - - - | 48 | 12 |
| Sum | 218 | Sum of the angles B and C | 131 | 48 |
| Difference | 38 | Half sum do. | 65 | 54 |

| | |
|---|---|
| As the sum of the sides, AB, AC, 218 - - | 2.338456 |
| Is to their difference, 38 - - - - - - - | 1.579784 |
| So is the tangent of half the sum of the angles B and C, 65° 54′ - - - - - | 10.349380 |
| | 11.929164 |
| To tang. of half their difference, 21° 17′ - | 9.590708 |

| | | |
|---|---|---|
| Half sum of the angles B and C - - | 65° | 54′ |
| Add and subtract half their difference - | 21 | 17 |
| Angle C - - - - - - - - - - - | 87 | 11 |
| Angle B - - - - - - - - - - - | 44 | 37 |

To find the side BC.

| | |
|---|---|
| As sine of B, 44° 37′ - - - | 9.846560 |
| Is to sine of A, 48° 12′ - - - | 9.872434 |
| So is AC, 90 - - - - - | 1.954243 |
| | 11.826677 |
| To BC, 95.52 - - - - - | 1.980117 |

2. In a triangle ABC are given AB = 109, BC = 76, and the contained angle B = 101° 30′, to find the other angles and side.

*Ans.* The angle A = 30° 57′, C = 47° 33′, and the side AC = 144.8.

3. Given, in a right-angled triangle, the base AB = 890 and the perpendicular BC = 787, to find the angles and hypothenuse.

*Ans.* The angle A = 41° 29′, C = 48° 31′, and the hypothenuse AC = 1188.

This problem may be used to find the distance between two points when separated by a swamp, river or other impassable barrier. Let B and C, Fig. 47, be two such points. Measure AC and AB and the angle A, whence BC may be calculated.

### CASE 4.

*Given the three sides, to find the angles.*

### RULE 1.

Consider the longest side of the triangle as the base, and on it let fall a perpendicular from the opposite angle. This perpendicular will divide the base into two parts, called segments, and the whole triangle into two right-angled triangles. Then,

As the base, or sum of the segments,
Is to the sum of the other two sides;
So is the difference of those sides,
To the difference of the segments of the base.*

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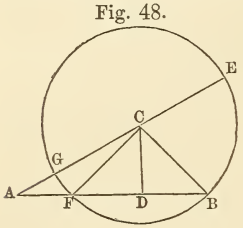
Fig. 48.




* Demonstration. Let ABC, Fig. 48, be a triangle, and CD be perpendicular upon AB About C as a centre, with the less side BC for a radius, describe a circle, meeting AC produced, in G and E, and AB in F. Then it is evident that AE is equal to the sum of the sides AC, BC, and that AG is equal to their difference; also because CD bisects FB (III. 3,) it is plain that AF is the difference of the segments of the base; but $AB \times AF = AE \times AG$ (V. 27 cor.;) therefore AB : AE : : AG : AF (V. 14;) that is, the base, is to the sum of the sides, as the difference of the sides, is to the difference of the segments of the base.

Cor. If AF be considered the base of the triangle AFC, then will CD be a perpendicular on the base produced; AE will be equal to the sum of the sides AC, FC, and AG will be equal to their difference; also AB will be equal to the sum of the segments AD, FD. But by the preceding demonstration, alternately AF : AE : : AG : AB; hence, when the perpendicular falls without the triangle, the base is to the sum of the sides, as the difference of the sides is to the sum of the segments of the base.

A rule might, therefore, be given, making either side of a triangle the base; and such a rule would be rather more convenient, in some cases, than the one above: but then, on account of the perpendicular, sometimes falling within and sometimes without the triangle, it would require two cases, and consequently would be less simple.

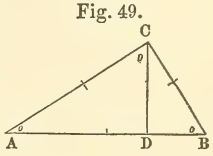
To half the base, add half the difference of the segments, and the sum will be the greater segment; also from half the base, subtract half the difference of the segments, and the remainder will be the less segment.

Then, in each of the two right-angled triangles, there will be known two sides, and an angle opposite to one of them; consequently the other angles may be found by Case 2.

1. In the triangle ABC, are given AB=426, AC=365, and BC=230; required the angles.

*By Construction,* Fig. 49.

Fig. 49.




Draw AB=426; with AC=365 in the dividers, and one foot in A, describe an arc, and with BC=230, and one foot in B describe another arc, cutting the former in C; join AC, BC, and ABC will be the triangle required. The angles measured by a scale of chords, will be A=32° 39′, B=58° 56′, and C=88° 25′.

*By Calculation.*

| | |
|---|---|
| AC - - - - - - - - | 365 |
| BC - - - - - - - - | 230 |
| Sum - - - - - - - - | 595 |
| Difference - - - - - | 135 |

| | | |
|---|---|---|
| As the base AB - - - - | 426 | 2.629410 |
| Is to the sum of the sides AC, BC | 595 | 2.774517 |
| So is the diff. of the sides AC, BC | 135 | 2.130334 |
| | | 4.904851 |
| To the diff. of the segments AD, DB | 188.6 | 2.275441 |
| Half diff. of the segments - - | 94.3 | |
| Half base - - - - - | 213. | |
| Segment AD - - - - | 307.3 | |
| Segment BD - - - - | 118.7 | |

The other angles may then be worked out by Case 2.

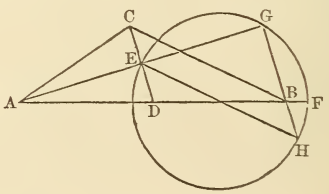
### RULE 2.

Add together the arithmetical complements of the logarithms of the two sides containing the required angle, the logarithm of the half sum of the three sides, and the logarithm of the difference between the half sum and the side opposite the required angle. Then half the sum of these four logarithms will be the logarithmic cosine of half the required angle.*

---

* DEMONSTRATION.—Let ABC be a triangle, of which the side AB is greater than AC; make AD = AC, join DC, bisect it in E, and join AE; draw EH parallel and equal to CB; join HB, and produce it to meet AE produced in G.

Now in the triangles, AED, AEC, all the sides of the one are equal to the sides of the other, each to each; therefore (I. 4) the angle EAD = EAC, and AED = AEC; consequently AED is a right angle.



EXAMPLES.

I. In the triangle ABC are given AB = 426, AC = 365 and BC = 230; required the angle A.

*By Calculation.*

| | | | |
|---|---|---|---|
| BC | 230 | | |
| AC | 365 | Ar. Co. | 7.437707 |
| AB | 426 | Ar. Co. | 7.370590 |
| | 2)1021 | | |
| Half sum | 510.5 | log. | 2.707996 |
| Difference | 280.5 | log. | 2.447933 |
| | | | 2)19.964226 |
| Cos. ½ A | 16° 20′ | | 9.982113 |
| | 2 | | |
| Angle A | 32° 40′ | | |

---

Because EH is equal and parallel to BC, BH is also equal and parallel to EC (I. 31;) now in the triangles EDF and HBF, the angle EFD=BFH, the angle FED=FHB (I. 27) and ED=EC=BH; therefore (I. 24) EF=FH, and FD=FB. Again, the angle HGE=DEA=a right angle; if therefore with the centre F, and radius FE=FH, a circle be described, it will pass through the point G.

Now 2 AF=2 AD + 2 DF=AD + AD + DB=AD + AB=AC + AB; therefore AF=½AC+½AB; also FK=½IK=½EH=½BC; hence, by adding equals to equals, AF+FK=½AC+½AB+½BC, or AK=½(AC+AB+BC;) again, AI=AK—IK=½(AC+AB+BC)—BC.

But (Dem. to rule, case 1st.) AD : AE : : sin. AED : sin. ADE : : rad. : cos. EAD (cos. ½BAC.) Also, AB : AG : : sin. AGB : sin. ABG : : rad.: cos. BAG (cos. ½BAC.)

Hence (c. 6) AB × AD : : AG × AE : : rad.$^2$ : (cos. ½BAC)$^2$. But AB × AD=AB × AC, and (cor. 36.3) AG × AE=AK × AI=½(AC+AB+BC) × [½(AC+AB+BC)—BC;] therefore AB × AC: ½(AC+AB+BC) × [½(AC+AB+BC)—BC] : : rad.$^2$ : (cos. ½BAC)$^2$.

Now it is evident, that in working this proportion by logarithms

If the other angles are required, they may be found by Case 1.

2. In a triangle ABC, are given AB = 64, AC = 47, and BC = 34, to find the angle B. *Ans.* Angle B = 45° 38′.

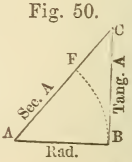
3. In a triangle ABC, are given AC = 88, AB = 108, and BC = 54, to find the angle C. *Ans.* C = 96° 4′.

The preceding rules solve all the cases of plane triangles, both right-angled and oblique. There are, however, other rules, suited to right-angled triangles, which are sometimes more convenient than the general ones. Previous to giving these rules, it will be necessary to make the following

*Remarks on right-angled triangles.*

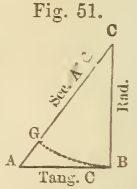
1. ABC, Fig. 50, being a right-angled triangle, make one leg AB radius, that is, with the centre A, and distance AB, describe an arc BF. Then it is evident that the other leg BC represents the tangent of the arc BF, or of the angle A, and the hypothenuse AC the secant of it.

Fig. 50.




2. In like manner, if the leg BC, Fig. 51, be made radius; then the other leg AB will represent the tangent of the arc BG, or angle C, and the hypothenuse AC the secant of it.

Fig. 51.

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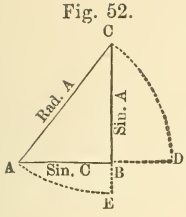
and taking the arithmetical complements of the logarithms of the first term, viz. of the two sides, including the required angle, if we omit the logarithm of the square of radius, which is 20, it is just equivalent to rejecting 20 from the sum of the logarithms, which would otherwise have to be done.

Fig. 52.




3. But if the hypothenuse be made radius, then each leg will represent the sine of its opposite angle; namely, the leg AB, Fig. 52, the sine of the arc AE or angle C, and the leg BC the sine of the arc CD, or angle A, and the cosine of C.

*The angles and one side of a right-angled triangle being given, to find the other sides.*

RULE.

Call any one of the sides radius, and write upon it the word radius; observe whether the other sides become sines, tangents, or cosines, and write these words on them accordingly. Call the word written upon each side the name of that side. Then,

As the name of the side given,
Is to the name of the side required;
So is the side given,
To the side required.*

*Two sides of a right-angled triangle being given, to find the angles and other side.*

RULE.

Call either of the given sides radius, and write on them as before. Then,

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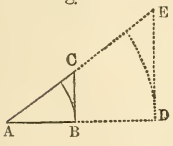
Fig. 53.




* Demonstration. Let ABC, Fig. 53, be a right-angled triangle; then it is evident that BC is the tangent, and AC the secant of the angle A, to the radius AB. Let AD represent the radius of the tables, and draw DE perpendicular to AD, meeting AC produced in E; then DE is the tangent, and AE the secant of the angle A, to the radius AD. But because of the similar triangles ADE, ABC, AD : DE : : AB : BC; that is, the tabular radius : tabular tangent : : AB : BC. Also AD : AE : : AB : AC; that is, the tabular radius : tabular secant : : AB : AC. These proportions correspond with the rule. When either of the other sides is made radius, the demonstration will be similar.

As the side made radius,
Is to the other given side;
So is radius,
To the name of that other side.*

After finding the angle, the other side is found as in the preceding rule.

### EXAMPLES.

1. In a right-angled triangle ABC, are given the base AB=208, and the angle A=35° 16′, to find the hypothenuse AC and perpendicular BC

*By Calculation.*

The hypothenuse AC being radius.

| | |
|---|---|
| As the sine of C, 54° 44′ - - - | 9.911942 |
| Is to radius - - - - - | 10.000000 |
| So is AB 208 - - - - - | 2.318063 |
| | 12.318063 |
| To AC 254.8 - - - - - | 2.406121 |

The base AB being radius.

| | |
|---|---|
| As radius - - - - - | 10.000000 |
| Is to the tangent of A, 35° 16′ - - | 9.849522 |
| So is AB 208 - - - - - | 2.318063 |
| To BC 147.1 - - - - - | 2.167585 |

* This is the converse of the preceding rule.

2. In a right-angled triangle, are given the hypothenuse AC=36.57, and the angle A=27° 46′, to find the base AB, and perpendicular BC.

*Ans.* Base AB=32.36, and perpendicular BC=17.04

3. In a right-angled triangle, there are given, the perpendicular=193.6, and the angle opposite the base 47° 51′; required the hypothenuse and base.

*Ans.* Hypothenuse=288.5, and base=213.9

4. Required the angles and hypothenuse of a right-angled triangle, the base of which is 46.72, and perpendicular 57.9.

*Ans.* { Angle opposite the base 38° 54′, angle opposite the perpendicular 51° 6′, and hypothenuse 74.4.

When two sides of a right-angled triangle are given, the other side may be found by the following rules, without first finding the angles.

### 1. *When the hypothenuse and one leg are given, to find the other leg.*

#### RULE.

Subtract the square of the given leg from the square of the hypothenuse; the square root of the remainder will be the leg required.* Or by logarithms thus,

To the logarithm of the sum of the hypothenuse and given side, add the logarithm of their difference; half this sum will be the logarithm of the leg required.

---

* DEMONSTRATION. The square of the hypothenuse of a right-angled triangle is equal to the sum of the squares of the sides (I. 42.) Therefore the truth of the first part of each of the rules is evident.

2. *When the two legs are given to find the hypothenuse.*

RULE.

Add together the squares of the two given legs; the square root of the sum will be the hypothenuse.* Or by logarithms thus,

From twice the logarithm of the perpendicular, subtract the logarithm of the base, and add the corresponding natural number to the base; then, half the sum of the logarithms of this sum, and of the base, will be the logarithm of the hypothenuse.

EXAMPLES.

1. The hypothenuse of a right-angled triangle is 272, and the base 232; required the perpendicular.

*Calculation by logarithms.*

| | | | |
|---|---|---|---|
| Hypothenuse - - - | 272 | | |
| Base - - - - | 232 | | |
| Sum - - - - | 504 | log. | 2.702431 |
| Difference - - - | 40 | —— | 1.602060 |
| | | | 2)4.304491 |
| Perpendicular - - | 142 | —— | 2.152245 |

* Put $h$ = the hypothenuse, $b$ = the base, and $p$ = the perpendicular, then (I. 42) $p^2 = h^2 - b^2 =$ (II. 6) $\overline{h+b} \times \overline{h-b}$, or $p = \sqrt{h+b \times h-b}$, whence, from the nature of logarithms, the latter part of the first rule is evident.

Also $h^2 = b^2 + p^2 = b \times \overline{b + \frac{p^2}{b}}$ or $h = \sqrt{\left(b \times \overline{b + \frac{p^2}{b}}\right)}$,

which, solved by logarithms, will correspond with the latter part of the second rule.

2. Given the base 186, and the perpendicular 152, to find the hypothenuse.

*Calculation by Logarithms.*

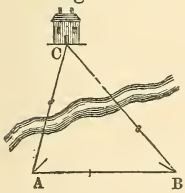
| | | | |
|---|---|---|---|
| Perpendicular | 152 | log. 2.181844 | |
| | | 2 | |
| | | 4.363688 | |
| Base - - - | 186 | 2.269513 | 2.269513 |
| | 124.2 | 2.094175 | |
| | 310.2 | | log. 2.491642 |
| | | | 2)4.761155 |
| Hypothenuse | 240.2 | | log. 2.380577 |

3. The hypothenuse being given equal 403, and one leg 321; required the other leg. *Ans.* 243.7.

4. What is the hypothenuse of a right-angled triangle, the base of which is 31.04 and perpendicular 27.2? *Ans.* 41.27.

The following examples, in which trigonometry is applied to the mensuration of inaccessible distances and heights, will serve to render the student expert in solving the different cases, and also to elucidate its use.

Fig. 54.




## THE APPLICATION OF PLANE TRIGONOMETRY TO THE MENSURATION OF DISTANCES AND HEIGHTS.

### EXAMPLE 1, Fig. 54.

Being on one side of a river and wanting to know the distance to a house on the other side,

I measured 500 yards along the side of the river in a right line AB, and found the two angles* between this line and the object to be CAB = 74° 14′ and CBA = 49° 23′. Required the distance between each station and the object.

*Calculation.*

The sum of the angles CAB and CBA is 123° 37′, which subtracted from 180° leaves the angle ACB = 56° 23′. Then by Case 1;

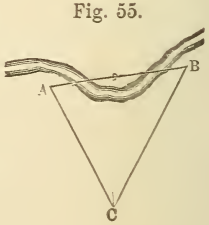
| s. ACB | : | s. CBA | : : | AB | : | AC |
|---|---|---|---|---|---|---|
| 56° 23′ | | 49° 23′ | | 500 | | 455.8 |

and

| s. ACB | : | s. CAB | : : | BA | : | BC |
|---|---|---|---|---|---|---|
| 56° 23′ | | 74° 14′ | | 500 | | 577.8 |

EXAMPLE 2, Fig. 55.

Fig. 55.




Suppose I want to know the distance between two places, A and B, accessible at both ends of the line AB, and that I measured AC = 735 yards, and BC = 840; also the angle ACB = 55° 40′. What is the distance between A and B?

*Calculation.*

The angle ACB = 55° 40′, being subtracted from 180°, leaves 124° 20′; the half of which is 62° 10′. Then by Case 3,

* The angles may be taken with a common surveyor's compass; or more accurately with an instrument called a theodolite.

$$BC+AC : BC-AC :: \text{tangent } \frac{CAB+CBA}{2} : \text{tangent } \frac{CAB-CBA}{2}$$

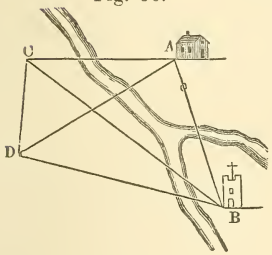
1575 105 62° 10′ 7° 12′

To and from $\frac{CAB+CBA}{2} = 62°\ 10'$ add and sub. $\frac{CAB-CBA}{2} = 7°\ 12'$, and we shall have CAB = 69° 22′, and CBA = 54° 58′. Then,

| s. ABC | : | s. ACB | :: | AC | : | AB |
|---|---|---|---|---|---|---|
| 54° 58′ | | 55° 40′ | | 735 | | 741.2 |

EXAMPLE 3, Fig. 56.

Fig. 56.




Wanting to know the distance between two inaccessible objects A and B, I measured a base line CD = 300 yards: at C the angle BCD was 58° 20′ and ACD 95° 20′; at D the angle CDA was 53° 30′ and CDB 98° 45′. Required the distance AB.

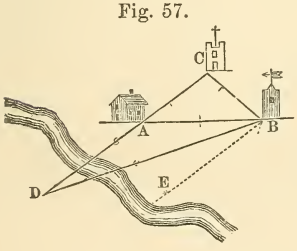
*Calculation.*

1. In the triangle ACD, are given the angles ACD = 95° 20′, ADC = 53° 30′, and the side CD = 300, to find AC = 465.98.

2. In the triangle BCD, are given the angle BCD = 58° 20′, BDC = 98° 45′, and side CD = 300, to find BC = 761.47.

3. In the triangle ACB we have now given the angle ACB = ACD—BCD = 37°, the side AC = 465.98 and BC = 761.47, to find AB = 479.8 yards, the distance required.

Fig. 57.




EXAMPLE 4, Fig. 57.

Being on one side of a river and observing three objects, A, B and C stand on the other side, whose distances apart I knew to be, AB = 3 miles, AC

$=2$, and $BC=1.8$, I took a station D, in a straight line with the objects A and C, being nearer the former, and found the angle $ADB=17°\ 47'$. Required my distance from each of the objects.

*Construction.*

With the three given distances, describe the triangle ABC; from B, draw BE parallel to CA, and draw BD making the angle $EBD=17°\ 47'$ (the given angle ADB) and meeting CA produced, in D: then AD, CD and BD will be the distances required.*

*Calculation.*

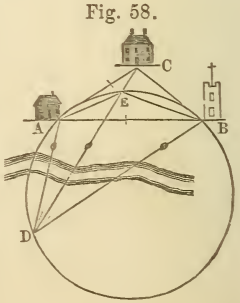
1. In the triangle ABC we have all the sides given, to find the angle $C=104°\ 8'$.

2. Subtract the sum of the angles D and C from 180°, the remainder $58°\ 5'$ will be the angle DBC; then in the triangle BCD we know all the angles and the side BC to find $DC=5.002$ and $BD=5.715$; therefore $DA=DC-AC=3.002$.

EXAMPLE 5, Fig. 58.

From a station at D, I perceived three objects, A, B and C, whose distances from each other I knew to be as follows: $AB=12$ miles, $BC=7.2$ miles, and $AC=8$ miles; at D, I took the angle $CDB=25°$ and $ADC=19°$. Hence it is required to find my distance from each of the objects.

Fig. 58.




* DEMONSTRATION. By construction, the distances AB, BC and AC are equal to the given distances; also the angle (I. 27) BDC = the angle DBE = the given angle.

*Construction.*

With the given distances describe the triangle ABC; at B, make the angle EBA=19°=the given angle ADC, and at A, make the angle EAB=25°=the given angle BDC; draw AE, and BE meeting in E, and (by prob. 10,) describe a circle that shall pass through the points A, E and B: join CE and produce it to meet the circle in D, and join AD, BD, then will AD, CD, and BD be the distances required.*

*Calculation.*

1. In the triangle ABC, all the sides are given, to find the angle BAC=35° 35′.

2. In the triangle AEB, are given all the angles, viz. EAB=25°, EBA=19°, and AEB=136°, and the side AB=12, to find AE=5.624.

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* Demonstration. The angle ADC standing on the same arc with the angle ABE is equal to it (III. 17.) For the same reason the angle BDC is equal to the angle BAE; but by construction the angles ABE and BAE are equal to the given angles; therefore the angles ADC and BDC are equal to the given angles.

*Note.*—When the given angles ADC, BDC are respectively equal to the angles ABC, BAC, the point E will fall on the point C, the circle will pass through the points A, C, and B, and the point D may be any where in the arc ADB; consequently, in this case, the situation of the point D, or its distance from each of the objects A, B, C, cannot be determined from the data given.

It may not be improper also to observe, that even when the angle ADB, which is the sum of the given angles, is equal to the sum of the angles ABC, BAC, or which is the same thing, is the supplement of the angle ACB, the circle passes through the points A, C, B; but then the angles ADC, BDC, unless they have been erroneously taken, will be respectively equal to the angles ABC, BAC.

3. In the triangle CAE we have given the side AC $=8$, AE$=5.624$, and the angle CAE$=$BAC$-$EAB$=10°\ 35'$, to find the angle ACE$=22°\ 41'$.

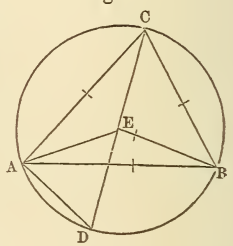
4. In the triangle DAC, all the angles are given, viz. ADC$=19°$, ACD$=22°\ 41'$ and CAD$=180°$—the sum of the angles ADC and ACD$=138°\ 19'$, and the side AC$=8$, to find AD$=9.47$ miles, and CD$=16.34$.

In the triangle ABD, we have the angle ADB$=$ADC $+$BDC$=44°$, the angle BAD$=$CAD$-$BAC$=102°\ 44'$, and the side AB$=12$, to find BD$=16.85$ miles.

EXAMPLE 6, Fig. 59.

A person having a triangular field, the sides of which measure AB$=50$ perches, AC$=46$ perches, and BC$=40$ perches, wishes to have a well dug in it, that shall be equally distant from the corners A, B and C. What must be its distance from each corner, and by what angle from the corner A, may its place be found?

Fig. 59.




*Construction.*

With the given sides construct the triangle ABC, and (by Prob. 10.) describe a circle that shall pass through the points A, B, and C; then the centre E of this circle is the required place of the well.

*Calculation.*

1. In the triangle ABC, all the sides are given, to find the angle ABC$=60°\ 16'$.

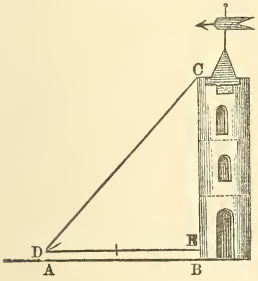
2. Join CE and produce it to meet the circumference in D; also join AE and AD; then the angles ADC, ABC

being angles in the same segment, are equal; also the angle DAC being an angle in a semicircle, is a right-angle: therefore in the right-angled triangle DAC, we have the angle ADC = ABC = 60° 16′, and the side AC, to find CD = 52.98 perches. The half of CD is = 26.49 perches = CE = the distance of the well from each corner.

3. The angle ACD = 90° — ADC = 29° 44; but because AEC is an isosceles triangle, the angle CAE = ACE = 29° 44′ the angle required.

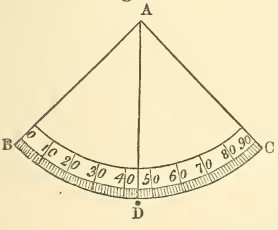
EXAMPLE 7, Fig. 61.

Fig. 61.




Wishing to know the height of a steeple situated on a horizontal plane, I measured 100 feet in a right line from its base, and then took the angle of elevation* of the top, which I found to be 47° 30′, the centre of the quadrant being 5 feet above the ground: required the height of the steeple.

Fig. 60.




* Angles of elevation, or of depression, are usually taken with an instrument called a quadrant, the arc of which is divided into 90 equal parts or degrees, and those, when the instrument is sufficiently large, may be subdivided into halves, quarters, &c. From the centre a plummet is suspended by a fine silk thread. Fig. 60 is a representation of this instrument.

To take an angle of elevation, hold the quadrant in a vertical position, and the degrees being numbered from B towards C, with the eye at C, look along the side CA, moving the quadrant till the top of the object is seen in a range with this side; then the angle BAD made by the plummet with the side BA, will be the angle of elevation required.

A Theodolite or Transit affords a more accurate method, but the quadrant may be made by any one.

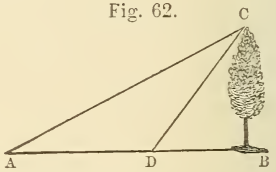
*Note.*—In finding the height of an object, it is best to take such a position that the observed angle of altitude may be about 45°; for when the observed angle is 45°, a small error committed in taking it, makes the least error in the computed height of the object.

*Calculation.*

In the right-angled triangle DEC, we have the angle CDE=47° 30′, and the base DE=AB=100 feet, to find CE=109.13 feet; to CE add EB=DA=5 feet, the height of the quadrant, and it will give BC=114.13 feet, the required height of the steeple.

EXAMPLE 8, Fig. 62.

Fig. 62.




Wishing to know the height of a tree situated in a bog, at a station D, which appeared to be on a level with the bottom of the tree, I took the angle of elevation BDC=51° 30′; I then measured DA=75 feet in a direct line from the tree, and at A, took the angle of elevation BAC=26° 30′. Required the height of the tree.

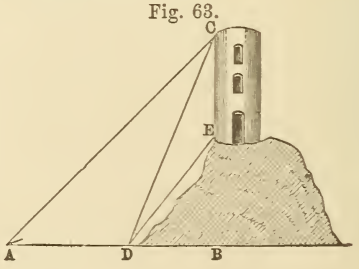
*Calculation.*

1. Because the exterior angle of a triangle is equal to the sum of the two interior and opposite ones, the angle BDC=DAC+ACD; therefore ACD=BDC—DAC=25°: now in the triangle ADC we have DAC=26° 30′, ACD=25°, and AD=75, to find DC=79.18.

2. In the right-angled triangle DBC are given DC=79.18, and the angle BDC=51° 30′ to find BC=61.97 feet, the required height of the tree.

EXAMPLE 9, Fig. 63.

Fig. 63.




Wanting to know the height of a tower EC, which stood upon a hill, at A, I took the angle of elevation CAB=44°; I then measured AD 134 yards, on level ground, in a straight line towards the

tower; at D the angle CDB was 67° 50′ and EDB 51°. Required the height of the tower and also of the hill.

*Calculation.*

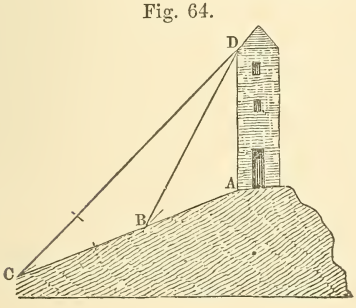
1. In the triangle ADC we have the angle DAC = 44°, the angle ACD = BDC—DAC = 23° 50′, and the side AD, to find DC = 230.4.

2. In the triangle DEC all the angles are given, viz. CDE = BDC—BDE = 16° 50′, DCE = 90°—BDC = 22° 10′, DEC = 180° = the sum of the angles CDE and DCE = 141°, and CD = 230.4, to find CE = 106 yards, the height of the tower.

3. In the right-angled triangle DBC, we have the angle BDC = 67° 50′, and the side DC = 230.4, to find BC = 213.4; then BE = BC—CE = 213.4—106 = 107.4 yards, the height of the hill.

EXAMPLE 10, Fig. 64.

Fig. 64.




An obelisk AD standing on the top of a declivity, I measured from its bottom a distance AB = 40 feet, and then took the angle ABD = 41°; going on in the same direction 60 feet farther to C, I took the angle ACD = 23° 45′: what was the height of the obelisk?

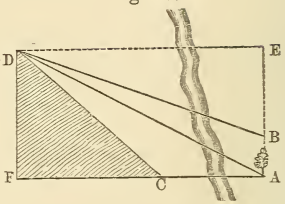
*Calculation.*

1. In the triangle BCD, we have given the angle BCD = 23° 45′, the angle BDC = ABD—BCD = 17° 15′, and side BC = 60, to find BD = 81.49.

2. In the triangle ABD are given the side AB = 40, BD = 81.49, and the angle ABD = 41°, to find AD = 57.64 feet, the height of the obelisk.

EXAMPLE 11, Fig. 65.

Fig. 65.



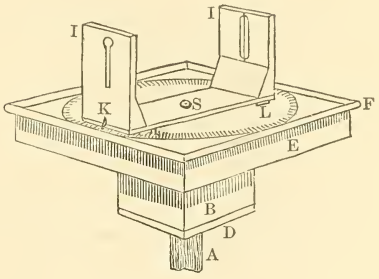

Wanting to know the height of an object on the other side of a river, but which appeared to be on a level with the place where I stood, close by the side of the river; and not having room to go backward on the same plane, on account of the immediate rise of the bank, I placed a mark where I stood, and measured in a direct line from the object, up the hill, whose ascent was so regular that I might account it a right line, to the distance of 132 yards, where I perceived that I was above the level of the top of the object; I there took the angle of depression of the mark by the river's side equal 42°, of the bottom of the object equal 27°, and of its top equal 19°: required the height of the object. *Ans.* 28.64.

*Simple Surveying Instrument.*

Since many students desire to perform some surveying and trigonometric measuring for themselves, but are unable to procure instruments for taking angles on account of their expensiveness, the following simple instrument is described in detail. The materials for it will cost from one to two dollars, and any careful worker with tools can construct it in a few hours. The results of it will be about as accurate as those of a surveyor's compass.

The board E is fastened to the block B, which turns around the staff A. The staff enters the block far enough to ensure stability. On the board is a graduated card-board circle (this may be had of J. W. Queen & Co., Philadelphia) 8 inches in diameter, graduated to half-degrees, or 13 inches in diameter, graduated to quarter-degrees. This circle is pro-

tected by a piece of glass the size of the board, perforated in the centre. The glass is held firmly by the moulding F. On this glass the sights I, I turn around the screw S, pieces of leather, L, L, being glued under each end to give uniform pressure and friction. A narrow slit is sawed down one sight, and a thread is stretched down the wider opening in the other. An index, K, is fitted in one of the sights, and works close to the glass.




The friction of the sights on the glass is so adjusted to that of the block on the staff that either may be moved without disturbing the other.

In taking an angle turn the sights so that the index K will stand over 0 on the circle. Turn the whole block round the staff till the sights point to one object. Then turn the sights around till they point to the other, and the index will read the angle.

In boring the hole in the block, it is better to make it larger than the staff, and partly fill it with perforated cork to secure more uniform friction. The cap D is screwed on to hold the cork and give it firmness.

## PRACTICAL QUESTIONS.

1. At 85 feet distance from the bottom of a tower, the angle of its elevation was found to be 52° 30′: required the altitude of the tower. *Ans.* 110.8 feet.

2. To find the distance of an inaccessible object, I measured a line of 73 yards, and at each end of it took

the angle of position of the object and the other end, and found the one to be 90°, and the other 61° 45′: required the distance of the object from each station. *Ans.* 135.9 yards from one, and 154.2 from the other.

3. Wishing to know the distance between two trees C and D, standing in a bog, I measured a base line AB= 339 feet; at A the angle BAD was 100° and BAC 36° 30′; at B the angle ABC was 121° and ABD 49°: required the distance between the trees. *Ans.* 697½ feet.

4. Observing three steeples, A, B and C, in a town at a distance, whose distances asunder are known to be as follows, viz. AB=213, AC=404, and BC=262 yards, I took their angles of position from the place D where I stood, which was nearest the steeple B, and found the angle ADB=13° 30′; and the angle BDC=29° 50′. Required my distance from each of the three steeples. *Ans.* AD=571 yards, BD=389 yards, and CD=514 yards.

5. A May-pole, whose top was broken off by a blast of wind, struck the ground at 15 feet distance from the foot of the pole: what was the height of the whole May-pole, supposing the length of the broken piece to be 39 feet? *Ans.* 75 feet.

6. At a certain place the angle of elevation of an inaccessible tower was 26° 30′; but measuring 75 feet in a direct line towards it, the angle was then found to be 51° 30′: required the height of the tower and its distance from the last station. *Ans.* Height 62 feet, distance 49.

7. From the top of a tower by the sea side, of 143 feet high, I observed that the angle of depression of a ship's

bottom, then at anchor, was 35°; what was its distance from the bottom of the wall? *Ans.* 204.2 feet.

8. There are two columns left standing upright in the ruins of Persepolis; the one is 64 feet above the plane, and the other 50; in a right line between these stands an ancient statue, the head of which is 97 feet from the summit of the higher, and 86 from that of the lower column; and the distance between the lower column and the centre of the statue's base is 76 feet: required the distance between the tops of the columns.

*Ans.* 157 feet

# SURVEYING.

## CHAPTER I.

### ON THE DIMENSIONS OF A SURVEY.

1. Surveying is the art of measuring, laying out and dividing land.

2. *A Four-Pole Chain* is an instrument used for measuring the boundaries of a survey. It is, as its name imports, 4 poles or 66 feet in length, and is divided into 100 equal parts or links. The length of a link is therefore 7.92 inches.

*Note.*—A Four-pole Chain is frequently called simply a chain.

3. *A Two-pole Chain* is 2 poles or 33 feet in length, and is usually divided into 50 equal parts or links. When it is thus divided, the links are of the same length as in a four-pole chain; and the measures taken with it are reduced to four-pole chains previous to using them in calculation.

Sometimes the two-pole chain is divided into 40 links; in which case, each two links is the one-tenth of a perch. Measures taken with a two-pole chain, thus divided, are usually expressed in perches and tenths.

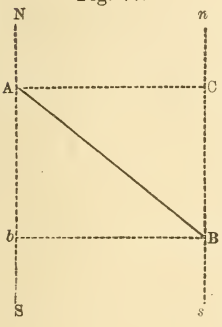
4. The *Distance* of a line in surveying, is its length, estimated in a horizontal direction. It is generally expressed either in chains and links, or in perches and tenths.

5. A *Meridian* or *Meridian Line* is any line that runs due north or south.

*Note.*—All the meridians passing through any survey of moderate extent may be considered as straight lines, parallel to one another.*

6. The *Bearing* or *Course* of a line, is the angle which it makes, with a meridian passing through one end; and it is reckoned from the North or South Points of the horizon towards the East or West Points.

Fig. 77.



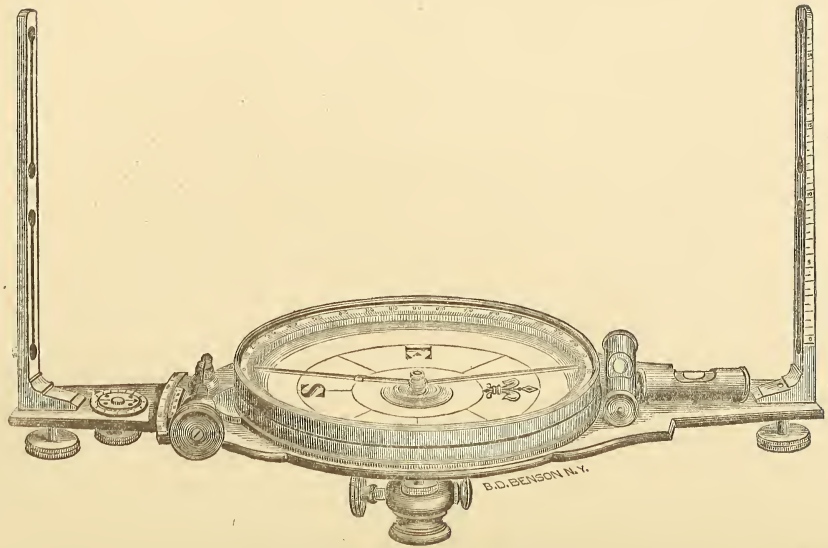

Thus, supposing the line NS, Fig. 77, to be a meridian, and the angle SAB to be 50°; then the bearing of AB from the point A, is 50° to the east of south; which is usually expressed thus: S. 50° E, and read, south, fifty degrees east.

7. The *Reverse Bearing* of a line is the bearing taken from the other end of the line.

*Note.*—The bearing and the reverse bearing of a line, are angles of the same magnitude,† but lying between

* The meridians are, in reality, curve lines which meet in the north and south poles of the earth. No two of them are therefore exactly parallel; but in usual surveys their deviation from parallelism is so very small, that there is no sensible error in considering them so.

† As the meridians are not exactly parallel, this is not strictly true, except in a few cases; but the difference is too small to be observed in practice. In




SURVEYOR'S COMPASS.

directly opposite points. Thus, if the bearing of AB, from the end A, is S. 50° E., the bearing of the same line from the end B, is N. 50° W.

8. A *Circumferentor* or *Surveyor's Compass*, is an instrument used to take the bearings of lines.

The circumference of its face is divided into degrees, and in some of the larger ones into half degrees, in such manner that two opposite points may be exactly in the direction of the sights with which the instrument is furnished. These points are the north and south points of the instrument. Midway between them, on the circumference, are the east and west points. The degrees are numbered from 0° to 90°, each way from the north and south points to the east and west ones. In the centre of the face is a pin, finely pointed, which supports a Magnetic Needle, moving freely within the instrument. The instrument, when used, is placed on a staff, having a pointed iron at the bottom, and a ball and socket at the top, or on a tripod.

The Chain and Compass are the instruments with which the dimensions of surveys in this country are frequently taken. It is important to have them accurately made. In the selection of a compass, particular attention should be directed to the settling of the needle. If, when the needle has been moved out of its natural position, it settles very soon, it is defective; either its magnetic virtue is weak, or it does not move with sufficient freedom on the pin.

9. The *Difference of Latitude*, or the *Northing* or

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the latitude of Philadelphia the greatest difference between the bearing and reverse bearing of a line, a mile in length, is only 44″. In higher latitudes the difference is greater.

*Southing* of a line, is the distance that one end is further north or south than the other end; or it is the distance which is intercepted on a meridian passing through one end, between this end and a perpendicular to the meridian, from the other end.

Thus, if NS, Fig. 77, be a meridian passing through the end A, of the line AB, and B*b* be perpendicular to NS, then is A*b* the difference of latitude or southing of AB.

10. The *Departure* or the *Easting* or *Westing* of a line is the distance that one end is further east or west than the other end; or it is the distance from one end, perpendicular to a meridian passing through the other end.

Thus B*b*, Fig. 77, is the departure or easting of the line AB.

But if *ns* be a meridian, and AC perpendicular to it, and if the bearing of the line be taken from B to A, then is BC the difference of latitude or northing, and AC the departure or westing, of the line AB.

*Note.*—It is evident from the definitions, that the Distance, Difference of Latitude, and the Departure form the sides of a right-angled triangle; in which, considering the departure as the base, the perpendicular is the difference of latitude, the hypothenuse is the distance, and the angle at the perpendicular is the bearing.

11. The *Meridian Distance* of any station, is its distance from a meridian passing through the first station of the survey, or any other assumed point.

12. The *Traverse Table*, is a table containing the dif-

ferences of latitude and the departures, computed to different courses and distances.

13. The *Area* or *Content* of a tract of land is the *horizontal* surface included within its boundaries, expressed in known measures, as Acres, Roods, and Perches.

14. In going round a tract of land and returning to the place of beginning, it is evident that the whole northing which has been made, must be equal to the southing, and the easting to the westing; or in other words, that the sum of all the northings must be equal to that of the southings, and the sum of the eastings, to that of the westings.

This principle enables us to judge of the accuracy of a survey, when the bearings and distances of all the sides have been taken. If the sums of the computed northings and southings are equal, and also those of the eastings and westings; or, if, though not exactly equal, they are very nearly so, we may conclude that the survey has been correctly made; as very small differences in these sums may be imputed to little, unavoidable errors in taking the bearings and measuring the distances. But when the sum of the northings differs considerably from that of the southings, or that of the eastings from that of the westings, we must infer that an error has been made, too great to be admitted. In this case a re-survey should be taken.

It is a rule with some of our best practical surveyors, that when the difference between the sums of the northings and southings, called the *error in latitude*, or that between the sums of the eastings and westings, called the *error in departure*, exceeds one link for every ten chains in the sum of the distances, a re-survey ought to be made.

When the errors in latitude and departure fall within the limits just mentioned, they should be properly apportioned among the several latitudes* and departures; we shall thus obtain what are called the corrected latitudes and departures. The method of doing this will be given in one of the following problems.

## PROBLEM I.

*To reduce two-pole chains and links to four-pole chains and links.*

### RULE.

1. If the number of chains is *even*, divide it by 2, and to the quotient annex the given number of links.

2. If the number of chains is *odd*, divide by 2 as before, for the chains; and for the 1 that is off, add 50 to the given number of links.

### EXAMPLES.

1. In 16 two-pole chains and 37 links, how many four-pole chains and links? *Ans.* 8 *ch.* 37 *links*, or 8.37 *ch.*

2. How many four-pole chains and links are there in 17 two-pole chains and 42 links? *Ans.* 8.92 *ch.*

3. How many four-pole chains and links are there in 22 two-pole chains and 7 links? *Ans.* 11.07 *ch.*

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* In order to conciseness of expression, difference of latitude is frequently called simply, latitude.

## PROBLEM II.

*To reduce two-pole chains and links to perches and hundredths of a perch.*

### RULE.

Multiply the links by 4, for the hundredths, and the chains by 2, for the perches. If the hundredths exceed 100, set down the excess, and add 1 to the perches.

*Note.*—This rule supposes the two-pole chain to be divided into 50 links.

### EXAMPLES.

1. Reduce 17 two-pole chains and 21 links to perches and hundredths. *Ans.* 34.84 *per.*

2. Reduce 15 two-pole chains and 38 links to perches and hundredths. *Ans.* 31.52 *per.*

3. Reduce 57 two-pole chains and 49 links to perches and hundredths. *Ans.* 115.96 *per.*

## PROBLEM III.

*To reduce square four-pole chains to acres.*

### RULE.

Divide by 10, and the quotient will be the acres. If there is a decimal in the quotient, multiply it by 4, for the roods; and the decimal of these by 40, for the perches.

EXAMPLES.

1 Reduce 523.2791 square chains to acres.

10)523.2791

52.32791
4

1.31164
40

12.46560 *Ans.* 52 *ac.* 1 *r.* 12 *p.*

2. Reduce 41.9682 square chains to acres.

*Ans.* 4 *ac.* 0 *r.* 31 *p.*

3 Reduce 132.925 square chains to acres.

*Ans.* 13 *ac.* 1 *r.* 6.8 *p.*

## PROBLEM IV.

*To reduce acres, roods and perches to square chains.*

RULE.

Divide the perches by 40 and prefix the roods; divide the result by 4 and prefix the acres; then this latter result, multiplied by 10, will give the square chains.

Or reduce the given quantity to perches and divide by 16.

EXAMPLES.

1. Reduce 13 *ac.* 1 *r.* 10 *p.* to square chains

40)10

4)1.25

13.3125 $A$=133.125 *sq. ch.*

2. Reduce 127 *ac.* 3*r.* 23*p.* to square chains.

*Ans.* 1278.9375 *sq. ch.*

3. Reduce 35 *ac.* 0 *r.* 20 *p.* to square chains.

*Ans.* 351.25 *sq. ch.*

## PROBLEM V.

*To find the bearing of a line.*

1. Let a stake of six or eight feet in length be set up perpendicularly, at the far end of the line. Set up the compass firmly at the beginning of the line, and, freeing the needle so as to rest on the pivot, adjust it to a horizontal position; the ball and socket admitting a motion for that purpose. This position can be determined with sufficient accuracy, by observing whether, when the compass is turned round, the ends of the needle remain at the same height above the face of the instrument.

2. Turn the compass round so as to bring the south end of it towards the stake at the far end of the line Then applying the eye to the sight at the north end, move the compass gently round till the stake can be seen through the fine slits in both sights, and let it remain in this position.

3. When the needle has settled, observe the number of degrees and parts of a degree, that are intercepted between the south end of the needle and the north or south point of the compass, to whichever it is nearest; which will be the bearing, reckoning it from *that* point, towards the east if the south end of the needle is to the right hand, but towards the west if it is to the left hand

*Note* 1.—The bearing thus obtained may be, and should be, verified by going to the far end of the line,

and from thence taking the bearing to the first end; which, if both bearings are correct will be the reverse of the former.

*Note* 2.—When there is a fence on the side, or other obstacle in the way, preventing the stake at one end from being seen through the compass sights at the other end, the bearing may be obtained by setting up the compass and stake at small *equal* distances to the right or left, so that the line joining them may be parallel to the side.

*Note* 3.—The method of obtaining the bearing between two stations when there are obstacles in the way, which also prevent a parallel bearing being readily taken, or when the stations are too distant to be seen from each other, will be noticed in the next chapter.

## PROBLEM VI.

*To measure the distance of a line.*

For convenience in marking the termination of the chain in measuring, *ten* iron pins should be provided, about a foot in length, and terminated at top by a small ring, to which a piece of red flannel or other conspicuous substance should be tied, in order that the pins may be readily found, when set up among high grass or in other situations where they would not otherwise be easily discovered.

Let the person who is to go foremost in carrying the chain, take nine of the pins in his left hand, and one end of the chain and the other pin in his right hand; then he moving on in the direction of the line, let another person take the other end of the chain and hold it at the beginning of the line. When the leader has moved on till the chain is stretched tight, he must set down the pin, per-

pendicularly, exactly at the end of the chain, the hinder chain-man taking care that the chain is in the direction of the line; which is readily determined by observing whether it is in a range with a stake previously set up at the far end of the line. When the leader has not his end of the chain in the direction of the line, the hinder chain-man can direct him which way to move, by a motion of his left hand. When the distance of one chain or half chain* has been thus determined, the carriers, taking hold of the two ends of the chain, move on till the hinder one comes to the pin which was set up by the other; then the chain being stretched, the person at the fore end of it sets up another pin as before; the hinder chain-man then taking up the pin at his end, they proceed to a third distance of the chain; and so on. When the person at the fore end of the chain has set up all his pins, he still moves on another length of the chain, and then setting his foot on it to keep it in place, he cries "*out.*" The hinder chain man then comes forward, and counts to him the ten pins; and he setting up one of them at the end of the chain, again moves on, dragging the chain after him, till he is checked by the hinder chain-man, who, getting the hind end of the chain, applies it as before to the pin set up. The number of outs should be carefully noticed; each *out* being ten chains, when a four-pole chain is used, but only five, when the measuring is done, with a two-pole chain. When arrived at the end of the line, the number of pins, which the one at the fore end of the chain has set up since the last *out*, and the number of links from the last pin to the end of the line, must be carefully noted. From these, and

---

* When a two-pole chain is used, one length of it may properly be called a half chain

the number of *outs*, the distance measured is readily determined.

All slant or inclined surfaces, as the sides of a hill, should be measured horizontally, and not on the plane or surface of the hill. To effect this, the hind end of the chain, in ascending a hill, should be raised from the ground till it is on a level with the fore end, and, by means of a plummet and line, or when the hill is not very steep, by estimation, should be held perpendicularly above the termination of the preceding chain. In descending a hill, the fore end of the chain should be raised in the same manner, and the plummet being suspended from it will show the commencement of the succeeding chain.

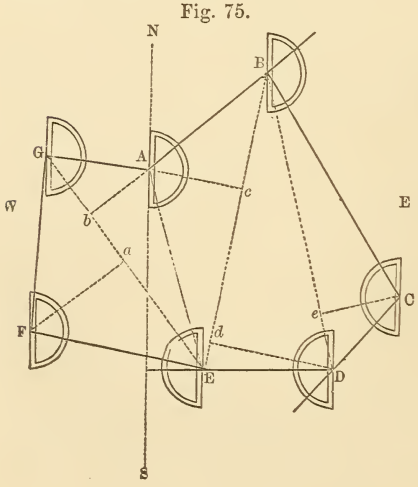
## PROBLEM VII.

*To protract a Survey, having the bearings and distances of the sides given.*

The method of doing this will be best understood by an example. Thus,

Suppose the following field notes to be given, it is required to protract the survey.

Fig. 75.




| | | Ch. |
|---|---|---|
| 1. | N. 50° E. | 9.60 |
| 2. | S. 32° E. | 16.38 |
| 3. | S. 41° W. | 6.30 |
| 4. | West | 8.43 |
| 5. | N. 79° W. | 10.92 |
| 6. | N. 5° E. | 11.25 |
| 7. | S. 83° E. | 6.48 |

Method 1st.

Draw NS, Fig. 75, to represent a meridian line; then N standing for the north and

S for the south, the east will be to the right hand, and the west to the left. In NS take any convenient point as A for the place of beginning, and apply the straight edge of the protractor to the line, with the centre to the point A, and the arch turned towards the east, because the first bearing is easterly; then holding the protractor in this position, prick off 50° the first bearing, from the north end, because the bearing is from the north; through this point and the point A, draw the line AB on which lay 9.60 chains, the first distance from A to B. Now apply the centre of the protractor to the point B, with the arch turned toward the east, because the second bearing is easterly, and move it till the line AB produced, cuts the first bearing 50°; the straight edge of the protractor will then be parallel to the meridian NS; hold it in this position, and from the south end prick off the second bearing 32°; draw BC and on it lay the second distance 16.38 chains. Proceed in the same manner at each station, observing always, previous to pricking off the succeeding bearing, to have the arch of the protractor turned easterly or westerly, according to that bearing, and to have its straight edge parallel to the meridian; this last may always be done by applying the centre to the station point, and making the preceding distance line produced if necessary, cut the degrees of the preceding bearing. It may also be done by drawing a straight line through each station, parallel to the first meridian.

When the survey is correct, and the protraction accurately performed, the end of the last distance will fall on the place of beginning.

### Case 2.

When angles and distances are given.

When the angles have been taken with the Theodolite or Transit, the following method is simpler:

Beginning at any convenient angle, lay down the first side. Place the flat edge of the protractor on this line, with the centre at its extremity, and prick off the angle between the first and second sides. Through this point and the end of the first side draw a line, and measure off on it the length of the second side, and so go round the tract.

When the survey is correct, and the protraction accurately performed, the last side should terminate at the starting point, and the angle at this point be equal to the first given angle.

Protract the figure of last Case from the following field notes:

| Angles. | | Distances. | |
|---|---|---|---|
| A | 227° | AB | 9.60 |
| B | 82° | BC | 16.38 |
| C | 107° | CD | 6.30 |
| D | 131° | DE | 8.43 |
| E | 169° | EF | 10.92 |
| F | 96° | FG | 11.25 |
| G | 88° | GA | 6.48 |

If the angles are taken, it is usual to take the bearing of one side. This may be used as the first side of the plot, and thus all directions are determined.

## PROBLEM VIII.

*The bearing of two lines from the same station being given, to find the angle contained between them.*

RULE.

When they run from the *same* point of the compass towards the *same* point, *subtract* the less from the greater.

When they run from the *same* point, towards *different* points, *add* them together.

When they run from *different* points, towards the *same* point, *add* them together, and take the *supplement* of the sum.

When they run from *different* points, towards *different* points, *subtract* the less from the greater, and take the *supplement* of the remainder.

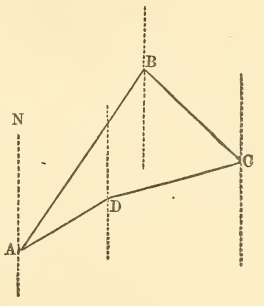
*Note.*—While these rules are useful to some surveyors, it is better, in any case, to draw a figure accurately representing the given lines, when the proper method of obtaining the angle will readily be seen.

EXAMPLES.

1. Given the bearing of the line AB, Fig. 67, N. 34° E., and AD, N. 58° E.; required the angle A.

$$\begin{aligned} \text{NAD} &= 58° \\ \text{NAB} &= 34° \\ \hline \text{Angle A} &= 24° \end{aligned}$$

Fig. 67.




2. Given the bearing of BA, Fig. 57, S. 34° W., and BC, S. 35° E.; required the angle B. *Ans.* B = 69°.

3. Given the bearing of BC, Fig. 67, S. 35° E., and CD, S. 87° W.; required the angle C. *Ans.* 58°.

4. Given the bearing of DC, Fig. 67, N. 87° E., and DA, S. 58° W.; required the angle D. *Ans.* 151°.

## PROBLEM IX.

*To change the bearings of the sides of a survey in a corresponding manner, so that any particular one of them may become a Meridian.*

RULE.

*Subtract* the bearing of the side that is to be made a meridian, from those bearings that are between the *same* points that it is, and also from those that are between points directly *opposite* to them. If it is *greater* than either of the bearings from which it is to be subtracted, take the difference, and change E. to W., or W. to E.

*Add* the bearing of the side which is to be made a meridian, to those bearings which are neither between the same points that it is, nor between the points that are directly opposite to them. If either of the sums exceeds 90°, take the *supplement* and change N. to S., or S. to N.*

*Note.*—When the bearings of some, or all, of the sides of a survey have been thus changed, and by calculation the changed bearing of another side or line has been

* The changing of the bearings so as to make a given side become a meridian, may be illustrated by means of a protracted survey. If a protracted survey or plot is held horizontally, with the meridian in a north and south direction, the north end being towards the north, the bearings of the sides of the plot will then correspond with the bearings of the sides of the survey. If then, keeping the paper horizontal, it be turned round till any particular side of the plot has a north and south direction, or becomes a meridian, the bearings of all the other sides of the plot will have been changed by a like quantity. But it is evident, that neither the relation of the different parts of the plot to one another, the area nor the lengths of the sides will have been altered by this change. In any problem of this kind let the student draw a plot, and note the effect of turning each of the sides in the proper direction through the given angle. He can thus easily see what additions or subtractions must be made to find the new bearings.

found, its *true* bearing will be obtained by applying to the changed bearing, the bearing of the side which was made a meridian, in a contrary manner to what is directed in the rule; that is, by adding in the case in which the rule directs to subtract, and by subtracting in the case in which it directs to add.

EXAMPLES.

1. Given the bearings of the sides of a survey as follow; 1st. S. 45½° W.; 2d. N. 50° W.; 3d. North; 4th. N. 85° E.; 5th. S. 47° E.; 6th. S. 20½° W.; and 7th. N. 51¼° W. Required the changed bearings, so that the 5th side may be a meridian.

| | |
|---|---|
| 1st. | S. 45½° W. |
| | 47 |
| | 92½ |
| | 180 |
| chang. bear. | N. 87½° W. |
| 2d. | N. 50° W. |
| | 47 |
| chang. bear. | N. 3° W. |
| 3d. | N. 0° E. |
| | 47 |
| chang. bear. | N. 47° E. |
| 4th. | N. 85° E. |
| | 47 |
| | 132 |
| | 180 |
| chang. bear. | S. 48° E. |

5th. side, changed bearing, South.

6th. S. 20½° W.
47

chang. bear. S. 67½

7th. N. 51° W.
47

chang bear. N. 4¼ W.

2. Given the following bearings of the sides of a survey; 1st. S. 40½° E.; 2d. N. 54° E.; 3d. N. 29¼° E.; 4th. N. 28¾° E.; 5th. N. 57° W.; and 6th. S. 47° W.; to find the changed bearings so that the 2d. side may be a meridian. *Ans.* 1st. N. 85½° E.; 2d. North; 3d. N. 24¾° W.; 4th. N. 25¼° W.; 5th. S. 69° W.; 6th. S. 7° E.

3. Given the bearings as in the 1st. example; viz. 1st. S. 45½° W.; 2d. N. 50° W.; 3d. North; 4th. N. 85° E.; 5th. S. 47° E.; 6th. S. 20½° W; 7th. N. 51¼° W.; to find the changed bearings so that the 6th side may be a meridian. *Ans.* 1st. S. 25° W.; 2d. N. 70½° W.; 3d. N. 20½° W.; 4th N. 64½° E.; 5th. S. 67½° E.; 6th. South; 7th. N. 71¾° W.

## PROBLEM X.

*Of the bearing, Distance, Difference of Latitude and Departure, any two being given, to find the other two*

RULE.

*When the bearing and distance are given.*

As Rad. : cos. of bearing : : distance : dif. of latitude.
Rad. sin. of bearing : : distance ; departure.

*When the bearing and difference of latitude are given.*

As cos. of bearing : rad. : : diff. lat. : distance.
Rad. : tang. of bearing : : diff. lat. : departure.

*When the bearing and departure are given.*

As sin. of bearing : rad. : : departure : distance.
Rad. : cotang. of bearing : : departure : diff. lat.

*When the difference of latitude and the departure are given.*

As diff. lat. : departure : : rad. : tang. of bearing.
Cos. of bearing : rad. : : diff. lat. : distance.

*When the distance and difference of latitude are given.*

As distance . diff. lat. : : rad. : cos. of bearing.
Rad. : tang. of bearing : : diff. lat. : departure.

*When the distance and departure are given.*

As distance : departure : : rad. : sin of bearing.
Rad. : cos. of bearing : : distance : diff. lat.

*Note.*—It is evident the above proportions are the solutions of a right-angled triangle, having for its sides the distance, difference of latitude, and departure; they may be proved by demonstrations similar to that on page 63.

The following examples may be worked out by the rules. The reason of each rule should be clearly seen before it is applied to the example.

1. Given the bearing of a line, S. 32° 30′ E., and the departure 10.96 ch. to find the distance and difference of latitude. *Ans.* Dist. 20.40 ch.; diff. lat. 17.20 S.

2. Given the distance of a line, running between the north and east, 44 ch. and its difference of latitude 34.43 ch.; to find the bearing and departure.

*Ans.* Bearing, N. 38° 30′ E.; dep. 27.39 ch. E.

3. The bearing of a line S. 32° 30′ E., and the difference of latitude 17.21 ch. being given, to find the distance and departure. *Ans.* Dist. 20.41 ch.; dep. 10.96 E.

4. Given the difference of latitude of a line 27.92 N., and the departure 5.32 E.; to find the bearing and distance. *Ans.* Bearing, N. 10° 47′ E.; dist. 28.42.

5. The distance of a line, running between the north and west, is 35.35 ch., and its departure 15.08 ch., required the bearing and difference of latitude.

*Ans.* Bearing N. 25° 15′ W.; diff. lat. 31.97 N.

## PROBLEM XI.

*To find the difference of latitude and departure corresponding to any given bearing and distance, by means of the Traverse Table.*

### RULE.

*When the distance is any number of whole chains or perches, not exceeding* 10.

Find the given bearing at the *top* or *bottom* of the column, according as it is *less* or *more* than 45°. Then

against the given distance, found in the column of distances at the side of the table, and under or over the given bearing, is the difference of latitude and departure, which must be taken as marked at the top of the column when the bearing is at the top, but as marked at the bottom when the bearing is at the bottom.

*When the distance is any number of chains or perches and decimals of a chain or perch.*

Take out the latitude and departure corresponding to each figure, moving the decimal point one place to the right or left, as the number is to the left or right of the unit's place, and add all the results.

EXAMPLES.

1. Given the bearing of a line S. 35¼ E., distance 79 ch.; required the latitude and departure.

| | Lat. | Dep. |
|---|---|---|
| 70. | 57.165 | 40.400 |
| 9. | 7.3498 | 5.1943 |
| | 64.5148 S. | 45.5943 E. |

2. Given the bearing N. 56½ W., distance 27.04.

| | Lat. | Dep. |
|---|---|---|
| 20. | 11.039 | 16.678 |
| 7. | 3.864 | 5.837 |
| .04 | .022 | .034 |
| | 14.925 N. | 22.549 W. |

3. Given bearing N. 29¾ E. and distance 29.84.

| | Lat. | Dep. |
|---|---|---|
| 20. | 17.364 | 9.924 |
| 9. | 7.814 | 4.466 |
| .8 | .695 | .397 |
| .04 | .035 | .020 |
| | 25.908 N. | 14.807 E. |

1. Given the bearing and distance of a line, N. 39½° W. 15.20 ch., to find its difference of latitude and departure. *Ans.* Diff. lat. 11.72 N., and dep. 9.67 W.

2. The bearing and distance of a line are N. 46° E., 27.25 ch.; required its difference of latitude and departure. *Ans.* Diff. lat. 18.93 N. and dep. 19.60 E.

3. The bearing and distance of a line are S. 37¼° W., 137.50 ch.; required its difference of latitude and departure. *Ans.* Diff. lat. 109.45 S., and dep. 83.23 W.

4. Required the difference of latitude and departure of a line, whose bearing and distance are S. 6½° E., 5.60 ch. *Ans.* Diff. lat. 5.56 S., and dep. 0.63 E.

## PROBLEM XII.

*Given the bearings and distances of all the sides of a tract of land to obtain the corrected latitudes and departures.*

### RULE.

1. Rule a table as in the annexed example, in the first vertical column of which, place the letters designating the sides, or the numbers denoting the stations at the beginning of each side; in the second column, place the bearings; and, in the third, the distances.

2. Find, by the last problem, the difference of latitude and the departure, corresponding to each side, and place them in the next four columns, under their proper heads of N. or S., E. or W. Add up the northings and southings; and if the sums are not equal, find their difference;

which will be the error of the survey in difference of latitude; which call by the same name as the *least* sum. Proceed in the same manner with the eastings and westings, and find the error in departure. Also add up the column of distances. Then it will be,

As the sum of the distances,
Is to any particular distance,
So is the *error* in latitude or departure
To the *correction* of latitude or departure, corresponding to that distance.

3. Find, by the above proportion, the corrections of latitude and departure corresponding to all the sides calculating them to the nearest two decimal figures, and place them in the next two columns, heading them with the *same names* as the *errors* in latitude and departure. If the sums of these corrections, are not respectively equal to the errors in latitude and departure, which, in consequence of the fractions neglected, will sometimes be the case, alter some of them by a unit in the second decimal figure, to make them so.

4. Apply these corrections to their corresponding differences of latitude and departures, by *adding* when of the *same* name, but by *subtracting* when of *different* names, and the corrected differences of latitude and departures will be obtained; which may be placed in the four succeeding columns.

In these the sums of the northings and southings will be equal, and also those of the eastings and westings.*

* The directions given in the rule, for correcting the errors in difference of latitude and departure, are deduced from the rule given and demonstrated in No. 4, of a periodical work, called the Analyst, by *Nathaniel Bowditch*, A. M., and also by the editor, *Professor Adrain*. The demonstration is too long, and not of a nature for insertion here.

*Note* 1.—In the proportion for finding the correction of the latitude or departure, the decimal parts of the sum of the distances and of the particular distance may be omitted, taking, in each case, the nearest number of whole chains.

2. The corrections may be frequently estimated with sufficient accuracy without the trouble of working out the proportions.

3. When one or two of the sides are hilly, or when there are other difficulties in the way of obtaining their bearing or distances with accuracy, it is better to allow a considerable part of the errors, on the latitudes and departures corresponding to them, and afterwards to apportion the remaining part among the others.

### EXAMPLES.

1. Given the bearings and distances of the sides of a tract of land as follow: 1st. S. 40½° E. 31.80 ch.; 2nd. N. 54° E. 2.08 ch.; 3rd. N. 29¼° E. 2.21 ch.; 4th. N. 28¾° E. 35.35 ch.; 5th. N. 57° W. 21.10 ch.; 6th. S. 47° W. 31.30 ch. Required the corrected differences of latitude and departures.

| Sta. | Courses. | Dist. Ch. | N. L. | S. L. | E. D. | W. D. | Cor. S. | Cor. E. | N. L. | S. L. | E. D. | W. D. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | S. 40½ E. | 31.80 | | 24.18 | 20.65 | | .03 | .05 | | 24.21 | 20.70 | |
| 2 | N. 54 E. | 2.08 | 1. 23 | | 1.68 | | .00 | .00 | 1.23 | | 1.68 | |
| 3 | N. 29¼ E. | 2.21 | 1.92 | | 1.08 | | .00 | .00 | 1.92 | | 1.08 | |
| 4 | N. 28¾ E. | 35.35 | 31.00 | | 17.00 | | .04 | .05 | 30.96 | | 17.05 | |
| 5 | N. 57 W. | 21.10 | 11.49 | | | 17.69 | .02 | .03 | 11.47 | | | 17.66 |
| 6 | S. 47. W. | 31.30 | | 21.34 | | 22.89 | .03 | 04 | | 21.37 | | 22.85 |
| | | 123.84 | 45.64<br>45.52 | 45.52 | 40.41 | 40.58<br>40.41 | .12 | .17 | 45.58 | 45.58 | 40.51 | 40.51 |

12 Er. S. .17 Er. E

As 124 : 32 :: .12 : .03
124 : 2 :: .12 : .00
24 : 35 :: .12 : .03 or .04
124 : 21 :: .12 : .02
124 : 31 :: .12 : .03

As 124 : 32 :: .17 : .04 or .05
124 : 2 :: .17 : .00
124 : 35 :: .17 : .05
124 : 21 :: .17 : .03
124 : 31 :: .17 : .04

2. Given the bearings and distances of the sides of a tract of land as follow: 1st. N. 75° E. 13.70 ch.; 2d. N. 20½ E. 10.30 ch.; 3d. East 16.20 ch.; 4th. S. 33½° W. 35.30 ch.; 5th. S. 76 W. 16 ch.; 6th. North 9 ch.; 7th. S. 84° W. 11.60 ch.; 8th. N. 53¼° W. 11.60 ch.; 9th. N. 36¾° E. 19.36 ch.; 10th. N. 22½° E. 14 ch.; 11th. S. 76¾° E. 12 ch.; 12th. S. 15° W. 10.85 ch.; 13th. S. 18° W. 10.62 ch.; to the place of beginning. Required the corrected latitudes and departures.

*Ans.* 1st. 3.56 N. 13.26 E.; 2d. 9.66 N. 3.62 E.; 3d. 0.02 N. 16.22 E.; 4th. 29.39 S. 19.44 W.; 5th. 3.85 S. 15.50 W.; 6th. 9.01 N. 0.01 E.; 7th. 1.19 S. 11.52 W.; 8th. 6.96 N. 9.27 W.; 9th. 15.54 N. 11.61 E.; 10th. 12.95 N. 5.38 E.; 11th. 2.73 S. 11.70 E.; 12th. 10.46 S. 2.80 W.; 13th. 10.08 S. 3.27 W.

## PROBLEM XIII.

*To find the difference of latitude and departure corresponding to any given bearing and distance, by means of the table of natural sines and cosines.*

### RULE.

Find from the table the natural sine of the angle representing the bearing, and multiply it by the distance, this will give the departure; find its cosine, and

multiply by the distance, this will give the difference of latitude.*

*Note.*—This rule will be found convenient when the bearings are given to minutes.

EXAMPLES.

1. Given the bearing and distance of a line N. 28° 42′ E. 26 ch., to find the difference of latitude and departure.

Natural cosine of 28° 42′ = .87715
.87715 × 26. = 22.81 = diff. of lat. N.
Natural sine of 28° 42′ = .48022
.48022 × 26. = 12.49 = dep. E.

2. Given the bearing and distance of a line S. 57° 28′ W. 11.40 ch., to find the difference of latitude and departure.

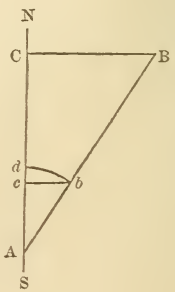
*Ans.* Diff. lat. = 6.13 S. Dep. = 9.61 W.

*Note.*—It is often more convenient to add to the *logarithmic* sine and cosine of the bearing the logarithm of the distance, and find the natural number corresponding.

---

* Demonstration.—Let NS be a meridian and AB a given line. Then CAB will be its bearing, AC its difference of latitude, and BC its departure. Lay off A*b* = 1, and describe the arc *db*; draw *bc* at right angles to AC. Then *bc* will be the natural sine of A and A*c* the cosine, as given in the table.

Now, A*b* or 1 : *bc* :: AB : BC,
and A*b* or 1 : A*c* :: AB : AC;
whence BC = AB sin A,
and AC = AB cos A,
which agree with the rule.



## PROBLEM XIV.

*To measure the angles and sides of a tract with a transit or theodolite and chain, and from the angles to calculate the bearings.*

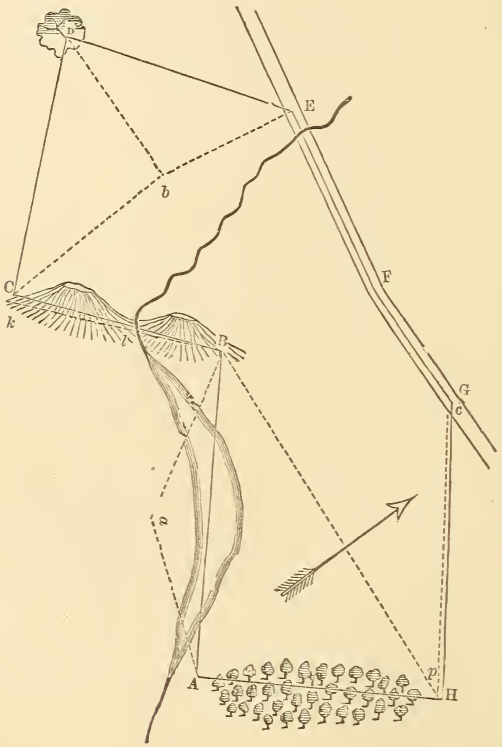
*Note.*—If the student is not familiar with the use of the transit or theodolite, he should now study Chapter VII. Practice with the instruments under a competent instructor is, however, by far the best and easiest way to understand them. The solution of this problem explains the field-work now most commonly used, by those who can command the necessary implements, to find the area of a farm.

In theory the problem of surveying a farm is a very simple one. The instrument must be carried around the tract from corner to corner, and the horizontal angles read, while the chainmen measure the distance from one angle to another. In practice difficulties are frequently met with, owing to inequalities of the ground, the impassable condition of the lines to be chained, or the thickets which prevent sighting from one station to another.

The first step is to determine accurately the positions of the corners. If these are not marked by stones, they must be established by mutual consent of the landowners interested, or by the old deeds.

Let us suppose the accompanying figure to represent a tract to be surveyed. Beginning with A, the instrument is placed directly over the corner stone, with the plumb-bob exactly at the angle. It being impossible to see H on account of an intervening woods, the angle A is left for the present. A pond of water makes it impossible to chain in a direct line to B, so, poles being set up at B and some convenient point as *a*, the angle, BA*a*, is measured, and the distance A*a* chained. The

instrument being conveyed to $a$, the angle A$a$B is read. Then by trigonometry the side AB may be calculated. At B the station C cannot be seen, nor can B and C be seen from any intermediate point. A random line, B$k$, is therefore run as near as possible in the direction of BC. This is done by pointing the telescope in the supposed direction, while an assistant places a pole as




far along the line as it can be seen, as at $l$. The angle AB$l$ is now read. The instrument is moved to $l$, pointed to B, and reversed so as to point in direction of $k$. This is kept up till the chainmen are at $k$, in the line DC produced, when the distance $k$C and the angle D$k$B are measured. Then in the triangle $k$BC

the side BC and the angle *b*BC can be calculated, and the true angles ABC, BCD are known. The angle D is inaccessible; hence the point *b* is taken, the angles DC*b*, D*b*C, D*b*E and DE*b* and the sides C*b*, *b*E are measured; whence the angle D and sides CD and DE may be calculated. The stations E, F and G being in the middle of a public road, no difficulty is found in measuring sides and angles. The line GH is covered by a fence which obstructs the sight. A point *c* is therefore taken in the line FG, and a point *p* is taken so that the line *cp* is parallel to GH. The angle F*cp* may now be read. The chaining may also be done along the line *cp*, care being taken to start directly opposite the station G, and not in the line FG. Between H and A there is a thick woods. If not too difficult, it is better to clear a track between the stations, so that one can be seen from the other; or, a random line may be run, and points in the line AH interpolated, to which the instruments may be directed in reading the angles A and H; (the method of performing this is shown on page 111); or, the diagonal BH and the angle ABH may be measured, whence AH and the angles A and H can be calculated. (Other methods of finding the length of lines connecting inaccessible points are given on page 112, &c.) It is well to measure several diagonals as a check on the other work; also angles should be taken, at several stations, to certain well-defined points within the tract, for the same purpose; and a few bearings should be read, to enable us to calculate the remainder. It should be remembered that direct measurements are always preferable to trigonometric calculations when they can be performed.

Many other difficulties will arise in practice which

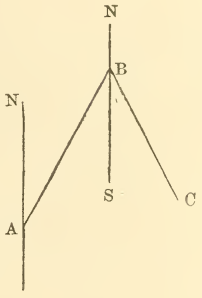
an ingenious and thoughtful person will overcome by methods improvised for the occasion. A little practice with a good surveyor is the best means of acquiring facility.

The greatest care should be taken in keeping notes of the work done. No calculations but those necessary to the continuance of the work should be made in the field; hence, it is very important that no confusion should arise in reading the notes made, and some systematic method should be employed to keep them. Every measurement should be noted down as soon as made, and the memory never relied on. Keeping field-notes will be described on page 113.

Having ascertained the various angles and sides of the tract, the next step is to test the accuracy of the work. The interior angles are added, and their sum should equal twice as many right angles as the figure has sides, less four right angles. Should they not be equal, or not within a very few minutes, a remeasurement of the angles becomes necessary, beginning with those in which, from the nature of the intervening ground, an error is most likely to occur. Having these correct, the survey is plotted by Case 2, page 93. Should the plot not close properly, a rechaining of the sides must be performed till the error is found and corrected.

From the angles the bearings may be easily deduced. One of the bearings should be taken in the field with great care if a deed of the property is needed. Or in case the area only is wanted, one of the sides may be assumed to run north and south, and the other bearings may be calculated by addition and subtraction.

If we know the bearing of a line and the angle it makes with the next, the second bearing may easily be determined; thus, let AB run N. 40° E., and the angle ABC be 58° to the right; draw AB, making the angle NAB = 40°. Make ABC = 58°. Then the bearing of BC = SBC = ABC − ABS = 58° − 40° = 18°. If a figure be drawn in this way, the student will readily see how to combine the angles to obtain the bearings.




Having the bearings and distances of the sides, the area is found by Problem IX., page 141.

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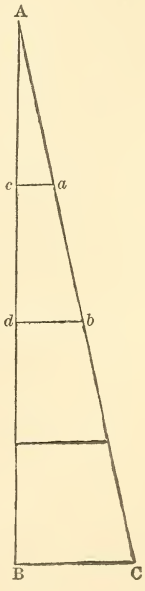
## MISCELLANEOUS PROBLEMS.

1. *To interpolate points in a line.*

It is sometimes necessary to find points in a direct line between two stations which are separated from each other by a woods, a hill or other obstacle, preventing the surveyor from seeing one station from the other. This is done by means of a random line. Let AB be the line in which it is desired to find a number of points. Starting from one end A, a line is run as near in the direction desired as can be ascertained. Let this be the line AC. Chain to various points in this line, as *a*, *b*, &c., and to C, and drive stakes. Measure also the distance CB between the ends of the two lines.

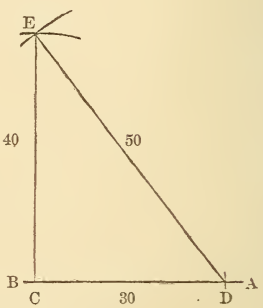


Then say, As AC : A$a$ : : CB : $ac$,
and AC : A$b$ : : CB : $bd$, &c., &c.

Calculate from these proportions the true length of $ac$ and $bd$, and from the stakes at $a$ and $b$ lay off the calculated distances parallel to CB; $c$ and $d$ will be points in the line AB.

2. *To run a perpendicular to a given line by the use of the chain.*

Let AB be the line, and C a point in it. Lay off CD on AB 30 links. With a radius of 40 links, sweep a circle with C as a centre. With a radius of 50 links, sweep a circle with D as a centre. They cut in E. Then will CE be at right angles to AB, and may be continued to any length. Any numbers in the ratio of 3, 4 and 5 will be the sides of a right-angled triangle.



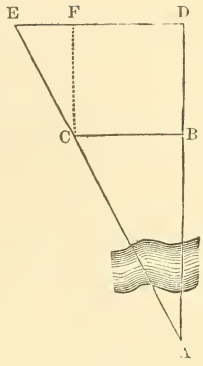

3. *To measure the distance to an inaccessible point by chain surveying.*

It is frequently necessary to know the distance to a point which cannot be reached, as the distance across a river, to a point out in the ocean, on a mountain, &c. If an instrument for taking angles can be commanded, it is best to measure a base line and angles, and calculate the distance by trigonometry. If not, the following methods are available:

1st Method.—Required the distance AB. Chain BC; run CE and BD directly from A, and chain CE, ED and BD. Then, supposing CF to be drawn, we have

$$ED - CB : DB :: CB : BA,$$

whence BA becomes known.

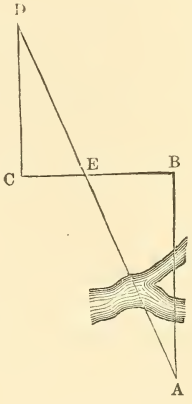



2d Method.—Draw BC at right angles to BA, and CD at right angles to BC; run DE towards A, and chain BE, EC and CD. Then

$$EC : CD :: EB : BA,$$

whence BA becomes known.

*Note.*—It is not necessary to make the angles B and C right angles if CD be drawn parallel to BA.




## FIELD-NOTES.

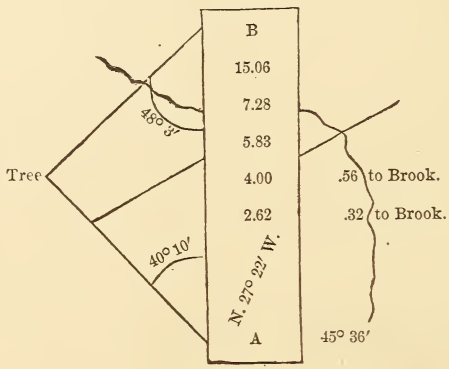
It is of great importance to the surveyor that he have a system of keeping a record of his field operations at once simple and full. Almost every one has his own method, which is better for him than any other. If the field-work is performed at a distance from the office, or lasts for several days, the records must be especially full and lucid.

One method is to draw, as accurately as possible, an outline of the tract to be surveyed, and to mark on

this map, in the proper places, the figures representing the magnitudes of the lines and angles measured. Another is to place letters at the angles of map, and write out on the margin the various dimensions; thus,

$$AB = 90 \text{ ch.} \quad A = 37°, \text{ \&c., \&c.}$$

The following plan is adopted by many surveyors: Two lines are drawn, about an inch apart, up and down the middle of the page of the note-book. All measurements in a certain line are kept between these lines, while surrounding measures and notes are kept outside. Thus, the accompanying notes would be interpreted as follows:




It is most convenient to begin at the bottom, for then the right side of the line agrees with the right side of the page. At A the angle with the last side was 45° 36′ to the right. The bearing of the line AB was N. 27° 22′ W. At 2.62 ch. from A there was an offset of .32 ch. to a brook. At 4.00 ch. from A there was an offset of .56 ch. to a brook. At 5.83 ch. from A the side AB crossed a fence, the inclination of the line denoting the angle of the fence with AB, which may be measured if desired. At 7.28 ch. from A the

line crossed the brook. At 15.06 ch. from A the other end of the line B was reached. A tree stood to the left of AB. The lines joining this with A and B made angles with AB of 40° 10′ and 48° 3′ respectively.

The student can elaborate this system as much as desired.

## CHAPTER II.

*On supplying omissions in the dimensions of a survey.*

When the bearings and distances of all the sides of a survey are known, except one bearing and one distance, or two bearings, or two distances, these can be obtained by calculation, provided those that are known can be depended on, as sufficiently accurate. This may sometimes be necessary when there are obstacles in the way of obtaining one or two of the bearings or distances; or when, after they have all been taken on the ground, the notes of one or two of them happen to be obliterated. As, however a bearing, or distance thus obtained, must be affected by any error or errors that may have been made in taking the others, it is better, when practicable, to have the bearings and distances of all the sides, as taken on the ground.

### PROBLEM I.

*The bearings and distances of all the sides of a tract of land, except the bearing and distance of one side, being given, to find these.*

#### RULE.

Find by prob. 11, of the preceding chapter, the differences of latitude and the departures for the sides whose bearings and distances are given, and place them in their proper columns in a table ruled for the purpose: Add up the northings and southings, and taking the dif-

ference of their sums, place it opposite the unknown side, in the column whose sum is the least. The sums of the two columns will then be equal. This is called *balancing* the latitudes. Do the same with the eastings and westings. The two numbers inserted to make the latitudes and the departures balance, will be the difference of latitude and the departure of the unknown side; with which its bearing and distance may be found, by prob. 10, of the preceding chapter.

*Note* 1.—By the application of this rule, the bearing and distance of a line joining two corners or stations, may be found, when there are obstacles in the way which prevent our going directly from one corner to the other, or when one cannot be seen from the other. To do this, let one or two, or more stations, if necessary, be taken out of the line, and take the bearing and distance from the first corner to the first assumed station; from this station to the second; and so on, to the second corner. Then considering these bearings and distances, as the bearings and distances of the sides of a survey, the required bearing and distance of the line may be found by the above rule. The bearing thus found must be *reversed*, in order to have the bearing from the *first* corner to the *second*.

2. In the same way the bearing and distance of a *straight road* to run between two given places, may be found, by taking the several bearings and distances of the old road if there is one; or of lines joining assumed stations and extending from one of the places to the other.

EXAMPLES.

1. The bearings and distances of the side of a tract of land, except the bearing and distance of one side which

are not known, are as in the following field-notes; required the unknown bearing and distance.

| | Chains. | | Chains. |
|---|---|---|---|
| 1. S. 45°½ W. | 15.16 | 5. ——— | ——— |
| 2. N. 50° W. | 22.10 | 6. S. 20½ W°. | 23.80 |
| 3. North | 18.83 | 7 N. 51¼ W°. | 26.47 |
| 4. N. 85° E. | 35.65 | | |

| Sta. | Bearings. | Dist. | N. | S. | E. | W. |
|---|---|---|---|---|---|---|
| 1 | S. 45½ W. | 15.16 | | 10.62 | | 10.81 |
| 2 | N. 50 W. | 22.10 | 14.20 | | | 16.93 |
| 3 | North | 18.83 | 18.83 | | | |
| 4 | N. 85 E. | 35.65 | 3.11 | | 35.52 | |
| 5 | | | | (19.79) | (21.20) | |
| 6 | S. 20½ W. | 23.80 | | 22.29 | | 8.33 |
| 7 | N. 51¼ W. | 26.47 | 16.56 | | | 20.65 |
| | | | 52.70 | 52.70 | 56.72 | 56.72 |

As diff. of lat. 19.79 S. Ar. Co. 8.703553
: dep. 21.20 E. - - - - - 1.326336
:: rad. - - - - - - - - - - - 10.000000

: tang. bear. S. 47° E. - - - - - 10.029889

As cos. bear. - - - - - Ar. Co. 0.166217
: rad. - - - - - - - - - - - 10.000000
:: diff. lat. 19.79 - - - - - 1.296447

: dist. - - 29.02 - - - - - 1.462664

*Ans.* S. 47° E. 29.02 ch.

2. Given the bearings and distances of the sides of a tract of land, as follow: 1st. N. 15¾ W 9.40 ch.; 2d N 63¾ E. 10.43 ch.; 3d. S. 49° E. 8.12 ch.; 4th. S. 13½ E. 8.45 ch.; 5th. S. 16¾ E. 6.44 ch.; 6th. Unknown; 7th. N. 60° W. 9.72 ch.; 8th. N. 17¼ E. 7.65 ch.; required the bearing and distance of the 6th. side.

*Ans.* S. 60° 8′ W. 12.27 ch.

3. One side of a tract of land of which a survey is to be taken, passes through a pond. Two stations are therefore taken on one side of the pond as represented in Fig. 80. The bearings and distances from the first end of the side to the first station, from that to the second, and thence to the other end of the side are; 1st. S. 52° W. 10.70 ch.; 2d. S. 7½° W. 13.92 ch.; and 3d. S. 34¼ E. 9 ch. Required the bearing and distance of the side. *Ans.* S. 10° 33′ W. 28.31 ch.

4. Given the bearings and distances of an old road, running between two places, as follow; 1st. S. 10° E. 92.20 ch.; 2d. S. 15° W. 120.50 ch.; 3d. S. 18½ W. 205. ch.; 4th. S. 71½ E. 68 ch. Required the bearing and distance of a straight road, that shall connect the two places. *Ans.* S. 2° 8′ W. 423.47 ch.

## PROBLEM II.

*Given all the bearings and distances of the sides of a survey, except the distances of two sides, to find these.*

### RULE.

By prob. 9, of the preceding chapter, change all the given bearings, in a corresponding manner, so that one of the sides whose bearings only are given, may become a meridian. With the changed bearings and given distances find the corresponding differences of latitude, and the departures. Add up the eastings and westings, and take the difference of their sums, which will be the

departure of that unknown side, which is not made a meridian. With this departure and the changed bearing find by prob. 10, of the preceding chapter, the distance and difference of latitude of this side, which place in their proper columns. Now add up the northings and southings, and take their difference, which will be the distance of the side made a meridian.*

### EXAMPLES.

Given the following bearings and distances of the sides of a survey; 1st. S. 45½ W. 15.16 ch.; 2d. N. 50° W. 22.10 ch.; 3d. North 18.83 ch.; 4th. N. 85° E. 35.65 ch.; 5th. S. 47° E. dist. unknown; 6th. S. 20½ W. dist. unknown; 7th. N. 51¼ W. 26.47 ch. to the place of beginning. Required the unknown distances.

| Sta. | Bearings. | Changed bearings. | Dist. | N. | S. | E. | W. |
|---|---|---|---|---|---|---|---|
| 1 | S. 45½° W. | N. 87½° W. | 15.16 | 0.66 | | | 15.15 |
| 2 | N. 50 W. | N. 3 W. | 22.10 | 22.07 | | | 1.16 |
| 3 | North | N. 47 E. | 18.83 | 12.85 | | 13.77 | |
| 4 | N. 85 E. | S. 48 E. | 35.65 | | 23.85 | 26.49 | |
| 5 | S. 47 E. | South | (29.02) | | (29.02) | | |
| 6 | S. 20½ W. | S. 67½ W. | (23.80) | | (9.11) | | (21.99) |
| 7 | N. 51¼ W. | N. 4¼ W. | 26.47 | 26.40 | | | 1.96 |
| | | | | 61.98 | 61.98 | 40.26 | 40.26 |

* The reason of the rule is obvious. For as the side made a meridian has no departure, the difference of the sums of the departures, must be the departure of the other unknown side. And when the difference of latitude of this side has been found and placed in its proper situation, the difference of the sums of the latitudes must evidently be the difference of latitude of the side made a meridian; or which, in this case, is the same thing, its distance.

As sin. chang. bearing $67\frac{1}{2}°$ Ar. Co. 0.034385
: rad. - - - - - - - - - - - 10.000000
:: dep. - - - - - - 21.99 - - - 1.342225

: dist. 6th side - - 23.80 - - - 1.376610

As rad. - - - - - - - - - - - 10.000000
: cotang. chang. bearing $67\frac{1}{2}°$ - - - 9.617224
:: dep. - - - - - - 21.99 - - - 1.342225

: diff. lat. 6th side - 9.11 - - - 0.959449

*Ans.* 5th side 29.02 ch. and 6th side 23.80 ch.

2. Given the bearings and distances of a tract of land as follow: 1st. S. $40\frac{1}{2}$ E. 31.80 ch.; 2d. N. 54° E. dist. unknown; 3d. N. $29\frac{1}{4}$ E. 2.21 ch.; 4th. N. $28\frac{3}{4}$ E. 35.35 ch.; 5th. N. 57° W. dist. unknown; 6th. S. 47° W. 31.30 ch.; to the place of beginning. Required the distances of the 2d. and 5th. sides.

*Ans.* 2d. side. 2.08 ch. and 5th. side 20.90 ch.

## PROBLEM III.

*Given the bearings and distances of all the sides of a survey except two; one of which has only its bearing given, and the other, the distance and the points of the compass between which it runs; to find the unknown bearing and distance.*

### RULE.

As in the last problem, change all the given bearings, so that the side whose bearing only is given, may become a meridian. Find the differences of latitude and the departures, corresponding to the changed bearings and the given distances. Take the difference of the sums of the eastings and westings, which will be the de-

parture of the side whose bearing is not given With the given distance and this departure, find by chap. 1. prob. 10. the changed bearing and difference of latitude, and place them in their proper columns. From the changed bearing, the true bearing may be readily found by note to prob. 9. chap. 1. Lastly, take the difference of the sums of the northings and southings, and it will be the distance of the side, changed to a meridian.

*Note.*—The changed bearing as found by the rule, must be reckoned from the *north*, or the *south* point of the compass, according as the one, or the other, will render the true bearing when found from it, conformable to the given points. The point from which the changed bearing must be reckoned determines also the column in which the difference of latitude must be placed. Sometimes the changed bearing when reckoned from either north or south, will render the true bearing conformable to the given points. In such cases, there are two different bearings and distances that will answer the conditions of the problem; and we can only know which of them is the right one by previously knowing the required bearing nearly.

EXAMPLES.

1. Given the bearings and distances of a survey as follow: 1st. S. *unknown* W. 15.16 ch.; 2d. N. 50° W. 22.10 ch.; 3d. N. 18.83 ch.; 4th. N. 85° E. 35.65 ch.; 5th. S. 47° E. 29.02 ch.; 6th. S. 20½ W. dist. unknown; 7th. N. 51¼° W. 26.47 ch. Required the unknown bearing and distance.

| Sta. | Bearings. | Changed bearings. | Dist. | N. | S. | E. | W. |
|---|---|---|---|---|---|---|---|
| 1 | S. (45° 36′) W. | (S. 25° 6′ W.) | 15.16 | | 13.73 | | (6.43) |
| 2 | N. 50 W. | N. 70½ W. | 22.10 | 7.37 | | | 20.83 |
| 3 | North | N. 20½ W. | 18.83 | 17.64 | | | 6.59 |
| 4 | N. 85 E. | N. 64½ E. | 35.65 | 15.35 | | 32.18 | |
| 5 | S. 47 E. | S. 67½ E. | 29.02 | | 11.11 | 26.81 | |
| 6 | S. 20½ W. | South | (23.81) | | (23.81) | | |
| 7 | N. 51¼ W. | N. 71¾ W. | 26.47 | 8.29 | | | 25.14 |
| | | | | 48.65 | 48.65 | 58.99 | 58.99 |

As dist. 1st side 15.16 Ar. Co. 8.819301
: dep. do. 6.43 - - - 0.808211
:: rad. - - - - - - - - - - - 10.000000

: sin. chang. bear. 25° 6′ - - - - 9.627512

As rad. - - - - - - - - - - - 10.000000
: cos. chang. bear. 25° 6′ - - - - 9.956921
:: dist. - - - - 15.16 - - - - 1.180699

: diff. lat. - - - 13.73 - - - - 1.137620

*Ans.* 1st, S. 45° 36′ W.; 6th, 23.81 ch.

2. Given the following bearings and distances of a survey: 1st. S. 40½° E. 31.80 ch.; 2d. N. 54° E. dist. *unknown* 3rd. N. 29¼° E. 2.21 ch.; 4th. N. *unknown* E. 35.35 ch.; 5th. N. 57° W. 20.90 ch.; 6th. S. 47° W. 31.30 ch.; to place of beginning. Required the bearing of the 4th. side and distance of the 2d. side.

*Ans.* Bearing of 4th. side N. 28¾° E.; dist. of 2d. side, 2.09 ch.

## PROBLEM IV.

*Given all the bearings and distances of the sides of a tract of land, except the bearings of two sides, to find these bearings.*

RULE.

1. Find the difference of latitude and the departure of each side, whose bearing and distance are both given. Take the difference of the sums of the northings and southings of these sides, and also the difference of the sums of the eastings and westings. These differences will be the difference of latitude, and the departure of a line, which, with those sides, would form a closing survey; and which may therefore be called a *closing line.*

2. With the difference of latitude and departure of the closing line, find, by prob. 10. chap. 1, its bearing and distance. Take the closing line and the two sides whose bearings are not given, for the three sides of a triangle, and calculate the angles.

3. To the bearing of the closing line, apply, by addition or subtraction, as the case may require, the angle contained between it, and the side which is the one coming first in the order of the survey; and it will give the bearing of that side. Then to the reverse bearing of that side, apply in a proper manner, the angle contained between the two sides which are sides of the survey, and it will give the bearing of the second of those sides.*

* It is easy to see the reason of the rule, by considering that the two sides whose bearings are not given, being made to form with the closing line, the

EXAMPLES.

1. Given the bearings and distances of the sides of a tract of land as follow: 1st. S. *unknown* W. 15.16 ch.; 2d. N. 50° W. 22.10 ch.; 3d. North 18.83 ch.; 4th. N. 85° E. 35.65 ch.; 5th. S. *unknown* E. 29.02 ch.; 6th S 20½° W. 23.80 ch.; and 7th. N. 51¼ W. 26.47. ch. Required the unknown bearings.

| Sta. | Bearings. | Dist. | N. | S. | E. | W. |
|---|---|---|---|---|---|---|
| 1 | S. W. | 15.16 | | | | |
| 2 | N. 50° W. | 22.10 | 14.20 | | | 16.93 |
| 3 | North | 18.83 | 18.83 | | | |
| 4 | N. 85 E. | 35.65 | 3.11 | | 35.52 | |
| 5 | S. E. | 29.02 | | | | |
| 6 | S. 20½ W. | 23.80 | | 22.29 | | 8.33 |
| 7 | N. 51¼ W. | 26.47 | 16.56 | | | 20.65 |
| | | | 52.70 | 22.29 | 35.52 | 45.91 |
| | | | 22.29 | | | 35.52 |
| | | | 30.41 S. | | | 10.39 E. |

sides of a triangle, the sum or difference of their differences of latitude, will necessarily be equal to the difference of latitude of the closing line; and that, therefore, their differences of latitude will be such as to make the sums of the northings and southings of the whole survey equal; and the same for the departures.

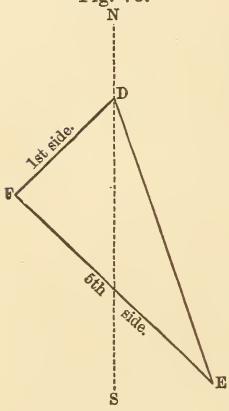
As diff. lat. - - - - 30.41 S. Ar. Co. 8.516984
: dep. - - - - - 10.39 E. 1.016616
:: rad. - - - - - - - - - - - - - 10.000000

: tang. of bear. of clos. line, S. 18° 52′ - 9.533600

As cos. of bear. of clos. line 18° 52′ Ar. Co. 0.023983
: rad. - - - - - - - - - - - - - 10.000000
:: diff. lat. - - - - 30.41 - - - - 1.483016

: dist. of clos. line - 32.14 - - - - 1.506999

Fig. 78.




Let DE, Fig. 78, represent the closing line, DF the 1st side of the survey, and FE the 5th side. Then

DE 32.14
DF 15.16 Ar. Co. 8.819301
FE 29.02 ——— 8.537313

2)76.32

Half sum 38.16 log. 1.581608

Rem. 6.02 — 0.779596

Cos. ½ F 43° 44′ 2)19.717818

F 87° 28′ 9.858909

As DE 32.14 Ar. Co. 8.492954
: FE 29.02 1.462697
:: sin. F. 87° 28′ 9.999575

sin. D. 64° 26′ 9.955226
DE, S. 18° 52′ E. FD, N. 45° 34′ E.
Angle D 64° 26′ Angle F 87° 28′

1st side, S. 45° 34′ W. 133° 2′
180° 00′

5th side, S. 46° 58′ E.

2. Given the bearings and distances of the sides of a tract of land as follow: 1st. S. *unknown* E. 31.80 ch.; 2d. N. 54° E. 2.08 ch.; 3d. N. 29¼ E. 2.21 ch.; 4th. N. 28¾° E. 35.35 ch.; 5th. N. 57° W. 20.90 ch.; and 6th. S *unknown* W. 31.30 ch. to the place of beginning. Required the unknown bearings.

*Ans.* 1st. S. 40° 29′ E.; and 6th. S. 47° W.

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## CHAPTER III.

### *Problems for finding the Content of Land.*

When the sides of a survey are right lines, and all the bearings and distances are given, the area may be found by a problem that will be given in this chapter. If one or two of the bearings or distances are not known, they may be found by the problems in the last chapter. Although the problem alluded to, is general, and may be applied whatever number of sides there may be, yet there are some particular rules for finding the areas of triangles and quadrilaterals, which are often useful. These, and also rules for finding the areas of circles and ellipses, are given in the first part of the chapter.

When a part of the boundary of a tract of land, is irregular, as is frequently the case, if one or more of the sides are bounded by water, it is sometimes very troublesome to take all the bearings and distances requisite to obtain the area with accuracy. In these cases, it is usual to run one or more straight lines, called *stationary* lines, near to such boundary, and so as to connect the straight sides of the survey. In measuring these stationary lines, perpendicular distances are measured from them, to each bend in the irregular boundary. These perpendicular distances are called *off-sets*. The lengths

of the off-sets, and the distance of the foot of each, from the commencement of the stationary line, should be carefully noted in the field book; observing also that such a number of off-sets should be taken, that the part of the irregular boundary intercepted between each adjacent two, may, without material error, be considered a straight line. From these notes, the area or areas of the land contained between the stationary line or lines, and the irregular boundary, may readily be calculated. This area added to the area enclosed by the stationary lines, and straight sides of the survey, when they are on the outside of the stationary lines, or subtracted from it, when on the inside, will give the area of the survey.

In those cases in which water is a boundary of a tract of land, if that water is a brook or rivulet, it is usual to consider a line running through its *middle* as the true boundary; and the off-sets must be measured accordingly. When tide water is the boundary, the land is considered as extending to the line of *low water mark.*

If the bearings of all the corners of a tract of land from two stations, taken either within or out of the tract are given, and also the bearing and distance of these stations from each other, the area may be calculated. It is however necessary, that the two stations should be so taken that they shall not be in a straight line, or very nearly in a straight line, with either of the corners of the land. This method of obtaining the area, though not practically so accurate as where the bearings and distances of the sides are correctly given, may sometimes be found useful.

Some surveyors, in order to calculate the area of a survey, first protract it; then dividing the plot into tri-

angles and trapeziums by lines joining opposite corners, they measure with the scale and dividers the lengths of such lines and perpendiculars as are requisite for calculating the areas of these. The sum of the areas thus obtained, is the area of the survey. When the survey is carefully protracted, and proper attention is given to take the measures with the utmost precision, this method serves to give a near value of the content; but is by no means to be depended on as equally accurate with the general problems mentioned above.

The area of a field or small tract of land, the corners of which can be seen from one another, may readily be found by means of the chain only. To do this, the lengths of the sides must be measured, and also the length of diagonals joining opposite corners, so as to divide the field into triangles. Or instead of the diagonals, the distances from some assumed point within the field, to the several corners, may be used. Having then all the sides of the several triangles, the area of each may be found; and the sum of these areas will be the area of the tract.

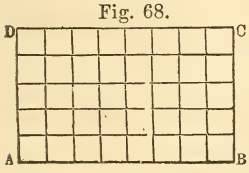
## PROBLEM I.

*To find the area of a Parallelogram, whether it be a Square, a Rectangle, a Rhombus, or a Rhomboid.*

RULE.

Multiply the length by the height or perpendicular breadth, and the product will be the area.*

Fig. 68.




* Demonstration. Let ABCD (Fig. 68) be a rectangle; and let its length AB and CD, and its breadth AD and BC, be each divided into as many equal parts, as are expressed by the number of times they contain the lineal measuring unit; and let all the opposite points of division be connected by right

*Note.*—Because the length of a square is equal to its height, its area will be found by multiplying the side by itself.

EXAMPLES.

1. Required the area of a square field, a side of which measures 7.29 four-pole chains.

7.29 Ch.
7.29

6561
1458
5103

10)53.1441 Area 5 A. 1 R. 10 P.

5.31441
4

1.25764
40

10.30560

2. Required the area of a rectangular field whose length is 13.75 chains, and breadth 9.5 chains.

---

lines. Then, it is evident that these lines divide the rectangle into a number of squares, each equal to the superficial measuring unit; and that the number of these squares is equal to the number of lineal measuring units in the length, as often repeated as there are lineal measuring units in the breadth, or height; that is, equal to the length multiplied by the breadth. But the area is equal to the number of squares or superficial measuring units; and therefore the area of a rectangle is equal to the product of the length and breadth.

Again, a rectangle is equal to any oblique parallelogram of an equal length and perpendicular height (I. 33;) therefore the area of every parallelogram is equal to the product of its length and height.

13.75 Ch.
9.5

6875
12375

10)130.625 Area 13 A. 0 R. 10 P

13.0625
4

.2500
40

10.0000

3. Required the area of a field, in the form of a rhomboid, whose length AB is 42.5 perches, and perpendicular breadth CD is 32 perches.

42.5 P.
32

850
1275

4|0)136|0.0

4)34

8 A. 2 R.

**4.** What is the area of a square tract of land whose side measures 176.4 perches? *Ans.* 194 A. 1 R. 36.96 P.

**5.** What is the area of a rectangular plantation whose length is 52.25 chains, and breadth 38.24 chains?

*Ans.* 199 A. 3 R. 8.6 P.

**6.** The length of a field, in the form of a rhombus, measures 16.54 chains, and the perpendicular breadth 12.37 chains: required the area. *Ans.* 20 A. 1 R. 33.6 P.

7. Required the area of a field in the form of a rhomboid, whose length is 21.16 chains, and perpendicular breadth 11.32 chains. *Ans.* 23 A. 3 R. 32.5 P.

## PROBLEM II.

*To find the area of a triangle when the base and perpendicular height are given.*

### RULE.

Multiply the base by the perpendicular height, and half the product will be the area.*

### EXAMPLES.

1. The base AB of a triangular piece of ground, measures 12.38 chains, and the perpendicular CD 6.78 chains: required the area. Fig. 49.

12.38 Ch.
6.78

9904
8666
7428

2)83.9364

10)41.9682 Area, 4 A. 0 R. 31 P.

4.19682
4

.78728
40

31.49120

* DEMONSTRATION.—A triangle is half a parallelogram of the same base and altitude (I. 35, Cor.).

2. Required the area of a triangular field, one side of which measures 18.37 chains, and the distance from this side to the opposite angle, 13.44 chains.

*Ans.* 12 A. 1 R. 15 P.

3. What is the area of a triangle whose base is 49 perches and height 34 perches? *Ans.* 5 A. 0 R. 33 P

## PROBLEM III.

*To find the area of a triangle when two sides and their included angle are given.*

### RULE.

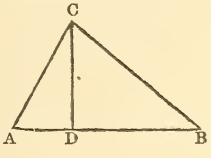
As radius,
Is to the sine of the included angle;
So is the rectangle of the given sides,
To double the area.*

### EXAMPLES.

1. In a triangular lot of ground ABC, the side AB measures 64 perches, the side AC 40.5 perches, and their contained angle CAB 30°: required the area. Fig. 49.

---

*Demonstration.—In the triangle ABC let AB and AC be the given sides, including the given angle A, and let CD be perpendicular on AB. Then by Trig. rad. : sin. A :: AC : CD; but (V. 1) AC : CD :: AC×AB : CD×AB; therefore rad. : sin. A :: AC×AB : CD×AB; but CD×AB is equal to twice the area of the triangle; hence the truth of the rule is evident.



| | | |
|---|---|---|
| As radius | | 10.000000 |
| Is to sin A, 30° | | 9.698970 |
| So is AB, AC | 64 | 1.806180 |
| | 40.5 | 1.607455 |
| | | 13.112605 |
| To double the area 1296 perches | | 3.112605 |

40)648

4)16 8

4 A. 0 R. 8 P.

2. What is the area of a triangle, two sides of which measure 15.36 chains and 11.46 chains respectively, and their included angle 47° 30′? *Ans.* 6 A. 1 R. 38 P.

3. One side of a triangular field bears N. 12° E. distance 18.23 chains, and at the same station the other adjacent side bears N. 78° 30′ E. distance 13.84 chains: required the area. *Ans.* 11 A. 2 R. 11 P.

4. Required the area of a triangular piece of ground, one side of which bears N. 82° 30′ W. dist. 19.74 chains, and at the same station the other adjacent side S. 24° 15 E. dist. 17.34 chains. *Ans.* 14 A. 2 R. 8 P

## PROBLEM IV.

*To find the area of a triangle when one side and the two adjacent angles are given.*

### RULE.

Subtract the sum of the two given angles from 180°; the remainder will be the angle opposite the given side. Then,

As the rectangle of radius and the sine of the angle opposite the given side,
Is to the rectangle of the sines of the other angles,
So is the square of the given side,
To double the area.*

EXAMPLES.

1. In a triangular field ABC, the side AB measures 76 perches, the angle A 60°, and the angle B 50°: required the area. Fig. 47.

The angle ACB=180° — the sum of the angles A and B,=70°.

| | | |
|---|---|---|
| As rad. × sin. C, | rad. | Ar. Co. 0.000000 |
| | sin. C, 70° | Ar. Co. 0.027014 |
| : sin. A × sin. B, | sin. A, 60° | 9.937531 |
| | sin. B, 50° | 9.884254 |
| :: $AB^2 = AB \times AB$, | AB 76 | 1.880814 |
| | AB 76 | 1.880814 |
| : double area in perches | 4078 | 3.610427 |
| | 40)2039 | |
| | 4)50 39 | |
| | 12 A. 2 R. 39 P. | |

* DEMONSTRATION.—Let AB (last Fig.) be the given side of the triangle ABC, and A and B the given angles; also let CD be perpendicular on AB. Then by Trig.

sin. ACB : sin. B :: AB : AC
rad. : sin. A :: AC : CD.

Therefore rad. × sin. ACB : sin. A × sin. B :: AB × AC : CD × AC :: AB : CD :: $AB^2$ : AB × CD; but AB × CD is equal to double the area of the triangle ABC; therefore rad. × sin. ACB : sin. A × sin. B :: $AB^2$ : double the area of the triangle ABC.

2. One side of a triangle measures 24.32 chains, and the adjacent angles are 63° and 74°: required the area.

*Ans.* 37 A. 0 R. 22 P.

3. What is the area of a triangular field, one side of which is 17.36 chains, and the adjacent angles 37° 30′, and 48° 15′?

*Ans.* 6 A. 3 R. 18 P.

## PROBLEM V.

*To find the area of a triangle when the three sides are given.*

RULE.

From half the sum of the three sides subtract each side severally; multiply the half sum and the three remainders continually together, and the square root of the last product will be the area.*

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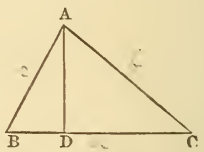
* Let $AB=c$, $BC=a$, and $AC=b$,




then (II. 8) $a^2+c^2=b^2+2a\,.\,BD$;

hence $$BD=\frac{a^2+c^2-b^2}{2a},$$

and $$AD=\sqrt{c^2-BD^2}=\sqrt{c^2-\frac{(a^2+c^2-b^2)^2}{4a^2}}=\frac{\sqrt{4a^2c^2-(a^2+c^2-b^2)^2}}{2a}.$$

Hence, $ABC=\frac{1}{2}BC\,.\,AD=\frac{1}{4}\sqrt{4a^2c^2-(a^2+c^2-b^2)^2}$. The quantity under the radical sign may be resolved into two factors, $[2ac+a^2+c^2-b^2]$ and $[2ac-(a^2+c^2-b^2)]$, and these again into

$$(a+c+b)(a+c-b) \text{ and } (b+a-c)(b+c-a).$$

Hence, the last equation becomes

$$ABC=\sqrt{\left[\frac{a+b+c}{2}\cdot\frac{a+c-b}{2}\cdot\frac{b+a-c}{2}\cdot\frac{b+c-a}{2}\right]};$$

and putting $S=\frac{a+b+c}{2}$, $ABC=\sqrt{S(S-b)(S-c)(S-a.)}$

Hence the rule.

EXAMPLES.

1. Required the area of a triangular tract of land whose three sides are 49.00, 50.25 and 25.69 chains.

| | | | |
|---|---|---|---|
| Half sum | 62.47 | log. | 1.795672 |
| Remainders | 13.47 | | 1.129368 |
| | 12.22 | | 1.087071 |
| | 36.78 | | 1.565612 |
| | | | 2)5.577723 |
| | 615 chains | | 2.788861 |

61.5 Acres = 61 A. 2 R.

2. What is the area of a triangular field whose sides measure 10.64, 12.28 and 9.00 chains?

*Ans.* 4 A. 2 R. 26 P.

3. What quantity of land is contained in a triangle, the sides of which are 20, 30 and 40 chains?

*Ans.* 29 A. 0 R. 7 P.

## PROBLEM VI.

*To find the area of a trapezium, when one of the diagonals and the two perpendiculars, let fall on it from the opposite angles, are given.*

RULE.

Multiply the sum of the perpendiculars by the diagonal, and half the product will be the area.*

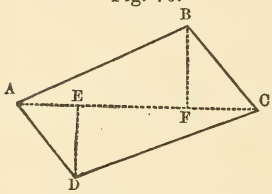
---

*DEMONSTRATION.—The area of the triangle ABC (Fig. 70) $= \frac{AC \times BF}{2}$, and the area of the triangle ADC $= \frac{AC \times DE}{2}$; therefore the sum of these is $\frac{1}{2}(BF+DE)AC$.

*Note.*—When all the sides and one of the diagonals are given, the trapezium will be divided into two triangles, the area of each of which may be found by the last problem. The sum of these areas will be the area of the trapezium.

EXAMPLES.

Fig. 70.




1. In a field ABCD, in the form of a trapezium, the diagonal AC measures 20.64 chains, the perpendicular BF 6.96 chains, and DE 5.92 chains; required the area. Fig. 70.

2. Required the area of a trapezium whose diagonal measures 16.10 ch. and the perpendiculars 6.80 ch. and 3.40 ch. *Ans.* 8 A. 0 R. 33¾ P.

3. The diagonal of a trapezium is 24 ch. and the perpendiculars are 8.27 ch., and 12.43 ch.; what is the area? *Ans.* 24 A. 3 R. 14 P.

## PROBLEM VII.

*To find the area of a trapezoid.*

RULE.

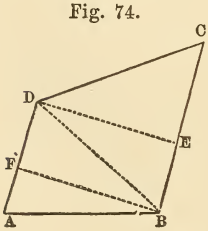
Multiply the sum of the parallel sides by their perpendicular distance, and half the product will be the area.*

EXAMPLES.

1. Required the area of a trapezoid ABCD, of which the parallel sides AD, BC measure 6.14 and 9.48 chains,

---

Fig. 74.




* Demonstration. The trapezoid ABCD, Fig. 74, = the triangle ABD + BDC = (by prob. 2,) $\frac{AD \times BF}{2} + \frac{BC \times DE}{2}$ = (because BF = DE,) $\frac{AD \times BF}{2} + \frac{BC \times BF}{2} = \frac{AD + BC \times BF}{2}$

respectively, and their perpendicular distance BF or DE, 7.80 chains.

Ch.
6.14
9.48
---
15.62
7.80
---
124960
10934
---
2)121.8360
---
60.9180 Ch.=6 A. 0 R. 15 P.

2. The parallel sides of a trapezoid are 12.41 and 8.22 chains, and their perpendicular distance 5.15 chains: required the area. *Ans.* 5 A. 1 R. 10 P.

3. Required the area of a trapezoid whose parallel sides are 11.34 and 18.46 chains, and their perpendicular distance 13.25 chains. *Ans.* 19 A. 2 R. 39 P.

## PROBLEM VIII.

*To find the area of a circle or of an ellipse.**

RULE.

Multiply the square of the circle's diameter, or the

* If two pins be set upright in a plane, and a thread, the length of which is greater than twice the distance between the pins, having the ends tied together, be put about the pins; and if the point of a pin or pencil applied to the thread, and held so as to keep it uniformly tense, be moved round, till it return to the place from which the motion began; then the point of the pin or pencil will have described on the plane, a curved line called an *Ellipse.*

product of the two diameters of the ellipse, by .7854, for the area.*

*Note* 1.—If the diameter of a circle be multiplied by 3.1416, the product will be the circumference; also if the circumference be divided by 3.1416, the quotient will be the diameter.

2. If the area of a circle be divided by .7854, the square root of the quotient will be the diameter.

EXAMPLES.

1. How many acres are in a circle a mile in diameter?

1 mile = 80 ch.

80

6400

.7854

3141600

47124

5026.5600 Sq. Ch. = 502 A. 2 R. 25 P. nearly.

*Or by Logarithms.*

| | | |
|---|---|---|
| Square of 80 | 80 | log. 1.903090 |
| | 80 | 1.903090 |
| | .7854 | −1.895091 |
| 5026.56 Sq. Ch. | | 3.701271 |

2. Required the area of an ellipse, the longer diameter of which measures 5.36 ch. and the shorter 3.28 ch.

* The demonstration of this rule in the case of the circle is given in Sharpless's Geometry, Book VI., $.7854 = \frac{1}{4}\pi$.

Ch.
5.36
3.28

4288
1072
1608

17.5808
.7854

703232
879040
1406464
1230656

13.80796032 Sq. Ch.=1 A. 1 R. 20.9 P.

## PROBLEM IX.

*The bearings and distances of the sides of a tract of land being given, to calculate the area.*

### RULE.

1. Rule a table and head it as in the annexed example, observing that the letters D. M. D. stand for Double Meridian Distance.

2. Find by prob. 12, chap. 1, the corrected differences of latitude and the departures, corresponding to the several sides, placing them in their proper places in the table.

3. Having drawn a plot of the tract, consider the most western station as the place of beginning. Count all east departures as positive, and all west departures as negative.

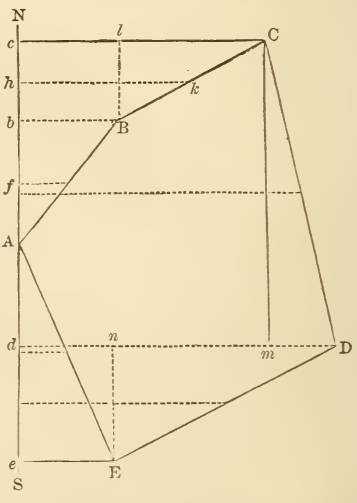
4. Take for the double meridian distance of the first side its corrected departure; take for the double meridian distance of any side the double meridian distance of the preceding side, plus its departure, plus the departure of the side itself.

5. Multiply each of the corrected differences of latitude by its corresponding double meridian distance. Place the product in the column of north or south areas, as the difference of latitude is north or south.

6. Add up the numbers in the columns of areas, take the difference of their sums and divide it by 2; the result will be the area of the survey.*

---

* Demonstration.—Let ABCDE be a plot of a survey, and let A be the most western corner. Through A draw the meridian NS.




The meridian distance of a side is the distance of its middle point from the meridian. Thus $hk$ is the meridian distance of BC, and its double meridian distance is $2hk = Bb + Cc$.

To prove clause 4 of the rule we proceed as follows:

The double meridian distance of AB is $Bb$, its departure. The double meridian distance of BC is $Bb + Cc = Bb + cl + lC$, or it is the double meridian distance of AB, plus the departure of AB, plus the departure of BC. The double meridian distance of CD is $Cc + Dd = (Cc + Bb) + lC + Dm$, which again agrees with the rule. The double meridian distance of DE is $Dd + Ee = (Dd + Cc) + Dm - Dn$, which also agrees with the rule, because $Dn$, being a western departure, is negative; and so on for any number of sides.

To show the meaning of clause 5 of the rule we proceed:

The double area of the triangle $ABb$ is $Ab \times Bb$, or the difference of latitude of AB multiplied by its double meridian distance. The double area of the trapezoid $BbCc$ is $(Bb + Cc) \times bc$, which is the product of the

*Note* 1.—The word plus in the 4th clause must be understood as having its algebraic meaning. If the quantity to be added be itself negative, as in the case of a west departure, the difference must be taken.

*Note* 2.—The last double meridian distance should be equal to the last departure; this affords a check to the accuracy of the work.

*Note* 3.—The eastern station could have been as well chosen as the point of beginning, in which case west departures would have been positive and east negative.

---

difference of latitude of BC and its double meridian distance. Hence the columns of N. areas and S. areas are the double areas of the various triangles and trapezoids between the various sides of the tract and the meridian.

To prove clause 6:

The double area ABCDE = 2C$c$D$d$+2D$d$E$e$−(2A$b$B+2B$b$C$c$+2A$e$E); that is, the sum of the south areas minus the sum of the north areas.

The whole may be arranged in tabular form as follows:

| Side. | N. Lat. | S. Lat. | D. M. D. | N. Areas. | S. Areas. |
|---|---|---|---|---|---|
| AB | A$b$ | | B$b$ | AB$b$ | |
| BC | $bc$ | | B$b$+C$c$ | B$b$C$c$ | |
| CD | | $cd$ | C$c$+D$d$ | | C$c$D$d$ |
| DE | | $de$ | D$d$+E$e$ | | D$d$E$e$ |
| EA | $e$A | | $e$E | AE$e$ | |

## EXAMPLES.

1. Given the bearings and distances, as in the following field-notes, to find the area.

| Sta. | Bear. | Dist. | N. | S. | E. | W. | Cor. N. | Cor. W. | N. | S. | E. | W. | D. M. D. | N. Areas. | S. Areas. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | N. 47 E. | 31.30 | 21.34 | | 22.89 | | .03 | .04 | 21.37 | | 22.85 | | 22.85 | 488.3045 | |
| 2 | S. 57 E. | 21.10 | | 11.49 | 17.69 | | .02 | .03 | | 11.47 | 17.66 | | 63.36 | | 726.7392 |
| 3 | S. 28¾ W. | 35.35 | | 31.00 | | 17.00 | .04 | .05 | | 30.96 | | 17.05 | 63.97 | | 1980.5112 |
| 4 | S. 29¼ W. | 2.21 | | 1.92 | | 1.08 | | | | 1.92 | | 1.08 | 45.84 | | 88.0128 |
| 5 | S. 54 W. | 2.08 | | 1.23 | | 1.68 | | | | 1.23 | | 1.68 | 43.08 | | 52.9884 |
| 6 | N. 40½ W. | 31.80 | 24.18 | | | 20.65 | .03 | .05 | 24.21 | | | 20.70 | 20.70 | 501.1470 | |
| | | | 45.52 | 45.64 | 40.58 | 40.41 | .12 | .17 | 45.58 | 45.58 | 40.51 | 40.51 | | 989.4515 | 2848.2516 |
| | | | | 45.52 | 40.41 | | | | | | | | | | 989.4515 |
| | | | Er. N. | .12 | .17 Er. W. | | | | | | | | | | 2)1858.8001 |
| | | | | | | | | | | | | | | | 929.40005 |
| | | | | | | | | | | | | | | | 92.940005 |
| | | | | | | | | | | | | | | | 4 |
| | | | | | | | | | | | | | | | 3.760020 |
| | | | | | | | | | | | | | | | 40 |
| | | | | | | | | | | | | | | | 30.40080 |

Area 92 A. 3 R. 30.4 P.

2. Given the boundaries of a tract of land as follows:

| | |
|---|---|
| 1. N. 23° 45′ W. 6.46 ch. | 4. S. 66° 22′ W. 4.895 ch. |
| 2. N. 65° 16′ E. 4.40 ch. | 5. N. 24° 2′ W. 2.625 ch. |
| 3. S. 41° 49′ E. 9.68 ch. | 6. S. 66° 59′ W. 2.49 ch. |

Required the area. *Ans.* 4 A. 2 R. 38.9 P.

*Note.*—In this case the latitudes and departures should be taken out by Problem XIII., p. 105. When a transit or theodolite is used, angles should be read to minutes. In the remaining examples of this problem the bearings were taken by a compass.

5. Required the area of a tract of land bounded as follows: 1st. S. 62° W. 7.57 ch.; 2d. N. 43½° W. 5.89 ch.; 3d North, 5.82 ch.; 4th. N. 33½° W. 8.83 ch.; 5th. N. 48° E. 4.81 ch.; 6th. N. 12° E. 4.66 ch.; 7th. N. 62½° E. 5.27 ch.; 8th. S. 6½° E. 5.60 ch.; 9th. S. 40½° E. 5.87 ch.; 10th. East, 6.54 ch.; 11th. North, 5.52 ch.; 12th. N. 68¼° E. 3.10 ch.; 13th. S. 30° E. 7.90 ch.; 14th. S. 23° W. 8.80 ch.; 15th. S. 31½° E. 6.42 ch.; 16th. S. 50° W. 8.40 ch.; 17th. N. 44° W. 6.85 ch. to the place of beginning.

*Ans.* 44 A. 2 R. 22 P.

6. Given the following field-notes to find the area of the survey; also the bearings and distance of the 3d side, which were omitted to be taken on account of obstacles in the way.

| | Ch. |
|---|---|
| 1. S. 85¼° E. | 23.30 |
| 2. S. 19 E. | 31.12 |
| 3. —— | —— |
| 4. N. 64 W | 29.72 |

| | | Ch. |
|---|---|---|
| 5. | N. 15½° | W. 22.46 |
| 6. | N. 58 | E. 25.94 |
| 7. | S. 27¾ | E. 6.60 |

*Ans.* Area 182 A. 0 R. 21.7 P. and the bearing and distance of the 3d side, S. 66° 23′ W. 28.06 ch.

7. Being furnished with the field-notes of a tract of land, and requested to calculate the area, I found on examining them, that the figures expressing the angles of bearing of the 4th and 5th sides were so defaced as to be illegible: but as the remaining data are sufficient, the area is required. The field-notes are as follows:

| | | Ch. |
|---|---|---|
| 1. | S. 60¾° | W. 10.34 |
| 2. | N. 27¼ | W. 17.88 |
| 3. | N. 51 | E. 15.85 |
| 4. | N. — | E. 9.61 |
| 5. | S. — | E. 19.18 |
| 6. | S. 16¾ | E. 22.21 |
| 7. | S. 71½ | W. 16.66 |
| 8. | N. 71¼ | W. 5.76 |

*Ans.* 81 A. 2 R. 23 P.

8. In a survey, represented Fig. 81, the corner at A was inaccessible, occasioned by the overflowing of water; but being a tree, it can be seen from the adjacent corners B and L. I therefore set my instrument at B and took the bearing to A, which I reversed, and set in my field-book as the first bearing. I then proceeded to take the bearings and distances of the several sides to L; and at L, I took the bearing of the side LA. The field-notes being as follows, the length of the sides AB and LA, and the area are required.

Fig. 81.

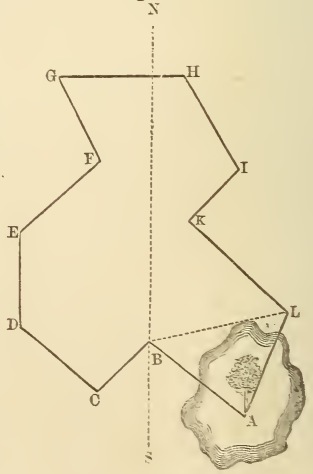
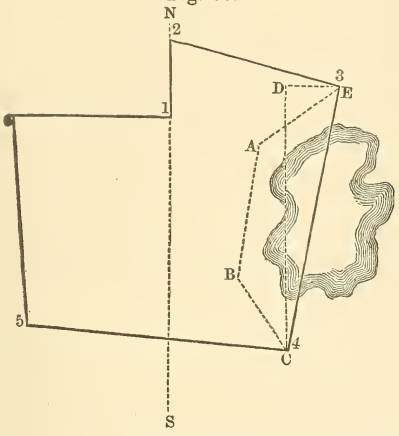


Fig. 80.




| | | |
|---|---|---|
| AB, | N. 51¼° | W. Ch. |
| BC, | S. 45½ | W. 15.16 |
| CD, | N. 50 | W. 22.10 |
| DE, | North, | 18.83 |
| EF, | N. 48 | E. 22.60 |
| FG, | N. 25½ | W. 20.17 |
| GH, | East, | 26.57 |
| HI, | S. 30½ | E. 22.86 |
| IK, | S. 44 | W. 15.04 |
| KL, | S. 47 | E. 28.55 |
| LA, | S. 20½ | W. |

*Ans.* AB, 26.47 ch.; LA, 23.81 ch.; and the area 244 A. 3 R. 13 P.

9. In taking a survey of a tract of land bounded by six straight sides, Fig. 80, I was prevented going directly from the 3d to the 4th corner by a pond of water. I therefore set up two stakes near the edge of the pond, and took the bearing and distance from the 3d corner to the first stake, from the first stake to the second, and from the second to the 4th corner, and noted them in my field-book as all belonging to the 3d station of the survey. The field-notes being as follows, the bearing and distance of the 3d side, and the area of the survey are required.

| | | | |
|---|---|---|---|
| 1. | | North, | 7.81 Ch. |
| 2. | | S. 76¼° | E. 18.15 |
| 3. | S. 52 | W. 10.70 | |
| | S. 7½ | W. 13.92 | |
| | S. 33¼ | E. 9.00 | |
| 4. | | N. 84¼ | W. 27.12 |
| 5. | | N. 4½ | W. 22.00 |
| 6. | | East, | 16.58 |

*Ans.* 3d side, S. 10° 47′ W. 28.42 ch.; and area 80 A. 0 R. 25 P.

## PROBLEM X.

*To find the area, when off-sets are taken.*

RULE.

1. Find by the last problem, the area enclosed by the stationary lines and straight sides of the survey.

2. Subtract the stationary distance of each off-set, from that of the one immediately following; the remainders will be the distances, intercepted on the stationary line, between each two adjacent off-sets. Place these under one another in a column as in the annexed examples. Also take the sums of each two adjacent off-sets, and place them in the next column, so as to correspond with the intercepted distances.

3. Multiply the sum of each two adjacent off-sets by their intercepted distance on the stationary line; then, half the sum of the products will be the area of the off-sets on that line.

4. If there are off-sets on more than one stationary line, proceed in the same manner with the others.

5. When the stationary lines are within the boundary of the survey, add the areas of the off-sets to the area enclosed by the stationary lines and straight sides; but when the stationary lines are without the boundary, subtract the areas of the off-sets.*

---

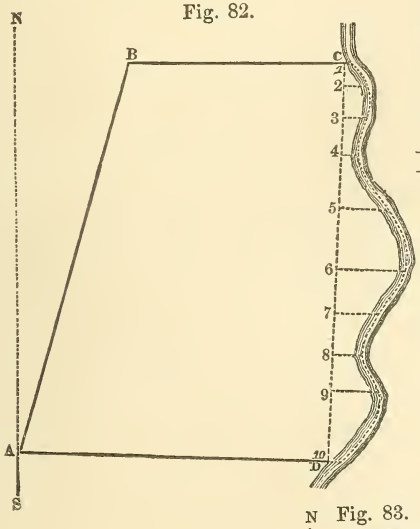
* Demonstration. Considering the boundary as straight between the ends of each two adjacent off-sets, it is plain that the area contained between the stationary line and boundary will be divided by the off-sets into trapezoids and triangles. Hence the truth of the rule is evident.

EXAMPLE 1, Fig. 82.

Required the area of a piece of meadow, bounded on one side by a brook; the field-notes being as follows:

1. N. 16½° E. 14.35 Ch.
2. East, 7.82
3. S. 3½ W. 14.45 Stat. line.
4. N. 86½ W. 11.07

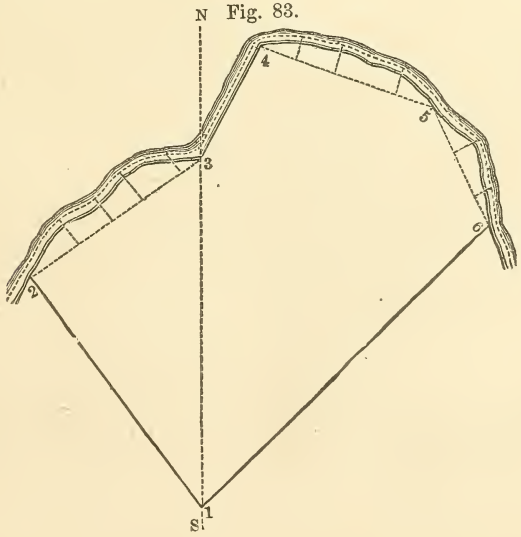
Fig. 82.




| | Left-hand off-sets on the stat line. Stat. | Dist. | Off-sets. |
|---|---|---|---|
| No. | 1. | 0.00 Ch. | 0.30 Ch. |
| | 2. | 0.95 | 0.84 |
| | 3. | 2.03 | 0.86 |
| | 4. | 3.28 | 0.50 |
| | 5. | 5.20 | 1.80 |
| | 6. | 7.43 | 2.35 |
| | 7. | 8.98 | 1.45 |
| | 8. | 10.46 | 1.08 |
| | 9. | 11.71 | 1.85 |
| | 10. | 14.45 | 0.35 |

The area of the part ABCD will be found, by the last problem, to be 13 A. 1 R. 11 P.

Fig. 83.



*To find the area of the off-sets.*

| No. | Sta. Dist. Ch. | Off-sets. Ch. | Intercep. Dist. | Sums of Off-sets. | Products. |
|---|---|---|---|---|---|
| 1 | 0.00 | 0.30 | | | |
| 2 | 0.95 | 0.84 | 0.95 | 1.14 | 1.0830 |
| 3 | 2.03 | 0.86 | 1.08 | 1.70 | 1.8360 |
| 4 | 3.28 | 0.50 | 1.25 | 1.36 | 1.7000 |
| 5 | 5.20 | 1.80 | 1.92 | 2.30 | 4.4160 |
| 6 | 7.43 | 2.35 | 2.23 | 4.15 | 9.2545 |
| 7 | 8.98 | 1.45 | 1.55 | 3.80 | 5.8900 |
| 8 | 10.46 | 1.08 | 1.48 | 2.53 | 3.7444 |
| 9 | 11.71 | 1.85 | 1.25 | 2.93 | 3.6625 |
| 10 | 14.45 | 0.35 | 2.74 | 2.20 | 6.0280 |
| | | | | | 2)37.6144 |

18.8072 Ch.
=1 A. 3 R. 21 P.

| | A. | R. | P. |
|---|---|---|---|
| Area of ABCD | 13 | 1 | 11 |
| Do. of off-sets | 1 | 3 | 21 |
| Whole area | 15 | 0 | 32 |

EXAMPLE 2. Fig. 83.

Required the area of a survey from the following field notes.

| | | Ch. | | Left hand off-sets. | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | 1st. Stationary Line. | | | 3d Stat Line. | | |
| | | | | Sta. No. | Dist. Ch. | Off-sets. Ch. | Sta. No. | Dist. Ch. | Off-sets. Ch. |
| 1 | N. $36\frac{3}{4}$° W | 30.00 | | 1. | 0.00 | 0.50 | 1. | 0.00 | 0.55 |
| 2. | N. $56\frac{1}{4}$ E. | 21.60 | stat. line. | 2. | 6.10 | 3.40 | 2. | 4.20 | 2.50 |
| 3. | N. $26\frac{1}{2}$ E. | 13.44 | Do. | 3. | 10.15 | 3.10 | 3. | 8.05 | 3.20 |
| 4. | S. $71\frac{1}{2}$ E. | 18.96 | Do. | 4. | 14.08 | 3.96 | 4. | 15.15 | 2.45 |
| 5. | S. $26\frac{1}{2}$ E. | 13.46 | Do. | 5. | 19.29 | 2.70 | 5. | 18.96 | 0.50 |
| 6. | S. 45 W. | 42.41 | | 6. | 21.60 | 0.55 | 4th Stat. Line. | | |
| | | | | 2d. Stat. Line. | | | 1. | 0.00 | 0.50 |
| | | | | 1. | 0.00 | 0.55 | 2. | 5.12 | 2.75 |
| | | | | 2. | 13.44 | 0.55 | 3. | 10.00 | 1.90 |
| | | | | | | | 4. | 13.46 | 0.70 |

The area within the stationary lines and straight sides, found by the last problem, is 1152.5381 square chains.

The area of the offsets is 134.4907 " "

Total area is 1287.0288 " "

or 128 A. 2 R. 32 P.

EXAMPLE 3.

The following field-notes of the survey of a pond are given, to find its area:

| Sides. | Angles. | Right-hand Off-sets. On AB. Dist. | Off-set. | On CD. Dist. | Off-set. |
|---|---|---|---|---|---|
| AB = 9.42 ch. | A = 73° 12′ | 1.00 | .35 | .34 | .41 |
| BC = 2.41 " | B = 88° 41′ | 1.59 | .26 | 1.00 | .00 |
| CD = 8.35 " | C = 112° 25′ | 2.25 | .50 | 2.15 | .68 |
| DA = 5.65 " | D = 85° 38′ | 4.17 | .46 | 3.75 | .00 |
| | | 4.58 | .54 | 4.40 | .00 |
| Stations A, B and C were | | 7.32 | .16 | 6.50 | .25 |
| on the edge of the pond. | | 7.66 | .06 | 8.00 | .00 |
| | | 8.25 | .11 | | |
| *Ans.* 3 A. 3 R. 37.43 P. | | 8.75 | .00 | | |
| | | 9.00 | .21 | | |

## PROBLEM XI.

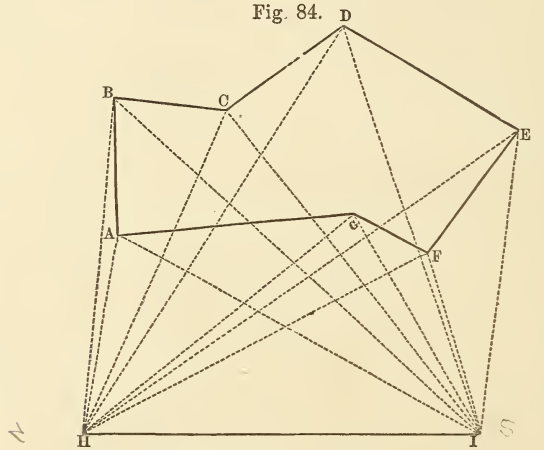
*Given the bearing and distance of two stations from each other and the bearings of all the corners of a tract of land from these stations, to find the area of the tract.*

The method of doing this will be best explained by an example.

EXAMPLE 1.

Let ABCDEFGA, Fig. 84, represent a field, all the angles of which can be seen from two stations, H and I, without it. The bearing and distance of the stations, and the bearings of all the angles of the field, from each station, being as follow, it is required to find the area.

Fig. 84.




The station H bears from the station I, North, dist. 28. Ch.

| | Bearings. | | Bearings. |
|---|---|---|---|
| HA | S. 81½° E. | IA | N. 28½° E. |
| HB | S. 85¾ E. | IB | N. 42¼ E. |
| HC | S. 68 E. | IC | N. 51½ E. |
| HD | S. 58¼ E. | ID | N. 71 E. |
| HE | S. 35½ E. | IE | S. 82½ E. |
| HF | S. 28½ E. | IF | N. 73½ E. |
| HG | S. 40 E. | IG | N. 60 E. |

*Construction.*

Draw HI according to the given bearing and distance; and from the points H and I, draw HA. HB. HC, &c.,

and IA, IB, IC, &c. according to the given bearings; then will the intersections A, B, C, &c. of the corresponding bearings HA and IA, HB and IB, HC and IC, &c. be the angular points of the field.

*Calculation.*

In each of the triangles IHA, IHB, IHC, &c. we have the side IH; and from the bearings of the sides, we have all the angles, to find the sides IA, IB, IC, &c.

Then in each of the triangles, IAB, IBC, ICD, &c. we have two sides, and the included angle; whence the areas may be found by prob. III.

From the sum of the areas of the triangles IAB, IBC, ICD, and IDE, which is equal to the area IABCDEI, subtract the sum of the areas of the triangles IAG, IGF and IFE, which is equal to the area IAGFEI; the remainder will be the area of the field ABCDEFGA.

*Note.*—In working the proportions for finding the sides IA, IB, &c. it will be unnecessary, when the area only is required, to take out the natural numbers corresponding to the logarithms of those sides; because in the proportions for finding the areas it will be sufficient to know the logarithms of the sides, without knowing their real lengths.

*Ans.* 21 A. 1 R. 8 P.

EXAMPLE 2.

Being required to calculate the area of a field, the owner of which refuses permission to go on it, I choose two stations, F and G, in the adjacent land, from whence all the angles of the field are visible. FG

was measured 20 chains, and the following angles were taken:

| | |
|---|---|
| GFA = 68½°, | FGA = 23°, |
| GFB = 24°, | FGB = 114°, |
| GFC = 38°, | FGC = 98¾°, |
| GFD = 59°, | FGD = 76½, |
| GFE = 103½° | FGE = 41°. |

What is the area? *Ans.* 33 A. 1 R. 7 P.

## PROBLEM XII.

*To find the area of a survey by protracting it, and dividing the plot into triangles and trapeziums.*

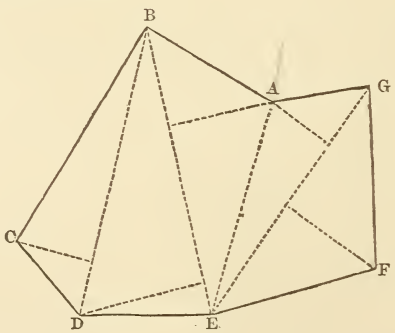
The method of doing this will be understood by the following example:

### EXAMPLE 1.

Given the bearings and distances as follows:

1. N. 50° W. 9.60 ch.;
2. S. 32° W. 16.38 ch.;
3. S. 41° E. 6.30 ch.;
4. East 8.43 ch.;
5. N. 79° E. 10.92 ch.;
6. N. 5° W. 11.25 ch.
7. S. 83° W. 6.48 ch.; to the place of beginning.

Required the area.




By drawing lines as in the plot it is divided into two trapeziums, AGFE, AEDB, and a triangle BDC.

Measure the several bases and perpendiculars, on the same scale that was used in the protraction, and find the double areas of the triangle and trapeziums by probs. 2 and 6; the sum of these will be the double area of the survey.

| Bases. | Perpens. | |
|---|---|---|
| EG 16.68 × | *Fa* 7.50, *Ab* 4.71 | = 203.6628 = 2 AGFE |
| EB 19.17 × | *Ac* 5.85, *Dd* 8.10 | = 267.4215 = 2 AEDB |
| BD 19.23 × | *Ce* 5.16 | = 99.2268 = 2 BDC |
| | | 2)570.3111 ch. = 2 ABCDEFG |
| | | 285.15555 ch. = 28A. 2R. 2P. |
| | | = the area required. |

EXAMPLE 2.

The following field-notes are given to protract the survey and find the area.

| | Ch. |
|---|---|
| 1. N. 15° 00′ E. | 20 |
| 2. N. 37° 30′ E. | 10 |
| 3. East | 7.50 |
| 4. S. 11° 00′ E. | 12.50 |
| 5. South | 13.50 |
| 6. West | 10. |
| 7. S. 36° 30′ W. | 10. |
| 8. N. 38° 15′ W. | 8.50 |

*Ans.* 46 A. 2 R. 9 P

## CHAPTER IV

## LAYING OUT AND DIVIDING LAND.

### PROBLEM I.

*To lay out a given quantity of land in a square form.*

RULE.

Reduce the given quantity to chains or perches, and extract the square root, which will be the length of a side, of the same denomination to which the given quantity is reduced.

EXAMPLES.

1. Required the side of a square that shall contain **9 A. 3 R. 28 P.**

40)28 Per.

4)3.7 R.

9.925 A.=99.25 ch.

Ch.
99.25(9.96 ch. the length of a side.
81

189)1825
1701

1986)12400
11916

484

2. Required the side of a square tract of land that shall contain 325 acres. *Ans.* 57 chains.

## PROBLEM II.

*To lay out a given quantity of land in a rectangular form, having one side given.*

### RULE.

Divide the given content by the length of the given side, the quotient will be the length of the required side.

### EXAMPLES.

1. It is required to lay out 120 acres in a rectangular form, the length of one side being given, equal 100 perches.

Acres.
120
4
480
40
1,00)192,00
192 Per. the length of the other side

2. The length of a rectangular piece of land is 8 chains; what must be its breadth, that the content may be 5 acres?

*Ans.* 6.25 chains.

## PROBLEM III.

*To lay out a given quantity of land in a rectangular form, having the length to the breadth in a given ratio.*

RULE.

As the less number of the given ratio,
Is to the greater;
So is the given area,
To a fourth term.*

The square root of this fourth term will be the length required. Having the length, the breadth may be found by the preceding problem. Or it may be found in the same manner as the length. Thus,

As the greater number of the given ratio,
Is to the less;
So is the given area,
To a fourth term.

The square root of this fourth term will be the breadth required.

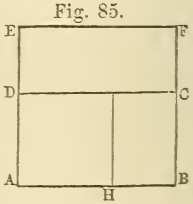
EXAMPLES.

1. It is required to lay out 864 acres in a rectangular form, having the length to the breadth in the ratio of 5 to 3.

---

* Demonstration. Let ABCD, Fig. 85, be a rectangle, and ABFE and AHGD be squares on the greater and less sides respectively: then (V. I) AD : AE (AB) : : the rectangle AC : square AF. Also AB : AH (AD) : : the rectangle AC : square AG. Hence the truth of the rule is evident.

Fig. 85.



864 A. = 138240 P.

Sq. P. Sq. P.

As 3 : 5 : : 138240 : 230400

$\sqrt{230400} = 480$ Perches, the length required.

Sq. P. Sq. P.

As 5 : 3 : : 138240 : 82944

$\sqrt{82944} = 288$ Perches, the breadth required.

2. It is required to lay out 27 A. 3 R. 20 P. in a rectangular form, having the length to the breadth in the ratio of 9 to 7. *Ans.* Length 75.725 P. Breadth 58.897 P.

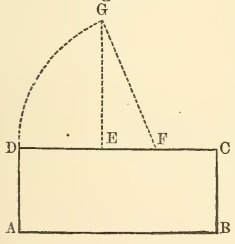
## PROBLEM IV.

*To lay out a given quantity of land in a rectangular form, having the length to exceed the breadth by a given difference.*

RULE.

To the given area, add the square of half the given difference of the sides, and extract the square root of the sum; to this root, add half the given difference for the greater side, and subtract it therefrom for the less.*

Fig. 86.




* Demonstration. Let ABCD, Fig. 86, be a rectangle; in DC let DE be taken equal DA or BC, and let EC be bisected in F; then (II. 6) $DF^2 = DC \times DE + FC^2 = DC \times AD + FC^2$ = the rectangle AC + the square of half the difference of the sides DC, DA; also DF + FC = DC, the greater side, and DF — FC = DE or DA, the less side.

This problem may be neatly constructed thus: take EC equal the given difference of the sides and bisect it in F; make EG perpendicular to EC and equal to the square root of the given area, and with the centre F and radius FG describe the arc DG meeting CE produced in D: make DA perpendicular to DC and equal to DE, and complete the rectangle ABCD which will be the one required. Since (I. 42) $FG^2 = EG^2 + EF^2$ = the given area + the square of half the given difference of the sides, the truth of the construction is plain from the preceding demonstration.

EXAMPLES.

1. It is required to lay out 47 A. 2 R. 16 P. in a rectangle, of which the length is to exceed the breadth by 80 perches.

| | |
|---|---|
| 2)80 P. | 47 A. 2 R. 16 P.=7616 Per. |
| — | 1600 |
| 40 | —— |
| 40 | $\sqrt{9216}=96$ |
| —— | half diff. add and subtract 40 |
| 1600 | — |
| | length 136 |
| | —— |
| | breadth 56 |

2. It is required to lay out 114 A. 2 R. 33.4 P. in a rectangular form, having the length to exceed the breadth by 15.10 ch. *Ans.* Length 42.25 ch. Breadth 27.15 ch.

## PROBLEM V.

*To lay out a given quantity of land in the form of a triangle or parallelogram, one side and an adjacent angle being given.*

RULE.

*For a triangle.*

As the rectangle of the given side and sine of the given angle,
Is to twice the given area:
So is radius,
To the other side, adjacent to the given angle.

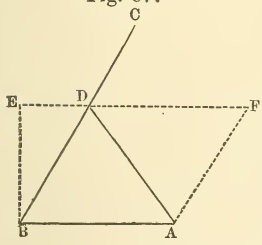
Then having two sides and the included angle given, the other angles and side, if required, may be found by trig. case 3.

*For a parallelogram.*

As the rectangle of the given side and sine of the given angle,
Is to the given area;
So is radius,
To the other side, adjacent to the given angle.*

EXAMPLES.

Fig. 87.




1. Let AB, BC, Fig. 87, be two sides of a tract of land; the bearing of AB is S. 87½° W. dist. 16.25 ch. and the bearing of BC, N. 27½° E.; it is required to lay off 10 acres by a straight line AD, running from the point A, to the side BC.

Bearing of BA, N. 87½° E.
——— BC, N. 27½ E.

Angle B, 60°

| | | |
|---|---|---|
| As AB × sin. B | AB 16.25 ch. - - | Ar. Co. 8.789147 |
| | sin. B, 60° - - - | ——— 0.062469 |
| : twice the given area 200 sq. ch. - - - | | 2.301030 |
| : : rad. - - - - - - - - - - - - | | 10.000000 |
| : BD 14.21 ch. - - - - - - - - - - | | 1.152646 |

* Demonstration.—It is demonstrated, prob. 3, chap. 3, Content of Land, that rad. : sin. B : : AB × BD : 2ABD (see Fig. 87); therefore rad. × AB : sin. B × AB : : AB × BD : 2ABD, or sin. B × AB : 2ABD : : rad. × AB : AB × BD : : rad. : BD. Since ABDF is equal to 2ABD, the truth of the rule for the parallelogram is evident.

This problem may be constructed as follows; take AB equal the given side and draw BC making the angle B equal to the given angle; make BE perpendicular to AB, and equal twice the given area of the triangle divided by the given side, or equal the given area of the parallelogram divided by the given side; and parallel to AB, draw EF cutting BC in D, and join DA; then will ABD be the triangle required; or complete the parallelogram ABDF, for the one required. The reason of the construction is plain.

2. Given the side AB, *Fig.* 15, of a parallelogram, equal 20 ch. and the angle A 63° 30′; required the side AC, that the content may be 21½ acres.

| | | |
|---|---|---|
| As AB × sin A | { AB 20 ch. - - - | Ar. Co. 8.698970 |
| | { sin. A, 63° 30′ - - | 0.048209 |
| : the given area 215 sq. ch. - - - - - - | | 2.332438 |
| : : rad. - - - - - - - - - - - - - | | 10.000000 |
| : AC 12.01 ch. - - - - - - - - - - | | 1.079617 |

3. Given one side of a triangle, equal 30 perches, an angle adjacent to this side 71° 15′, and the area 2 acres; required the other side adjacent to the given angle.

*Ans.* 22.53 perches.

4. Given one side of a parallelogram, equal to 32.26 ch., an angle adjacent to this side 83° 30′, and the area 74 acres; required the other side adjacent to the given angle. *Ans.* 23.09 ch.

## PROBLEM VI.

*The area and base of a triangle being given, to cut off a given part of the area by a line running from the angle opposite the base.*

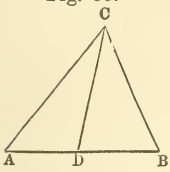
### RULE.

As the given area of the triangle,
Is to the area of the part to be cut off;
So is the given base,
To the base corresponding to that area.*

* The truth of the rule is manifest from V. 1, Cor. 2.

EXAMPLES.

Fig. 88.




1. Given the area of the triangle ABC, Fig. 88, equal 650 square perches, and the length of the base AB, 40 perches; it is required to cut off 290 perches towards the angle A, by a line running from the angle C to the base.

ABC. ADC. AB. AD.
As 650 : 290 : : 40 : 17.85 per.

2. In a triangle ABC, there are given the area 27 A. 1 R. 16 P. and the base AB 35.20 ch., to cut off 10 acres towards the angle B, by a line CD running from the angle C to the base: the part BD of the base is required.

*Ans.* 12.87 ch.

## PROBLEM VII.

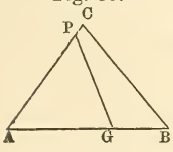
*The area and two sides of a triangle being given, to cut off a triangle containing a given area, by a line running from a given point in one of the given sides, and falling on the other.*

RULE.

As the given area of the triangle,
Is to the area of the part to be cut off;
So is the rectangle of the given sides,
To a fourth term.

Divide this fourth term by the distance of the given point from the angular point of the two given sides; the quotient will be the distance of the required point from the same angle.*

Fig. 89.




* DEMONSTRATION.—From the demonstration to prob. 3, chap. 3, we have, Fig. 89, rad. : sin. A : : AB × AC : 2ABC, and rad. : sin. A : : AP × AG : 2APG; therefore 2ABC : 2APG : : AB × AC : AP × AG, or ABC : APG : : AB × AC : AP × AG; hence the truth of the rule is manifest.

EXAMPLES.

1. Given the area of the triangle ABC, *Fig.* 89, 5 acres; the side AB 50 perches, the side AC 40 perches, and the distance of a point P from the angle A, 36 perches; it is required to find a point G to which, if a line be drawn from the point P, it shall cut off a triangle APG containing 3 A. 0 R. 20 P.

| | | | |
|---|---|---|---|
| As the triangle ABC | 800 sq. p. | Ar. Co. | 7.096910 |
| : the triangle APG | 500 - - - - - - - | | 2.698970 |
| :: AB × AC { AB | 50 - - - - - - - | | 1.698970 |
| :: AB × AC { AC | 40 - - - - - - - | | 1.602060 |
| : AP × AG - - - - - - - - - - | | | 3.096910 |
| AP 36 - - - - - - - | | log. | 1.556303 |
| AG 34.72 per. - - - - - - - | | | 1.540607 |

2. Given the area of a triangle ABC, 12 A. 1 R. 23 P. the side AB 20 ch., the side AC 16.25 ch., and the distance of a point P in the side AB, from the angle A 8.50 ch.; it is required to find the distance AG of a point G in the line AC, so that a line drawn from P to G may cut off a triangle APG containing 3 acres. *Ans.* 9.25 ch.

## PROBLEM VIII.

*The area and base of a triangle being given, to cut off a triangle containing a given area, by a line running parallel to one of the sides.*

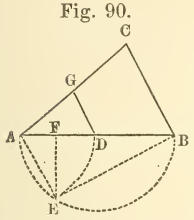
RULE.

As the given area of the triangle,
Is to the area of the triangle to be cut off;
So is the square of the given base,
To the square of the required base.

The square root of the result will be the base of the required triangle.*

EXAMPLES.

Fig. 90.




1. Given the area of the triangle ABC, Fig. 90, 500 square perches, and the base AB 40 perches; it is required to cut off 120 sq. per. towards the angle A, by a line DG running parallel to the side BC.

| | | | | |
|---|---|---|---|---|
| As the triangle ABC | | 500 | - | Ar. Co. 7.301030 |
| : the triangle ADG | | 120 | - - - | - 2.079181 |
| :: $AB^2$ | AB | 40 | - - - | - 1.602060 |
| | AB | 40 | - - - | - 1.602060 |
| : $AD^2$ - - - - - - - - - - | | | | 2)2.584331 |
| AD 19.6 per. - - - - - - - - | | | | 1.292165 |

2. Given the area of a triangle ABC, 10 acres, and the base AB 25 ch., to find BD a part of the base, so that a line DG running from the point D, parallel to the side AC, may cut off a triangle BDG containing $4\frac{1}{2}$ acres. *Ans.* BD = 16.77 ch.†

* The truth of this rule is manifest from V. 19.

This problem may be neatly *constructed* as follows: Let ABC, Fig. 90, be the given triangle, and AB the given base; on AB describe the semicircle AEB, and take AF to AB in the ratio of the part to be cut off, to the whole triangle; draw FE perpendicular to AB, meeting the semicircle in E, join AE, and make AD equal to AE; from D draw DG parallel to BC, and the thing is done. For, join EB, and we have, by similar triangles, AB : AE : : AE : AF, or $AB^2 : AE^2$ : : AB . AE : AE . AF : : AB : AF.

But (V. 19) $AB^2 : AE^2(AD^2)$ : : ABC : ADG;

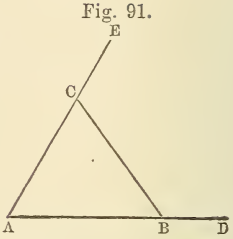
$\therefore$ AB : AF : : ABC : ADG.

† If it be required to produce two sides of a given triangle so far that the triangle formed by these sides produced, and a line drawn between them parallel to the third side, may contain a given area, it may be done by the above rule. Thus, Fig. 90. ADG : ABC : : $AD^2 : AB^2$.

## PROBLEM IX.

*The bearings of two adjacent sides* AD, AE, Fig. 91, *of a tract of land being given, to cut off a triangle* ABC *containing a given area by a line* BC *running a given course.*

Fig. 91.




RULE.

From the given bearings of the lines, find the angles A, B, and C; then,

As the rectangle of the sines of the angles A and B,
Is to the rectangle of radius and sine of the angle C;
So is twice the given area,
To the square of the side AB.*

In like manner the other sides may be found; or having found one side, the others may be found by trig. case 1.

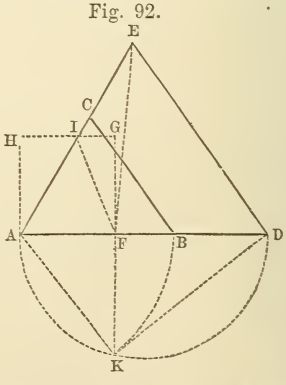
EXAMPLES.

1. Let the bearing of AD, Fig. 91, be N. 87° 30′ E. and of AE, N. 27° 30′ E.; it is required to cut off 10 acres by a line BC running N. 38° W.

---

* The truth of this rule is evident from the demonstration to prob. 4, chap. 3.

Fig. 92.




*Construction.* Draw AD, AE, (Fig. 92,) according to the given bearings, and in AD take AF equal the square root of the given area, and on it describe the square AFGH; make IE = AI, and draw ED, according to the reverse bearing of the division line BC, meeting AD in D; on AD describe a semicircle, and produce GF to meet it in K, join AK and make AB equal to it; draw BC parallel to DE, and ABC will be the triangle required. For join IF, EF and KD; then (III. 21 and V. 8, Cor.) AD : AK (AB) : : AK (AB) : AF; or (last proof) AD : AF : : ADE : ABC; but (V. 1) AD : AF : : ADE : AFE; therefore ADE : ABC : : ADE : AFE, and consequently ABC = AFE; but because AI = IE, AFE = 2AFI = AFGH; therefore ABC = AFGH = the given area of the triangle.

| AD, N. 87° 30′ E. | BA, S. 87° 30′ W. | CA, S. 27° 30′ W. |
|---|---|---|
| AE, N. 27 30 E. | BC, N. 38 00 W. | CB, S. 38 00 E. |

Angle A, 60 00 — 125 30 — Angle C, 65 30

180 00

Angle B, 54 30

As sin. A × sin. B, { A 60° 00′ — Ar. Co. 0.062469; B 54 30 — 0.089314

: rad. × sin. C, { C 65 30 - - - - - 9.959023; rad. - - - - - - 10.000000

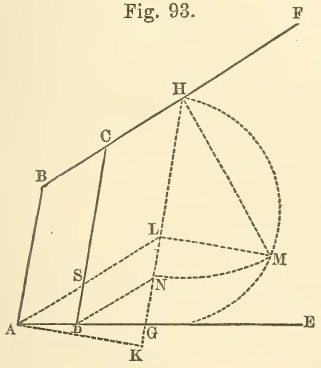
:: twice the given area, 200 sq. ch. - - 2.301030

: $AB^2$ - - - - - - - - - - - - 2)2.411836

AB 16.07 - - - - - - - - - - - 1.205918

2. Given the bearing of one side of a tract of land, S. 53° 15′ E., and the bearing of an adjacent side taken at the same angle, N. 55° 00′ E., to cut off 4 acres by a line running N. 4° 00′ W.; required the distance on the first side. *Ans.* 9.76 ch.

Fig. 93.




## PROBLEM X.

*The bearings of three adjacent sides*, EA, AB, BF, Fig. 93 *or* 94, *of a tract of land, and the length of the middle side* AB, *being given, to cut off a trapezoid* ABCD, *containing a given area, by a line* DC, *parallel to* AB.

RULE.

From the given bearings find the angles A and B; add these together, and take the difference between their sum and 180°, and call it P. Then,

As the product of the sines of A and B,
Is to the product of radius and sine of P;

So is twice the area to be cut off,
To a fourth term.

When the sum of the angles A and B is greater than 180°, add this fourth term to $AB^2$; but when the sum of these angles is less than 180°, take the difference between this fourth term and $AB^2$. The square root of the result will be DC. Then,

As the sine of P,
Is to the sine of B,
So is the difference between DC and AB,
To AD.*

---

*Demonstration.—Produce EA and FB, Fig. 95, to meet in P. Then (V. 19) PDC : PAB : : $CD^2$ : $AB^2$, or (IV. 10) ABCD : PAB : : $CD^2-AB^2$ : $AB^2$, or PAB : ABCD : : $AB^2$ : $CD^2-AB^2$, or 2PAB : $AB^2$ : : 2ABCD : $CD^2-AB^2$. But by the demonstration to prob. 4, chap. 3, 2PAB : ($AB^2$) : : sin. A × sin. B : rad. × sin. P. Consequently,

sin. A × sin. B : rad, × sin. P : : 2ABCD : $CD^2$—$AB^2$.

Fig. 94.

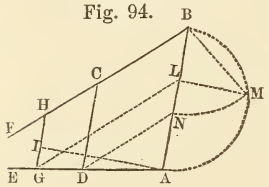
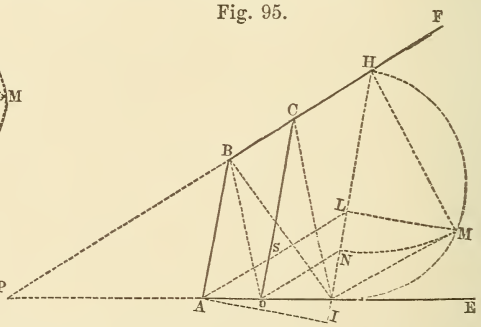



Fig. 95.




Now it is plain that $AB^2$, added to this 4th term, gives $CD^2$. A similar demonstration applies when the sum of the angles A and B is less than 180°, as in Fig. 94. The latter part of the rule does not require demonstration.

*Construction.*—Draw AI perpendicular to AB, and make it equal to the quotient of twice the given area divided by AB. From I, draw IH parallel to AB meeting AE and BF, in G and H, and on GH, describe the semicircle GMH. From A, draw AL parallel to BF; and make LM perpendicular to GH. With the distance HM and centre H, describe the arc MN; and from N, draw ND parallel to AL. Lastly, draw DC parallel to AB, and it will be the division line required. For join BD, BG, and CG, Fig. 95. Then by similar triangles PG : PD : : GH : DC : : GH : HM : : HM : HL : : DC : AB : : PC : PB. Hence (V. 2) CG is parallel to BD; and consequently the triangle BDC is equal to BGD. To each of these, add ABD. Then we have ABCD = ABG. But it is plain from the construction that ABG is equal to the given area. Hence ABCD is equal to the given area.

When the sum of the angles A and B is *less* than 180°, as in Fig. 94,

EXAMPLES.

1. Given the bearing of EA, Fig. 93, West, AB, N. 10° E. dist. 15 ch.; and BF, N. 58° 30′ E. to cut off 10 acres by a line CD, running parallel to AB. Required the length of the division line and the distance AD.

| AE, N. 90° E. | BF, N. 58° 30′ E. | A, 80° 0′ |
|---|---|---|
| AB, N. 10 E. | BA, S. 10 0 W. | B, 131 30 |
| A = 80° | 48° 30 | 211 30 |
| | 180 0 | 180 0 |
| | B=131 30 | P=31° 30′ |

As sin. A × sin. B, { A, 80° 00′ Ar. Co. 0.006649
B, 131 30 ——— 0.125544
: rad. × sin. P. { Rad. - - - - - - 10.000000
P, 31 30 - - - 9.718085
:: twice the given area 200 sq. ch. - - - 2.301030
: fourth term 141.68 - - - - - - 2.151308

$AB^2 = 225.$

$DC = \sqrt{366.68} = 19.15$ ch.

As sin. P, 31° 30′ - - - Ar. Co. 0.281915
: sin. B, 131 30 - - - - - - - 9.874456
:: DC − AB, 4.15 - - - - - - - 0.618048
: AD 5.95 - - - - - - - 0.774419

---

the semicircle must be described on AB; the point L must be determined by drawing GL parallel to FB; and the arc MN must be described with the radius BM and centre B. The other parts of the construction are the same as before.

2. Given the bearings of three adjacent sides of a tract of land and the length of the middle one as follows: 1st. N. 20° W.; 2d. N. 60° 30′ E. dist. 6 ch.; 3d. S. 61° 30′ E.; to cut off a lot containing 2½ acres, by a line parallel to the 2d side. Required the length of the division line and the distance on the 1st side.

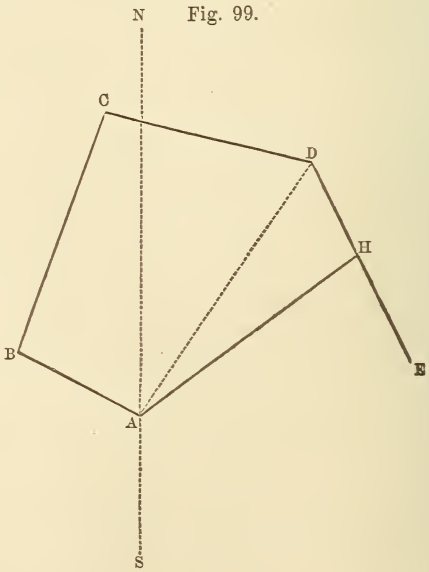
*Ans.* Division line 8.70 ch.; distance on 1st side 3.45 ch.

3. Given as follows: 1st side N. 31° 15′ W.; 2d N. 58° 45′ E. dist. 13.50 ch.; 3d S. 14° 45′ E.; to cut off 8 acres by a line parallel to the 2d side. The length of the division line and the distance on the 1st side are required. *Ans.* Division line 11.61 ch.; distance on the 1st side 6.38 ch.

## PROBLEM XI.

*The bearings of several adjacent sides,* AB, BC, CD, DE, Fig. 99, *of a tract of land, and the distance of each, except the last, being given, to cut off a given area by a line* AH *running from the angle* A, *and falling on the side* DE.

Fig. 99.




RULE.

Take out the latitudes and departures of the sides whose bearings and distances are known. Find the difference of the sums of the northings and

southings, and of the eastings and westings. The remainders will be the latitude and departure of the closing side AD.

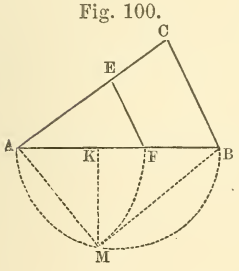
With this latitude and departure calculate the bearing and distance.

Calculate also the area of the figure cut off by this closing line, and subtract it from the given area. The remainder will be the area to be laid off in a triangular form by Problem V., page 160.

The angle DHA applied to the bearing of DE will give the bearing of HA.

## PROBLEM XII.

Fig. 100.




*The sides* AB, BC, CA, Fig. 100, *of a triangular piece of ground being given, to divide it into two parts having a given ratio, by a line* FE, *running parallel to one of the sides as* BC.

RULE.

As the sum of the numbers expressing the ratio of the parts,
Is to that number of the ratio which corresponds to the part to be adjacent to A;
So is the square BC,
To the square of FE.

Then, As BC : AB : : FE : AF.*

---

* Demonstration. Let $m$ to $n$ be the ratio of the part AFE to the part FECB; then (IV. 9) $m+n : m$ : : ABC : ADE : (V. 19) $BC^2 : FE^2$.

*Construction.* On AB describe the semicircle AMB, and by Prob. 16 Page 36, divide AB in K, so that AK may be to KB in the given ratio of the part AFE to the part FECB; draw KM perpendicular to AB, meeting the

EXAMPLES.

1. Let AB be 21.26 ch.; BC, 12.76 ch.; and AC, 19.30 ch.; it is required to divide the triangle by the line FE, parallel to BC, so that the part AFE may be to the part FECB as 2 to 3.

As $5 : 2 :: 12.76^2 : FE^2 = 65.12704$.
$FE = \sqrt{65.12704} = 8.07$.
As $12.76 : 21.26 :: 8.07 : AF = 13.45$.

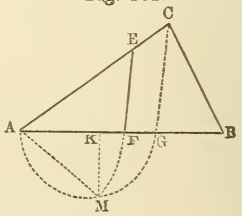
2. The three sides of a triangular piece of land, taken in order, measure 15, 10, and 13 chains respectively; it is required to divide it into two equal parts by a line parallel to the second side. What will be the length of the division line and its distance from the place of beginning, measured on the first side?

*Ans.* Division line 7.07 ch.; dist. on 1st side 10.61 ch

## PROBLEM XIII.

*The bearings and distances of the sides* AB, BC, CA, Fig. 101, *of a triangular piece of ground being given, to divide it into two parts having a given ratio, by a line* FE, *running a given course.*

Fig. 101.




RULE.

As the product of the sines of F and E,
Is to the product of the sines of B and C;

---

semicircle in M, and with the radius AM and centre A, describe the arc MF. From F draw the division line FE parallel to BC. Since AB : AM (AF) :: AM (AF) : AK, we have (see Demonstration, page 165) AB : AK :: $AB^2$ : $AF^2$ :: ABC : AFE. Hence the truth of the construction is evident.

So is the square of BC,

To a fourth term.

Multiply this fourth term by that number of the ratio which corresponds to the part to be adjacent to the angle A, and divide the product by the sum of the numbers expressing the ratio. The square root of the result will be FE. Then,

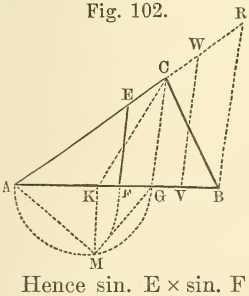
As sin. A : sin. E : : FE : AF.*

EXAMPLES.

1. Let the bearing of AB, be S. 82¼° E. dist. 14.17 ch.; BC, N. 18¾° W. 8.51 ch.; and CA, S. 61½° W. dist. 12.87 ch.; it is required to divide the triangle by the line FE, running N. 14¾° E. so that the part AFE may be to the part FECB in the ratio of 2 : 3.

---

Fig. 102.




* Demonstration.—Draw CG and BR, Fig. 102, parallel to EF; and let VW, also parallel to EF, make the triangle AVW equal to ABC. Then (V. 15, cor.) AB.AC = AV.AW; hence AB : AV : : AW : AC, but (V. 4) AB : AV : : BR : VW, and AW : AC : : VW : CG. ∴ BR . GC = VW². Now,

As sin. R (sin. E) : sin. C : : BC : BR,
sin. G (sin. F) : sin. B : : BC : GC.

Hence sin. E × sin. F : sin. B × sin. C : : BC² : BR × GC : : BC² : VW².

Also, $m+n : m$ : : ABC : AFE : : AVW : AFE : : (V. 19) VW² : FE².

*Construction.*—From C, Fig. 101, draw CG according to the reverse bearing of FE, and on AG describe the semicircle AMG. By prob. 16, page 36, divide AB in K, so that AK may be to KB in the given ratio of the part AFE to the part FECB. Draw KM perpendicular to AB, and with the radius AM and centre A describe the arc MF. From F, and parallel to GC, draw FE, the required division line. For join KC; Fig. 102. (V. 1, cor. 2) KC divides the triangle in the given ratio. Now AC : AE : : AG : AF (AM) : : (V. 8, cor.) AM (AF) : AK; therefore AC : AE : : AF : AK, and hence AC . AK = AE . AF. ∴ (V. 15, cor.) AFE = AKC. Consequently FE divides the triangle in the given ratio.

Angle A=36¼°, B=63½°, C=80¼°, E=46¾° and F=83°.

| | | | |
|---|---|---|---|
| As sin. E × sin. F, | E, 46° 45′ | Ar. Co. | 0.137647 |
| | F, 83 00 | | 0.003249 |
| : sin. B × sin. C, | B, 63 30 | | 9.951791 |
| | C, 80 15 | | 9.993681 |
| :: $BC^2$ | BC, 8.51 | | 0.929930 |
| | BC, 8.51 | | 0.929930 |
| : fourth term | 88.35 | | 1.946228 |
| | 2 | | |
| | 5)176.70 | | |

FE = √35.34 = 5.95

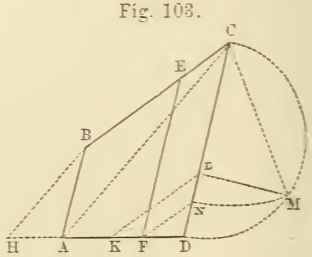
| | | | |
|---|---|---|---|
| As sin. A, 36° 15′ | | Ar. Co. | 0.228185 |
| : sin. E, 46° 45′ | | | 9.862353 |
| :: FE. 5.95 | | | 0.774517 |
| : AF, 7.33 | | | 0.865055 |

2. The bearings and distances of a triangular piece of land ABC are, AB. S. 69° E. 21.40 ch.; BC, N. 31½ E. 18.66 ch.; and CA, S. 74½ W. 30.85 ch.; and it is required to divide it by a line FE, running due north, so that the part AEF, may be to the part FECB, as 4 to 5. What will be the length of the division line FE, and the distance AF? *Ans.* FE 10.74, and AF 17.40.

## PROBLEM XIV.

*The bearings and distances of the sides* AB, BC, CD, DA, Fig. 103, *of a trapezoidal tract of land being given, to divide it into two parts having a given ratio, by a line* FE, *running parallel to the parallel sides* AB, CD.

Fig. 103.




RULE.

Multiply the square of AB, by that number of the

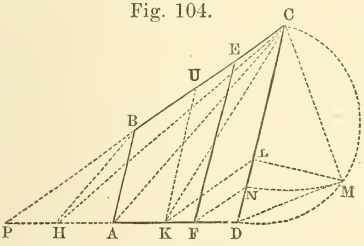
ratio which corresponds to the part, to be adjacent to CD, and the square of CD, by the other number of the ratio. Add the two products together, and divide the sum by the sum of the numbers expressing the ratio. The square root of the quotient will give FE. Then,

As DC—AB : FE—AB : : AD : AF.*

EXAMPLES.

1. Let the bearing of AB be N. 14° E. dist. 10 ch. BC, N. 55¼° E. dist. 18.67 ch.; CD, S. 14° W. dist. 20.98 ch.; and DA, W. 12.70 ch.; it is required to divide the trapezoid into two parts by a line FE, parallel to AB or DC,

Fig. 104.




* Demonstration. — Produce DA and CB, Fig. 104, to meet in P. Then (V. 19) PDC : PAB : : $CD^2 : AB^2$, or (IV. 10) ABCD : PAB : : $CD^2 - AB^2 : AB^2$, or ABCD : $CD^2 - AB^2$ : : PAB : $AB^2$. In like manner ABEF : $FE^2 - AB^2$ : : PAB : $AB^2$. Hence (IV. 15) ABCD : $CD^2 - AB^2$ : : ABEF : $FE^2 - AB^2$, or ABCD : ABEF : : $CD^2 - AB^2 : FE^2 - AB^2$. Therefore $m+n : m$ : : $CD^2 - AB^2 : FE^2 - AB^2$, or multiplying extremes and means, $\overline{m+n}$. $FE^2 - \overline{m+n}$. $AB^2 = m$. $CD^2 - m$. $AB^2$. But $(m+n)$ $AB^2 = m$. $AB^2 + n$. $AB^2$. Therefore, adding equals to equals, we have $\overline{m+n}$. $FE^2 = m$. $CD^2 + n$. $AB^2$. Hence the truth of the rule is evident.

*Construction.*—Join CA, Fig. 103, and parallel to it draw BH, meeting DA produced in H. Divide (prob. 16, page 36) HD in K, so that HK may be to KD in the given ratio of ABEF to FECD, and draw KL parallel to BC. On CD describe the semicircle CMD, and draw LM perpendicular to CD. With the radius CM and centre C, describe the arc MN, and from N draw NF parallel to KL. From F draw the division line FE parallel to AB or CD. For join KC, Fig. 104, and draw KU parallel to AB. Then since BH is parallel to AC, the triangle AHC is equal to ABC; and adding ADC to each, we have CHD = ABCD. Now (V. 4) PC : PE : : CD : FE (CM) : : CM (EF) : CL (UK) : : PF : PK. Therefore (IV. 15) PC : PE : : PF : PK; and hence (V. 15, cor.) the triangle PEF is equal to PCK. Consequently CKD = FECD. But it has been proved that CHD = ABCD; hence taking equals from equals we have HCK = ABEF. But HCK : CKD : : HK : KD : : $m : n$. Therefore ABEF : FECD : : $m : n$.

so that the part ABEF may be to the part FECD as 3 to 2.

2. $AB^2 = 200$
3. $CD^2 = 1320.4812$

$5)1520.4812$

$FE = \sqrt{304.0962} = 17.44$

As 10.98 : 7.44 : : 12.70 : AF = 8.61

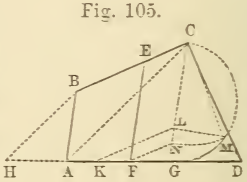
2. The boundaries of a trapezoidal field ABCD are given as follow; viz. AB, N. 80° W. 60 per.; BC, N. 39½° W. 45.5 per.; CD, S. 80° E. 89.4 per.; and DA, South, 30 per.; and it is required to divide it into two equal parts by a line FE parallel to AB or CD. What will be the length of the division line FE, and the distance AF?

*Ans.* FE 76.13 per., and AF 16.46 per.

## PROBLEM XV.

*The bearings and distances of the sides* AB, BC, CD, DA, Fig. 105, *of any quadrilateral tract of land being given, to divide it into two parts having a given ratio, by a line* FE, *running parallel to one of the sides as* AB *or* CD.

Fig. 105.




RULE.

Call the side to which the division line is to be parallel, the *parallel* side; and the one opposite to this, the *opposite* side. From the bearings, find the angles. Take the difference between the sum of the angles adjacent to the parallel side and 180°, and call it P. Then,

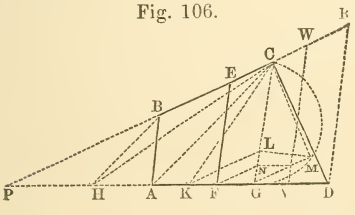
As the product of the sines of the angles adjacent to the parallel side,
Is to the product of the sines of the angles adjacent to the opposite side:
So is the square of the opposite side,
To a fourth term.

Multiply this fourth term by that number of the ratio which corresponds to the part to be adjacent to the parallel side, and to the product add the product of the square of the parallel side by the other number of the ratio; and divide the sum by the sum of the numbers expressing the ratio. The square root of the quotient will be the length of the division line FE. Then,

As the sine of P,
Is to the sine of E;
So is the difference between FE and the parallel side,
To the distance of F from the adjacent end of the parallel side.*

---

Fig. 106.




*Demonstration. — Produce DA and CB, Fig. 106, to meet in P. Draw DR and CG each parallel to AB; and let VW, also parallel to AB, make the triangle PVW equal to PCD. Then (V. 15, cor.) PD : PV :: PW : PC. But (V. 4) PD : PV :: DR : VW, and PW : PC :: VW : CG. Therefore DR : VW :: VW : CG; and hence $DR \times CG = VW^2$. But by trigonometry,

As sin. CRD (sin. B) : sin. C :: CD : DR,
sin. CGD (sin. A) : sin. D :: CD : CG.

Hence sin. A × sin. B : sin. C × sin. D :: $CD^2 : DR \times CG$.
Or, sin. A × sin. B : sin. C × sin. D :: $DC^2 : VW^2$.

But by the demonstration to the rule in the last problem, we have

$$\overline{m+n}.\ FE^2 = m.\ VW^2 + n.\ AB^2.$$

Hence the truth of the rule is evident.

*Construction.* From C, Fig. 105, draw CG parallel to AB, and on it describe the semicircle CMG.

EXAMPLE.

1. Let the bearing of AB be North, 12 ch.; BC, N. $56\frac{1}{2}°$ E. 20.78 ch.; CD, S. $33\frac{1}{2}°$ E. 22.21 ch.; and DA, S. $80\frac{1}{2}°$ W. 30 ch.; it is required to divide the tract into two parts by a line FE, parallel to AB, so that the part ABEF may be to the part FECD as 3 to 5.

Angle A = $80\frac{1}{2}°$, B = $123\frac{1}{2}°$, C = 90°, D = 66°, and P = 24°.

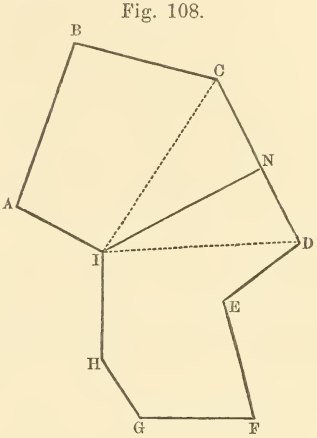
| | | | |
|---|---|---|---|
| As sin. A × sin. B, | A, 80° 30′ | Ar. Co. | 0.005997 |
| | B, 123 30 | | 0.078893 |
| : sin. C × sin. D, | C, 90 00 | - - - | 10.000000 |
| | D, 66 00 | - - - | 9.960730 |
| :: $CD^2$ | CD, 22.21 | - - - | 1.346549 |
| | CD, 22.21 | - - - | 1.346549 |
| : fourth term | 547.92 | - - - - - - | 2.738718 |
| | 3 | | |
| | 1643.76 | | |
| | $5AB^2 = 720.00$ | | |
| | 8)2363.76 | | |
| | $FE = \sqrt{295.47} = 17.19$ | | |
| As sin. P, 24° 00′ | - - - - - | Ar. Co. | 0.390687 |
| : sin. E, 123° 30′ | - - - - - - - - | | 9.921107 |
| :: FE − AB, 5.19 | - - - - - - - - | | 0.715167 |
| : AF, 10.64 | - - - - - - - - | | 1.026961 |

Join CA, and draw BH parallel to it meeting DA produced in H. Divide HD in K, so that HK may be to KD in the given ratio of the part ABEF to FECD. Draw KL parallel to BC and LM perpendicular to CG. With the radius CM and centre C describe the arc MN. Draw NF parallel to KL and FE parallel to AB. Then will FE be the division line. When the division line is to be parallel to CD, the semicircle must be described on CD, and the line CG need not be drawn. The demonstration of the construction is the same as for the last problem.

## PROBLEM XVI.

*The boundaries of a tract of land* ABCDEFGHIA, Fig. 108, *being given, to divide it into two equal parts by a line* IN *running from the corner* I, *and falling on the opposite side* CD.

Fig. 108.




RULE.

Suppose lines drawn from I, to C and D, and calculate the area of the whole tract.

Take the corrected latitudes and departures of IA, AB, and BC, and by balancing find the latitude and departure of CI; also calculate the area of the part IABCI; from half the area of the whole tract, subtract the area of the part IABCI, the remainder will be the area of the triangle ICNI.

Lay off this area by Problem V., page 160. The angle CNI applied to the bearing of CN will give the bearing of CI.

1. Given the boundaries of a tract of land as follows: *viz.* 1st. S. 35¼° W. 11.20 ch.; 2d. N. 45° W. 24.36 ch.; 3d. N. 15½° E. 10.80 ch.; 4th. S. 77° E. 16 ch.; 5th. N. 87½° E. 21.50 ch.; 6th. S. 60° E. 14.80 ch.; 7th. South, 10.91 ch.; 8th. N. 85° W. 29.28 ch. to the place of beginning; to divide the tract into two equal parts by a line running from the first station and falling on one of the opposite sides; the bearing and distance of the division line are required.

*Ans.* N. 7° 18′ E. 15.28 ch.

## CHAPTER V.

### MISCELLANEOUS QUESTIONS.

1. A circular fish-pond is to be dug in a garden, that shall take up just half an acre: what must be the length of the cord that strikes the circle?

*Ans.* 27.75 yards.

2. Two sides of a triangle are 20 and 40 perches respectively: required the third side, so that the content may be just an acre.

*Ans.* Either 23.099 or 58.876 perches.

3. In 110 acres of statute measure, in which the pole is 5.5 yards, how many Cheshire acres, where the customary pole is 6 yards; and how many of Ireland, where the pole in use is 7 yards?

*Ans.* 92 A. 1 R. 29 P. Cheshire; 67 A. 3 R. 25 P. Irish.

4. The ellipse in Grosvenor square, London, measures 840 links the longer way, and 612 the shorter, within the rails; now the wall being 14 inches thick, it is required to find what quantity of ground it encloses, and how much it stands upon.

*Ans.* It encloses 4 A. 0 R. 6 P. and stands on 1760½ square feet

5. Required the dimensions of an elliptical acre, with the greater and less diameters in the ratio of 3 to 2.

*Ans.* 17.481 by 11.654 perches.

6. The three sides of a triangular field, containing 6 A. 1 R. 12 P. are in the ratio of the three numbers, 9, 8, 6, respectively: required the sides.

*Ans.* 59.029, 52.47, and 39.353 perches.

7. In a pentangular field, beginning with the south side and measuring round towards the east, the first or south side is 27.35 ch., the second 31.15 ch., the third 23.70 ch., the fourth 29.25 ch., and the fifth 22.20 ch.; also the diagonal from the first angle to the third is 38 ch., and that from the third to the 5th. 40.10 ch.; required the area of the field. *Ans.* 117 A. 2 R. 39 P.

8. Required the dimensions of an oblong garden, containing three acres, and bounded by 104 perches of pale fence.* *Ans.* 40 P. by 12.

9. How many acres are contained in a square meadow, the diagonal of which is 20 perches longer than either of its sides? *Ans.* 14 A. 2 R. 11 P.

10. A gentleman has a garden 100 feet long and 80 broad, and a gravel walk is to be made of equal width half round it; what must be the width of the walk, so that it may take up just one fourth of the ground.

*Ans.* 11.8975 feet.

11. A person has a circular yard that is 150 feet in diameter, and wishes a walk of equal width made round it within the fence: required the width of the walk so that it may occupy a fifth part of the ground.

*Ans.* 7.918 feet.

---

* It may not be improper here to observe, that the 2d question, and all those following the 8th, admit of neat geometrical constructions.

12. From a point within a triangular field the sides of which were equal, I measured the distances to the three angles, and found them 12.5, 10, and 7.5 chains, respectively; required the area. *Ans.* 12 A. 1 R 23 P.

13. On examining the field-notes of a lot of ground of which I wished to know the content, I found them as follow: 1st. S. 72° W. 24 per., 2d. North, 38 per., 3d. N. 82½ E. 41 per., 4th. ——, 20 per., 5th. S. 80° E. 11.5 per., 6th. S. 26° W. 22 per., and 7th. ——, 37 per., to the place of beginning. The bearings of the 4th and 7th boundary lines were illegible; but the data remaining being sufficient, the area is required.

*Ans.* 12 A. 3 R. 2 P.

14. It is required to lay out 4½ acres of land in a triangular form, so that the length of one side may be 15 chains, and the lengths of the other sides in the ratio of 2 to 3; what must be the lengths of those sides?

*Ans.* 7.7914 and 11.6871 chains; or 29.58536 and 44.37804 chains

15. It is required to lay out five acres of ground in a triangular form to be bounded by 135 perches of fence; the length of one side is to be 50 perches; what must be the lengths of the other sides?

*Ans.* 33.3785 and 51.6215 perches.

16. The area of a rectangular field is 7½ acres and the length of the diagonal 50 perches: required the sides.

*Ans.* 30 and 40 perches.

17. In a rectangular tract of land, containing 58 A. 3 R. 8 P. the difference of the lengths of the sides is just equal to the difference between the lengths of the longer side and the diagonal; hence the sides are required.

*Ans.* 21 and 28 chains.

18. The boundaries of a tract of land are as follow: 1st. N. 14° W. 15.20 ch.; 2d. N. 70½° E. 20.43 ch.; 3d. S. 6° E. 22.79 ch.; 4th. N. 86½° W. 18 to the place of beginning; within the tract there is a spring, the bearing and distance of which, from the 2d corner, is S. 75° E. 7.90 ch. It is required to cut off 10 acres from the west side of this tract by a straight line running through the spring; what must be the distance of the division line from the 1st corner, measured on the fourth side.

*Ans.* 4.6357 chains.

19. The boundaries of a quadrilateral tract of land are as follow: 1st. N. 35¼° E. 23 ch.; 2d. N. 75½° E. 30.50 ch.; 3d. S. 3¼° E. 46.49 ch. and 4th. N. 66¼° W. 49.64 ch., to the place of beginning. This tract is to be divided into four equal parts by two straight lines, one of which is to run parallel to the 3d side; required the distance of the parallel division line from the first corner, measured on the 4th side; also the bearing of the other division line, and its distance from the same corner, measured on the first side.

*Ans.* Distance of the parallel division from the first corner 32.50 chains; the bearing of the other, S. 88° 22′ E. and its distance from the same corner 5.99 chains.

# CHAPTER VI.

## VARIATION OF THE COMPASS.

A MERIDIAN indicated by the magnetic needle is not, in general, a true one; for the needle does not point truly to the north point of the horizon, but varies from it, in some places to the eastward, and in others to the westward.

The angle contained between the true meridian and that indicated by the needle, is called the *variation of the compass*.

The variation is named *east* or *west*, according as the north end of the needle points to the eastward or westward of the true north.

As the variation is different in different places, so also, in the same place, it does not remain the same, but differs sensibly in the course of a few years. Hence, in running a line that was run a number of years previously, the bearing will be found different from what it was at that time; this, together with some difference in compasses, causes many difficulties, and frequently inaccuracies, in tracing old lines.

The easiest way to guard against those difficulties and inaccuracies would be to make and return the surveys according to the true, and not the magnetic bearings. In order to do this, it will be necessary to know the variation of the compass for the place in which the survey is made; and this may readily be found by first tracing a meridian line in the following manner.

*To draw a true meridian line by means of the greatest elongation of the pole star.*

The pole star is situated about $1\frac{1}{2}^{\circ}$ from the true pole, and therefore apparently revolves round it, in a small circle, once in about 23 h. 56 m. When at its greatest distance *east* or *west* from the true pole, it is said to be at its greatest east or west *elongation.* It is therefore evident that in the course of one apparent revolution it must be twice at its greatest elongation, once to the east and once to the west.

The following tables exhibit the times, nearly, of the greatest eastern elongations of the pole star for six months of the year, and of the greatest western elongations for the other six months. The other greatest elongations take place in the day time, and are therefore invisible. Some of those inserted in the tables are also invisible; because they occur, either before daylight is gone, in the evening, or after it has returned, in the morning. The most of those in the 3d, 4th, 9th, and 10th months are in this situation.

The time in the tables is reckoned from noon; and therefore when it is less than 12 hours, the greatest elongation takes place in the evening of the same day; but when it exceeds 12 hours, if 12 hours be subtracted from it, the remainder will be the time of greatest elongation in the morning of the following day.

*Eastern Elongations.*

| Day. | April. | | May. | | June. | | July. | | August. | | Sept. | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | H. | M. | H. | M. | H. | M. | H. | M. | H. | M. | H. | M. |
| 1..... | 18 | 37 | 16 | 39 | 14 | 37 | 12 | 39 | 10 | 37 | 8 | 36 |
| 7..... | 18 | 14 | 16 | 16 | 14 | 14 | 12 | 16 | 10 | 14 | 8 | 12 |
| 13..... | 17 | 50 | 15 | 52 | 13 | 50 | 11 | 52 | 9 | 50 | 7 | 48 |
| 19..... | 17 | 26 | 15 | 28 | 13 | 26 | 11 | 29 | 9 | 27 | 7 | 25 |
| 25..... | 17 | 03 | 15 | 05 | 13 | 03 | 11 | 05 | 9 | 03 | 7 | 01 |

*Western Elongations.*

| Day. | October. | | Nov. | | December. | | January. | | February. | | March. | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | H. | M. | H. | M. | H. | M. | H. | M. | H. | M. | H. | M. |
| 1..... | 18 | 27 | 16 | 25 | 14 | 28 | 12 | 26 | 10 | 24 | 8 | 30 |
| 7..... ..... | 18 | 04 | 16 | 02 | 14 | 04 | 12 | 02 | 10 | 00 | 8 | 06 |
| 13..... | 17 | 40 | 15 | 38 | 13 | 40 | 11 | 39 | 9 | 37 | 7 | 43 |
| 19..... | 17 | 17 | 15 | 15 | 13 | 17 | 11 | 15 | 9 | 13 | 7 | 19 |
| 25..... | 16 | 53 | 14 | 51 | 12 | 53 | 10 | 51 | 8 | 49 | 6 | 55 |

In order to determine a true meridian, by the method here used, it is necessary to know the bearing of the pole star, called its *azimuth*, at the time of its greatest elongation. This depends on the latitude of the place, and the distance of the star from the pole. This distance is called the *polar distance* of the star. It is subject to a small annual *diminution*, which is called its annual *precession*. The polar distance of the star on the 1st. of the 1st. month (January) 1830, was 1° 35′ 51″; and its annual precession is 19.3″.

The polar distance may be found for any subsequent time by multiplying 19.3″, by the interval between the 1st. of the year 1830, and the given time, and subtracting the product from 1° 35′ 51″. Thus, suppose the polar

distance of the pole star was required for the 1st of the 7th month (July), 1845. The interval is 15.5 years, and 19.3″×15.5 gives 299.15″=4′ 59″. From 1° 35′ 51″ take 4′ 59″, and we have 1° 30′ 52″ for the polar distance of the star at the time proposed.*

When the polar distance of the pole star is known, its azimuth, at the time of greatest elongation, may be found by the following proportion:

As the cosine of the latitude of the place
Is to radius
So is the sine of the polar distance
To the azimuth.

This azimuth will be *east* or *west*, according as the elongation is *east* or *west*, and consequently its name will be known from the preceding tables.

As an example, let the azimuth of the pole star at Philadelphia, latitude 39° 57′, be required for the 1st of the 7th month (July), 1845. The polar distance found above is 1° 30′ 52″; and it may be taken 1° 31′ without material error.

| | | |
|---|---|---|
| As cos. of lat. 39° 57′ - - - | Ar. Co. | 0.115428 |
| : radius - - - - - - - - - - | | 10.000000 |
| : : sin. of pol. dist. 1° 31′ - - - - | | 8.422717 |
| : sin. of azimuth 1° 59′ E. - - - | | 8.538145 |

* The polar distance obtained as above is called the *mean* polar distance, and it is sufficiently accurate for our present purpose. To obtain the *true* polar distance two small corrections, called *aberration* and *nutation*, would have to be applied. The true polar distance for January 1st, 1880, was 1° 19′ 24″.

In order to observe the greatest elongation of the pole star, it will be necessary to prepare the following simple apparatus.

Place two posts firmly in the ground, about three feet apart, and nearly east and west from each other; the heights of the posts, which should be the same, may be about two or three feet; on those posts, place a thick board or plank, five or six inches wide, and nail it fast to each of them, taking care that it be level or nearly so; take a piece of board, a foot or eighteen inches long and four or five wide, and near the middle of it fasten a compass-sight perpendicularly; this board is to slide on the horizontal one already mentioned.

Take a stiff pole 18 or 20 feet in length, and fix it in an inclined position, in such a manner that a plumb line suspended from the upper end, may be nearly north, from the middle of the horizontal board, and about ten feet distant from it; the elevation of the pole must be such that the pole star, when viewed through the compass sight placed on the horizontal board, may appear a few inches below its upper end; when in this position the lower end should be fastened in the ground, and the pole should be supported by a couple of crotchets placed near the middle. The plumb should weigh a pound or more, and should swing in a vessel of water, in order to prevent the line being agitated by the motion of the air.

The apparatus being prepared, proceed, about 15 or 20 minutes previous to the time of greatest elongation as indicated by the table, to make the observation as follows: Let an assistant hold a lighted candle near the

plumb line, so as to illuminate it and render it distinctly visible; place the small board with the compass-sight attached to it, on the horizontal one, and move it east or west as the case may require, till the pole star, plumb line, and aperture in the compass-sight are all in a direct range. If the star should be deviating to the east, it will leave the plumb-line to the west; and the contrary, if deviating to the west; keep therefore shifting the sight, till the star appears stationary behind the plumb-line; which it will do for several minutes at the time of its greatest elongation, and will then recede from the line on the contrary side from which it did before it became stationary. The compass-sight must not be moved after the star has attained its greatest elongation; but the aperture in it being then in a direct range with the plumb line and star, the board to which the sight is fixed, must be fastened to the one on which it slides, by a small tack passing through each end. This being done let an assistant take a straight stake, with a piece of lighted candle stuck on it, and go north to the distance of 30 or 40 perches; then looking through the compass-sight, direct him to set up the stake perpendicularly, and in such a situation that the candle stuck on the top may appear exactly behind the plumb-line; when thus placed, let it be firmly fixed in the ground. Next, let another straight stake be set up in the same manner near the plumb-line; the remaining part of the work may then be left till morning.

Measure accurately the distance between the two stakes. Then,

As radius,
Is to the tangent of the azimuth;
So is the distance between the stakes, in feet,
To a fourth term, in feet.

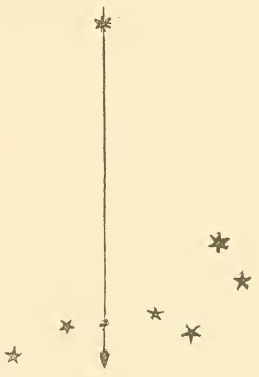
Lay off the distance contained in this fourth term from the northerly stake, and perpendicular to a line joining the two stakes; it must be laid off towards the *west* if the azimuth is *east*, but towards the *east* if the azimuth is *west*. Next remove the northerly stake, and set it up at the other extremity of the distance thus laid off; then a straight line joining the two stakes will be a true *meridian* line.

To obtain the variation, set up a compass in the place of the southerly stake, and direct the sights truly to the northerly one; the needle will then point out the variation, which will be *east* or *west*, according as the north end of the needle points to the *east* or *west* of the north point of the compass. The whole process is so simple, that an example is deemed unnecessary.

If the surveyor be provided with a theodolite or transit instrument, the true meridian may be found much more easily and accurately as follows: About half an hour before the time of greatest elongation level the instrument and point it to the star. Follow the star with the vertical spider line until it appears to have no farther side motion, but travels up or down the vertical line. Clamp the telescope firmly, and again examine to see if the star be on the line. Then depress the telescope, and direct an assistant who holds a pole with a light on it if necessary. Drive a stake under the instrument, another under the light. We may now calculate the deviation of this second stake from a meridian through the first as by the last method, or with the transit measure off to the right or left the known polar distance of the north star. The telescope will then point in the line of the true meridian.

If the spider lines cannot be seen, place a piece of white paper nearly in front of the telescope, so as to reflect the light from a lamp down the tube.

The position of the true meridian may also be determined quite accurately as follows: Hold a plumb-line a few feet in front of the telescope, and light up the spider lines as before; make the vertical spider line agree with the plumb-line. Polaris and Alioth (the first star in the handle of the dipper) come on the meridian about the same time; 17 minutes after the plumb-line passes through both stars Polaris is on the meridian. Point the telescope to it at this time, depress it and set a stake. The line from the instrument to the stake will be a true meridian.



*To obtain the true bearings of a survey, from the magnetic ones, the variation being given.*

This is a simple case of addition or subtraction, which the student can solve in any case.

*To find the difference between the present variation, and that at a time when a tract of land was formerly surveyed, in order to trace or run out the original lines.*

Go to any part of the premises, where any two adjacent corners are known; and if one can be seen from the other, take its bearing; which compared with that of the same line in the former survey, shows their difference. But if one corner cannot be seen from the other, run the line according to the given bearing, and measure

the nearest distance between the line so run, and the corner; then,

As the length of the given line,
Is to the said distance;
So is 57.3 degrees.*
To the difference of variation required.

EXAMPLE.

Suppose it be required to run a line, which some years ago bore N. 45° E. dist. 20 ch. and in running this line by the given bearing, the corner is found 20 links to the left hand; what allowance must be made on each bearing to trace the old lines; and what is the present bearing, by the compass, of this particular line?

| | L. | | L. | | Deg. |
|---|---|---|---|---|---|
| As | 2000 | : | 20 | : : | 57.3 |
| | | | | | 20 |
| | | | | | 2000)1146.0(0° 34′ |

Consequently 34 minutes, or a little more than half a degree, is the allowance required; and the line in question bears N. 44° 26′ E.

*Note.* The above rule is simple and sufficiently accurate when the distance between the sought corner and

* 57.3 is the radius (nearly) of a circle in such parts as the circumference contains 360

random line, is small. But when this distance is considerable, it will be better to find the angle by trigonometry.

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## ON LOCAL ATTRACTION.

It is well known that iron or any ferruginous substance attracts the magnetic needle, and consequently when near, will draw it aside from the position in which it would otherwise settle. And as the earth in many places contains, near its surface, substances of this kind, the needle will not unfrequently be attracted from its true direction. The surveyor ought therefore, at each station, to take a back sight to the preceding one; and if he arrive at one at which the compass does not reverse truly, he may conclude, provided no error was committed in taking the bearing at the last station, that at the present one, the needle is affected by some local attraction. In such a case, he should first determine whether any error was committed at the last station, and if none is found, take the difference between the bearing from the last station and the reverse bearing, which will be the local variation of the needle at the present station. This variation must be applied, according to its name, to the bearing of the following station.

If at the first and second station of a survey the compass is found not to reverse truly, the surveyor will be at loss to know which of them is affected by attraction. But by taking another station, either within or without the survey, and taking its bearing from each of those

stations, and the bearing of each of those from it, he may, in general, determine at which of them the attraction exists.

*Note.*—The area of the survey is not affected by the general variation, because it is the same at each station. But where local attraction exists and causes a variation in the position of the needle, as this variation will be different at different stations, it will, unless ascertained, and allowed on the corresponding bearings, materially affect the truth of the survey.

# CHAPTER VII.

## OF THE THEODOLITE.

A Vertical Angle is any angle in a plane perpendicular to the horizon. Consequently angles of elevation and depression are vertical angles.

A *Theodolite* is an instrument used to measure both horizontal and vertical angles.

By the aid of a theodolite, surveys may be much more accurately made than with the compass, especially when the tract is large or the ground hilly, or where there is local attraction. Before describing the theodolite, it will be best to notice separately one or two of its appendages.

### *Of the Spirit Level.*

The *Spirit Level* consists principally of a cylindrical glass tube, nearly, but not entirely, filled with alcohol, or some other fluid. The inner part of that side of the tube which, when in use, is to be uppermost, is ground from end to end into a regular curve, having its convexity upwards. In consequence of this curvature in the upper part of the cavity, the *air bubble*, or part not occupied by the fluid, must be at the middle of the tube when the *axis of the level*, that is, the straight line passing lengthwise through its middle, is in a horizontal position. And conversely the axis must be horizontal when the instrument is so placed that the bubble stands at the middle of the tube.

## *Of the Vernier.*

A *Vernier* is a graduated index which serves to subdivide the smallest divisions of a graduated arc or straight line.

Verniers are somewhat different according to the value of the divisions which they are to subdivide and the degree of precision they are designed to give; but the principle of construction is the same in all. The following description applies to the vernier attached to the common theodolite, the limb of which is divided into degrees and half degrees, numbered from 0° to 360°; and the vernier subdivides the half degrees to minutes.

In Fig. 115, AB represents a part of the graduated limb, and CD the vernier; the whole being drawn on an enlarged scale. The extent of the vernier is exactly equal to 29 half degrees, or 870 minutes, on the limb, and is divided into 30 equal parts, numbered from 0 to 30. Each division of the vernier contains therefore 29 minutes. Hence it is evident that when the 0, or *zero* line of the vernier exactly coincides, as in the figure, with a division line on the limb, the first division line of the vernier must be one minute behind the line on the limb first following that with which the zero of the vernier coincides; the second line of the vernier must be two minutes behind the second on the limb; the third line on the vernier must be three minutes behind the third on the limb; and so on. If then, the vernier were to be moved forward till its first division line coincided with the first on the limb, the zero of the vernier would be one minute past the line on the limb with which it before coincided; if moved till the second lines coincided, the zero of the vernier

would be two minutes past the same line; and in like manner for other coincidences. It therefore follows that for any position of the vernier, the number of the division line on it which coincides with one on the limb, must express the number of minutes that the zero of the vernier is past the division line on the limb, next preceding it.

The arc indicated by the vernier index in any position is the arc intercepted in the direction in which the degrees are numbered between the zero line of the limb and the zero line of the vernier. Hence to find the arc, or as it is technically expressed, to *read off* the arc, we must add to the arc expressed by the number of whole degrees intercepted and the odd half degree, if there is one, the number of minutes indicated by the *number* of that division line on the vernier which coincides, or is the nearest to coincidence, with one on the limb. Thus, as the zero of the vernier in Fig. 115 exactly coincides with the division line on the limb, numbered 50, the arc indicated is 50°. In Fig. 116. the zero of the vernier is past the line on the limb denoting 52° 30′, and the 21st division line of the vernier coincides with a line on the limb. The arc indicated is therefore 52° 51′.

Fig. 115.

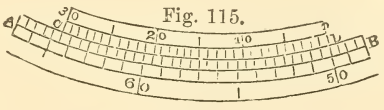
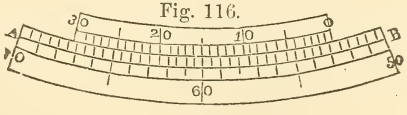


Fig. 116.



To determine the reading of a vernier, applied to a

graduated arc in which the value of the least division is more or less than half a degree, we must divide this value by the number of divisions on the vernier, and the quotient will indicate the reading that the vernier is intended to give. In the nautical instrument called a sextant, the value of the least division on the limb is usually 10 minutes, and the vernier is divided into 60 equal parts. Hence 10 minutes, or 600 seconds, divided by 60, gives 10 seconds for the reading of the vernier.

### *Description of the Theodolite.*

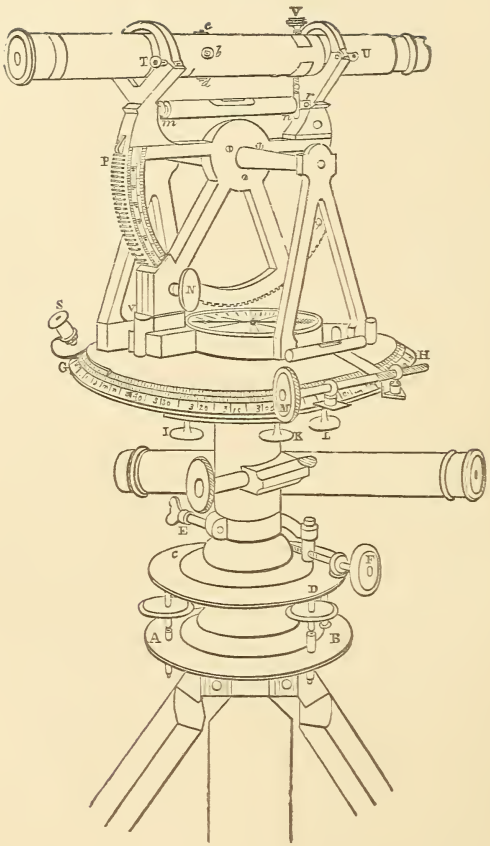
The Theodolite is represented in Fig. 109. When in use, the lower circular plate AB is screwed fast to a three-legged stand or tripod, the upper part of which is represented in the figure. The circular plate CD has a hollow axis passing through it at right angles at its centre, and firmly attached to it. The lower part of the axis terminates in a ball, to which it is fastened by a screw. This ball is partly enclosed in a socket projecting from the upper side of the plate AB; the opening in the upper part of the socket being larger than the axis, so as to allow the latter some motion in every direction. In the lower plate AB four screws, called *levelling screws*, are inserted, standing opposite each other in pairs, the tops of which press against the under side of the plate CD. These screws are turned by milled projections or heads, two of which are shown in the figure. By turning them in opposite directions, the position of the plate CD may be changed so that it may be made level, even when the lower plate AB has considerable inclination.

The part GH of the instrument consists of two circular plates in close contact with each other. The

lower one is called the *graduated plate*, and its chamfered edge, which is usually made of silver, is called the *limb* of the instrument. To the central part of this plate, a hollow axis is attached, the cavity of which fits to the outside of the axis of the plate CD, so that the former may move easily and steadily about the latter. The lower part of the axis is embraced by a clamping-piece that may be tightened or loosened by the screw E. When the clamp is loose, the graduated plate may be turned round by the hand; but when it is made fast, the axis of the graduated plate, and consequently the plate itself, becomes firmly connected with the plate CD, and also, when the levelling screws are tight, with the stand of the instrument. The graduated plate may however, when thus connected, be moved a small distance either way by turning the screw F, called a *tangent-screw*, which gives it a slow motion. Around the axis of the graduated plate is another hollow axis, with which a telescope, called the *lower* telescope, is connected. When the axis of the graduated plate is clamped fast, this axis and the telescope may be made to revolve by turning the milled head I, and may be secured in any position by the screw K, which clamps the axis to the graduated plate.

The plate immediately above the graduated plate is called the *vernier plate*. It has, at its centre, a solid axis which fits into the cavities of the hollow axes of both the graduated plate and plate CD. When the screw L, which serves to clamp the vernier plate to the graduated plate, is loosened, the former may be turned round by hand; and when this screw is made tight, a slow motion may be given to the vernier plate by the tangent-screw M. The vernier plate has two, or sometimes three, verniers at different parts of its

Fig 109.



edge, which are chamfered for the purpose. When there are only two, they are placed directly opposite to each other. When there are three, they are placed at equal distances around the edge. The microscope S is used to enable the eye more certainly to distinguish the line on the vernier that coincides with one on the limb. Two levels are placed on the vernier plate, at right angles to each other; one of which is shown in the figure.

The frame which supports the vertical semicircle and *upper* telescope with its attached level, is attached to the vernier plate by three screws at equal distances from one another. The immediate supports of this telescope are called *wyes*, or Y's, from their resemblance to the letter Y. The telescope is held in its place by two curved pieces, moveable on joints, which pass over it and are fastened by the pins T and U. When these pins are taken out, the pieces may be turned back and the telescope taken from its place and *reversed*, that is changed end for end.

In the tube of the telescope a flat ring is placed at right angles to its axis, and connected with the tube by four screws, opposite to each other in pairs; the heads of three of which, *b*, *c*, and *d*, are shown. Two spider's lines or very fine wires, at right angles to each other, are attached to the ring, in the directions respectively of each pair of screws, and intersecting each other in the centre of the ring. At each end of the principal tube, and fitting it on the inside, a short moveable tube is inserted. The one at the end to which the eye is applied, and which usually contains several glass lenses, may be moved out or in, by a small pin attached to its lower side, and may thus be so adjusted as to render the spider's lines distinctly

visible. The tube at the other end, which contains one lens, may be moved out or in, by turning the milled head V, and may be thus so placed as to render an object to which the telescope is directed, as distinct as its distance permits.

A straight line in the direction in which the intersection of the spider's lines or wires is seen by an eye placed at the eye end of the telescope, is the *line of sight* of the telescope, and is technically called the *line of collimation*. The telescope is said to be *directed* to any point in an object, when it is so placed that the line of collimation passes through the point; or which is the same, when the point is directly behind the intersection of the spider's lines.

The semicircle PQ is called the *vertical limb* of the instrument. That face of it which is not seen in the figure is divided into degrees and half degrees, numbered each way from a line taken as a zero or 0 line; and the arc is read by a vernier numbered as in Fig. 117. The other face has two sets of unequal divisions on it, numbered each way from a zero line. One of these denotes the difference between the real distance of an object to which the upper telescope is directed and its horizontal distance, expressed in 100th parts of the distance. The other denotes the vertical distance of the object, above or below a horizontal line, passing through the instrument, expressed in 100th parts of the horizontal distance. The vertical limb is moved so as to give to the telescope different inclinations to the horizon, by means of the milled head N.

At the bottom of the frame which supports the vertical limb and upper telescope, and directly over the centre of the vernier plate, there is a compass box, containing a magnetic needle. This part of the

instrument may be used to take the bearing of an object in the same manner as a common compass.

## ADJUSTMENTS OF THE THEODOLITE.

In order that the theodolite may be in a good state for use, the line of collimation of the upper telescope should exactly coincide with the axis of the telescope: the axis of the attached level should be parallel to this line; the axes of the levels on the vernier plate should be parallel to this plate; the line of collimation of the telescope should, when the milled head N is turned. move in a plane perpendicular to the vernier plate; and when this line is brought parallel to the vernier plate, the zero of the vertical limb should coincide with the zero of its vernier. Previous therefore to using the instrument, the different parts should be examined, and *adjusted* if necessary. This may be done by the following methods.

### FIRST ADJUSTMENT.

*To make the line of collimation coincide with the axis of the telescope.*

The instrument being firmly screwed on the tripod and the legs of the latter being sufficiently extended to ensure its remaining fixed in its position, loosen the clamp screw L, and turn the vernier plate till the telescope points to some distant object having on it a small well-defined point. Then, having fastened the screw L, move the telescope by the tangent-screw M, and milled head N, till the line of collimation is directed exactly to the point.

Revolve now the telescope in its Y's, half round, that is, till the level, from being directly below the tele-

scope is directly above it. If the horizontal spider's line still coincides with the point, it requires no adjustment; but if it does not, diminish the distance one-half, by loosening one of the screws *c* and *d* and tightening the other; and then bring the line to coincide with the point by means of the milled head N. Revolve the telescope round to its first position, and if the horizontal line and point do not then coincide, repeat the operation till the coincidence has place in both positions. In a similar manner the vertical line may be adjusted. When both adjustments are complete, the line of collimation should coincide with the same point during a complete revolution of the telescope in its Y's.

## SECOND ADJUSTMENT.

*To make the axis of the level attached to the upper telescope parallel to the line of collimation.*

Turn the vernier plate till the telescope comes directly over two of the levelling screws; and if the telescope is not nearly in a horizontal position, make it so by turning the milled head N. Then turn the levelling screws over which the telescope stands, in opposite directions, till the bubble of the level stands exactly at the middle of the tube; observing to keep the screws firm against the plate CD. When this is done, reverse the telescope in its Y's, and if the bubble does not stand at the middle of the tube, correct *half* of the deviation by one of the screws *m* and *n*, which serve to raise or depress the ends of the level, and then, by the levelling screws, bring the bubble to the middle. Again reverse the telescope and repeat the correction if necessary.

Revolve now the telescope in its Y's so as to bring the level a considerable distance from its proper or lowest position, and if the bubble deviates from the middle of the tube, make the requisite correction by means of two screws, *p* and *q*, which move the end of the level laterally, one of which is shown at *p*. When this part of the adjustment has been so made that the bubble will remain in the middle of the tube while the telescope is revolved either way, the first part of the adjustment should, by again reversing the telescope, be examined, and repeated if necessary.

### THIRD ADJUSTMENT.

*To make the axes of the levels on the vernier plate parallel to that plate.*

Turn the vernier plate till the upper telescope stands over two of the levelling screws; then one of the levels on this plate will be parallel to these two screws, and the other will be parallel to the other two. By means of these levelling screws bring the bubbles of both levels to stand in the middles of their respective tubes. Then move the vernier plate 180°, and if the bubble in either of the levels deviates from the middle, correct one-half of the deviation by one of the screws at its ends, and the other half by the levelling screws, parallel to it; and if the bubble of the other level also deviates from the middle, proceed in the same way to correct the deviation. Repeat the operations till the bubbles of both levels will remain at the middles of their respective tubes during a complete revolution of the vernier plate.

## FOURTH ADJUSTMENT

*To make the axis about which the vertical limb revolves, parallel to the vernier plate.*

Turn the vernier plate till the upper telescope is directed towards a well-defined, elevated point, on a house or other object, not very remote, and having clamped the plate, direct the telescope by means of the tangent-screw M and milled head N, exactly to the point. Then turn the milled head N till the line of collimation coincides with some well-defined point near the ground; or, if none such is found, let an assistant make a suitable mark in the direction of the line of collimation when thus brought nearly horizontal. This being done, reverse the telescope in its Y's; and, proceeding as before, direct it to the elevated point. Then, if by turning the milled head N, the line of collimation is brought to coincide with the lower point or mark, the axis of the vertical limb is parallel to the vernier plate; but if this is not the case, the adjustment must be made by the screws which attach the upper frame to this plate.

## FIFTH ADJUSTMENT.

*To adjust the vernier of the vertical limb, or determine the correction which should be made, to allow for its deviation from correct adjustment.*

Direct the upper telescope to some elevated point and note the angle of elevation indicated by the vernier. Reverse the telescope in its Y's, and again direct it to the same point, and note the angle of elevation If the angles are the same, the vernier is properly

adjusted. If they differ, the position of the vernier requires adjustment; and this may be effected by the screw *v*, which is one of those that fasten the upper frame to the vernier plate.

Instead of changing the position of the vernier, we may take half the difference of the two angles of elevation as a *correction*, to be added to all those angles of elevation and subtracted from those of depression, that are taken with the telescope in that position in its Y's, which gave the least of the two angles; but to be applied in a reverse manner when the position of the telescope is that which gave the greater angle.

## SIXTH ADJUSTMENT.

*To make the axis about which the lower telescope revolves vertically, perpendicular to the axis of the instrument.*

Turn the vernier plate till the levels on it are respectively parallel to a pair of opposite levelling screws, and by means of these screws bring the bubbles to stand in the middles of the tubes; the instrument is then levelled, and its axis is perpendicular to the horizon. Suspend a plumb-line of considerable length at a short distance from the instrument, and loosening the clamping screw K, turn the milled head I, till the telescope is directed to the line, and then fasten the clamping-screw. Now, applying the hand to the eye-end of the telescope, move it vertically, and observe whether it continues directed to the line, throughout its whole extent. If it does, the axis is properly adjusted; if it does not, the adjustment may be made by two small screws which move the remote end of the axis vertically.

The lower telescope being used only as a guard to

ascertain whether or not the horizontal limb of the instrument remains fixed while the vernier plate is made to revolve, in order to direct the upper telescope to an object, requires no other adjustment than the above, except those for distinct vision of the spider's lines and object.*

In extensive practical operations, the above adjustments should be examined, and corrected if necessary, not only before the commencement, but every day or two during their continuance.

### *Of the arc of the horizontal limb, corresponding to a given position of the vernier plate.*

When there is but one vernier, the arc indicated by it is regarded as the corresponding arc. When there are two or three, it is usual to distinguish them by calling one A, another B, and the third, if there is a third, C, and to consider the arc indicated by the ver-

* When the foregoing adjustments have been completed for the first time, there is another examination which it is well to make. Direct the upper telescope to some well-defined point near the horizon, and note the arcs on the horizontal limb, indicated by the different verniers. Then having directed the lower telescope to the same point, fasten its clamping-screw K; examining first, however, to see that the upper telescope has not changed its position, or if it has, bringing it back by the tangent-screw F. Reverse now the upper telescope in its Y's, and again direct it to the point; examining at the same time to see that the lower telescope has not changed its position, or, if it has, bringing it back by the tangent-screw F. Note the arcs indicated by each of the verniers; and subtract the arc indicated by each vernier in the former position from that in the latter, increasing that of the latter by 360°, if necessary. Add together the remainders, and divide the sum by the number of verniers. If the quotient is 180°, we infer that the axis about which the upper telescope revolves is, as it ought to be, at right angles to the line of collimation of the telescope. If the quotient differs materially from 180°, the instrument is imperfect, except one of the Y's is laterally adjustable. Where this is not the case, the imperfection can only be well remedied by an instrument-maker. If the remainders mentioned above differ much from each other, we infer that the instrument has not been well centered, or not well divided.

nier A, as expressing the position of the vernier plate, at least very nearly; the readings of the other verniers being used merely as a test, or means of correcting the former. We therefore note the arc indicated by the vernier A. Then note the minutes of the arc indicated by the vernier B; but instead of the number of degrees indicated by it, we take either the number indicated by the vernier A, or this number increased or diminished by a unit, so that the arc set down for the vernier B, may be very nearly the same as that for the vernier A. If there is a third vernier C, we proceed in the same manner. Then the sum of the arcs obtained for each vernier, divided by the number of verniers,* will give the value of the arc corresponding to the given position of the vernier plate, or, as it is sometimes expressed, the *arc indicated by the verniers.* Thus, if we suppose the instrument to have three verniers, and that for a given position of the plate, the reading of the vernier A is 142° 2′, of B 261° 58′, and of C, 22° 3′; then, instead of these quantities for B and C, we write for B, 141° 58′, and for C, 142° 3′. Adding together 142° 2′, 141° 58′, and 142° 3′, and dividing the sum by 3, we obtain 142° 1′, for the arc indicated by the verniers.

## PROBLEM I.

*To measure with the Theodolite the horizontal angula distance of two objects, as seen from a given station.*

By means of a plumb-line suspended from the centre of the plate which forms the top of the tripod, set the

* When, as is commonly the case, the number of the degrees of the arc for each vernier is the same, we need only divide the sum of the minutes of the arcs, by the number of verniers, and annex the quotient to that number of degrees.

centre of the instrument directly over the station-mark, and then level it, as directed in the sixth adjustment; make the zero of the limb agree with the zero of the vernier; direct the upper telescope to the object which stands to the *left*, when the face is turned towards the angle to be measured. Then, having directed the lower telescope to the same object, and fastened its clamping-screw K, direct the upper telescope to the other object, and note the readings of the verniers; observing first, however, whether there has been any change in the position of the lower telescope; and if there has, bringing it back to the object at the left by means of the tangent-screw F, and again by the tangent-screw M, adjusting the direction of the upper telescope to the object at the right. The verniers will read the angle required.

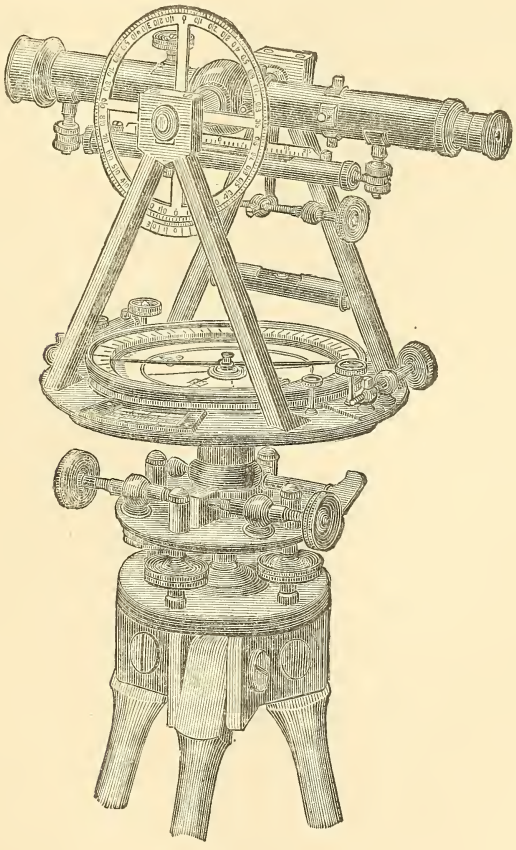
## PROBLEM II.

*To measure a vertical angle with the Theodolite.*

The instrument being placed and levelled, direct the upper telescope to the point, of which the angle of elevation or depression is to be taken. Then, if the vernier of the vertical limb is accurately adjusted, the angle indicated by it will be the angle required. When this is not known to be the case, after having noted the arc indicated by the vernier, move the vertical limb till the telescope is horizontal, as indicated by the bubble of its level standing in the middle of the tube, and note the arc then indicated by the vernier. If in both positions the zero of the limb is on the same side as the zero of the vernier, subtract one arc from the other; but if on different sides, add the two arcs together. The result will be the required angle.

## OF THE TRANSIT.

The greater part of the preceding descriptions and adjustments will apply as well to the surveyor's transit as to the theodolite. The main difference between the



two is that the telescope of the transit will turn through 360° in a vertical plane, thereby enabling the surveyor much more easily to reverse. If provided with a grad-

uated vertical circle, it will perform all the problems of the theodolite.

As it can be readily directed both forwards and backwards in the same line, while the telescope of the theodolite must be taken from its supports and reversed, the transit is the much more convenient instrument for surveyors.

The accompanying figure represents one of the most convenient and modern forms of transit instrument.

## CHAPTER VIII.

### LEVELLING.

The surface of an expanse of tranquil water or any similar surface concentric with it, is called a *Level Surface.* Any points situated in a level surface are said to be on the *same level;* and any line traced in such a surface is called a *line of level.*

If through any place, a level surface be conceived to pass, the distance which another place is from this surface, either above or below, measured on a line perpendicular to it, is called the *difference of level* of the two places.

*Levelling* is the art of determining the difference or differences of level of two or more places.

In consequence of the globular figure of the earth, a level surface is not, as it appears to be, a plane surface. It is nearly, though not exactly, spherical. In the operations of levelling we may, without sensible error, regard a level surface at any place, as being strictly a spherical surface, with a radius equal to 3956 miles, the mean radius of the earth, or, which is more exact, with a radius equal to 3968 miles;* the centre being

* The earth, if we disregard the inequalities in its surface, is an oblate spheroid. Its polar diameter is 7899 miles, and its equatorial diameter 7925 miles. A level surface is therefore a spheroidal surface. The radius of the spherical surface which most nearly coincides with any small portion of this spheroidal surface, changes slightly with the latitude of the place. For any place in the United States, the greatest error which can occur from

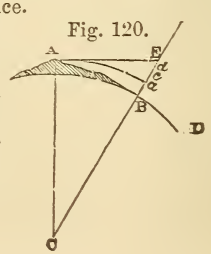
in a straight line conceived to be drawn downwards from the place, perpendicular to the level surface. And for places not very remote from each other, we may regard the spherical surfaces of level passing through them, as having a common centre.

Let A and B, Fig. 120, be two places, not very remote from each other, and C, the common centre of the spherical surfaces of level passing through them. With the centre C and radius CA, describe the arc A*a*. Then will A*a* be a line of level of the place A, and B*a* will be the difference of level of the two places A and B.

The line A*d*, drawn perpendicular to CA, is called a line of *apparent level* of the place A. It is the line of level that would be indicated by an accurately adjusted levelling instrument placed at A. The distance *ad* is called the *correction of the apparent level.* This correction must be *subtracted* from the height B*d* of the apparent level to obtain the height B*a*, of the true level. The correction varies as the square of the distance from the place.* The following table contains the

considering the level surface passing through it, as a spherical surface with a radius equal to the mean radius of the earth, is about $\frac{1}{3}$ of an inch for a distance of two miles. For a spherical surface with a radius 5 or 6 miles greater than the equatorial radius of the earth, the greatest error is about $\frac{1}{10}$ of an inch, at the same distance. It may be further observed, that the greatest error or greatest deviation of the spherical from the spheroidal surface, varies as the square of the distance from the place.

Fig. 120.




* We have (V. 27), $(2Ca+ad).ad=Ad^2$. But as *ad*, for any distance to which a single sight in levelling is ever extended, is extremely small in comparison with 2C*a*, we may, without sensible error, take 2C*a* instead of $(2Ca+ad)$. We shall thus have $2Ca.ad=Ad^2$; or $ad=\frac{Ad^2}{2Ca}$. Consequently, as C*a* is constant, *ad* varies as the square of the distance A*d* varies.

value of the correction in decimal parts of a foot, for each chain of distance, from 1 to 120.

## TABLE,

*Giving the differences between the true and apparent level, for distances from* 1 *to* 120 *chains.*

| Chains. | Feet. | Chains. | Feet. | Chains. | Feet. | Chains. | Feet. | Chains. | Feet. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | .000 | 25 | .065 | 49 | .250 | 73 | .554 | 97 | .978 |
| 2 | .000 | 26 | .070 | 50 | .260 | 74 | .569 | 98 | .998 |
| 3 | .001 | 27 | .076 | 51 | .270 | 75 | .585 | 99 | 1.019 |
| 4 | .002 | 28 | .082 | 52 | .281 | 76 | .600 | 100 | 1.040 |
| 5 | .003 | 29 | .087 | 53 | .292 | 77 | .616 | 101 | 1.060 |
| 6 | .004 | 30 | .094 | 54 | .303 | 78 | .632 | 102 | 1.082 |
| 7 | .005 | 31 | .100 | 55 | .314 | 79 | .649 | 103 | 1.103 |
| 8 | .007 | 32 | .106 | 56 | .326 | 80 | .665 | 104 | 1.124 |
| 9 | .008 | 33 | .113 | 57 | .338 | 81 | .682 | 105 | 1.146 |
| 10 | .010 | 34 | .120 | 58 | .350 | 82 | .699 | 106 | 1.168 |
| 11 | .013 | 35 | .127 | 59 | .362 | 83 | .716 | 107 | 1.190 |
| 12 | .015 | 36 | .135 | 60 | .374 | 84 | .734 | 108 | 1.213 |
| 13 | .018 | 37 | .142 | 61 | .387 | 85 | .751 | 109 | 1.235 |
| 14 | .020 | 38 | .150 | 62 | .400 | 86 | .769 | 110 | 1.258 |
| 15 | .023 | 39 | .158 | 63 | .413 | 87 | .787 | 111 | 1.281 |
| 16 | .027 | 40 | .166 | 64 | .426 | 88 | .805 | 112 | 1.304 |
| 17 | .030 | 41 | .175 | 65 | .439 | 89 | .823 | 113 | 1.327 |
| 18 | .034 | 42 | .183 | 66 | .453 | 90 | .842 | 114 | 1.351 |
| 19 | .038 | 43 | .192 | 67 | .467 | 91 | .861 | 115 | 1.375 |
| 20 | .042 | 44 | .201 | 68 | .481 | 92 | .880 | 116 | 1.399 |
| 21 | .046 | 45 | .211 | 69 | .495 | 93 | .899 | 117 | 1.423 |
| 22 | .050 | 46 | .220 | 70 | .509 | 94 | .919 | 118 | 1.447 |
| 23 | .055 | 47 | .230 | 71 | .524 | 95 | .938 | 119 | 1.472 |
| 24 | .060 | 48 | .240 | 72 | .539 | 96 | .958 | 120 | 1.497 |

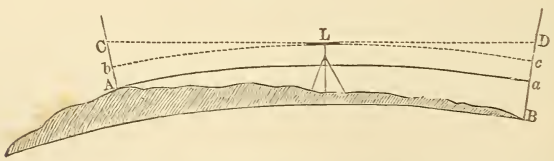
When a ray of light passes obliquely through portions of air of different densities, it becomes bent from a straight line, and enters the eye so as to make the point

from which it proceeds appear in a direction slightly different from its true direction. This effect is called *Refraction;* and when the point, or body from which the light proceeds, is on or near the earth, it is called *Terrestrial Refraction.* Terrestrial Refraction generally makes the point appear to be more elevated than it really is. Thus it is not actually the point *d* of the line CE, Fig. 120, that, to an eye at A, appears to be at *d*, but another point *c*, a little below *d*. If therefore we wish to notice the effect of refraction, we must take *ac* instead of *ad*, for the correction of apparent level. In temperate climates, *cd* is, in the usual state of the atmosphere, about $\frac{1}{6}$ of *ad*. Consequently we may obtain the correction of apparent level with allowance for refraction, by diminishing the correction given in the preceding table by a $\frac{1}{6}$ part.

As the effect of refraction in the common operations of levelling is always small, and is subject to considerable variations depending on the state of the air between the object and place of observation, it is commonly disregarded. It would, however, in general be better to allow for it as above, except when the object is but little distant from the place of observation, or when, by the method noticed in the next article, the desired result is obtained independently of the correction of apparent level.

Let CD, Fig. 121, be the line of apparent level indi-

Fig. 121.

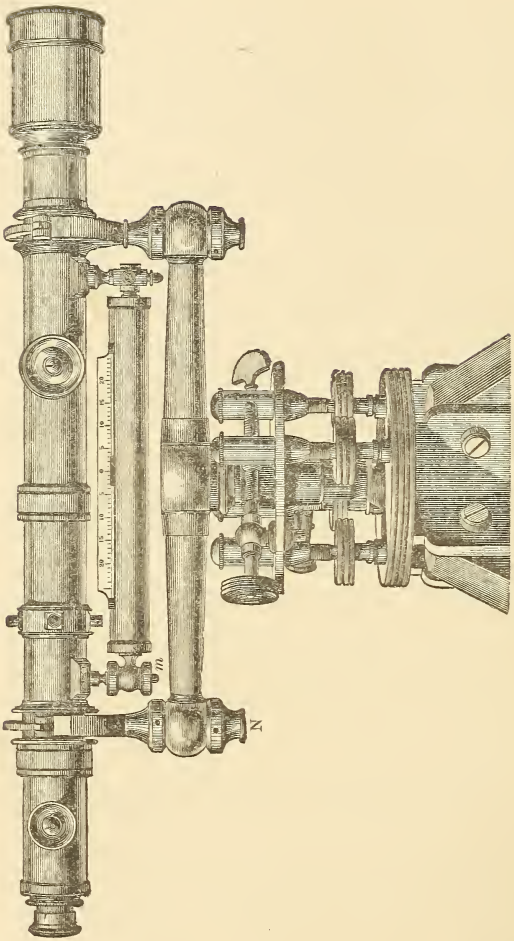



cated by a levelling instrument L, placed midway between the places A and B; also let the arc *bc* be the

line of true level passing through the instrument, the arc A*a*, concentric with the former, the line of true level of the place A, and AC and BD, lines perpendicular to A*a*. Then in consequence of the equality of the distances LC and LD, we have *b*C equal to *c*D; and therefore as A*b* is equal to *ac*, we have AC equal to *a*D. Consequently B*a*, the difference of level of the places A and B, is obtained by subtracting AC, the height of the apparent level of the instrument, at A, from BD, its height at B. We thus obtain the difference of level of the two places, independently of the correction of apparent level. It is also independent of refraction, as the effect of refraction would be sensibly the same for the two points C and D, and in the same direction. It may be observed that it is not necessary the instrument should be directly between the places. It may be placed in any convenient position on either side of the line joining them, provided its distances from the places are equal, or very nearly so.

## OF THE LEVELLING INSTRUMENT.

The *Levelling Instrument*, or *Level* as it is frequently called, is an instrument used to denote the line of apparent level. The instrument is represented on page 218. The tripod and levelling screws are similar to those of the transit. The spirit-level has a screw at one end, which gives it motion vertically, and one at the other end, which gives it motion horizontally. The object glass is moved in and out by a screw, and another screw gives a similar motion to the eye-piece. Thus the focus is adjusted for all distances, and the spider lines



are kept in distinct vision. Clamp and tangent-screws give horizontal motion to the telescope, but there is no opportunity for similar motion in a vertical plane.

## FIRST ADJUSTMENT.

*To make the line of collimation coincide with the axis of the telescope.*

The method of doing this is similar to that employed in the first adjustment of the theodolite.

## SECOND ADJUSTMENT.

*To make the axis of the level parallel to the line of collimation.*

By turning the screw N, bring the bubble of the level to stand at the middle of the tube. Reverse the telescope in its wyes, and, if the bubble does not then stand in the middle, correct one half of the deviation by the screw *m*, and the other half by the screw N. Again reverse the telescope in its wyes, and repeat the correction if necessary.

Now by revolving the telescope in its wyes, bring the level to some distance on one side of its lowest or proper position, and if the bubble then deviates from the middle, the deviation must be corrected by means of the screws, which move one end of the level laterally, the correction being continued till the bubble will remain at the middle while the telescope is revolved so as to bring the level to a considerable distance on either side of its lowest position. When this has been done, the first part of the adjustment should again be examined and corrected if necessary.*

* A method by which the second adjustment may be made for an instrument in which the telescope is not reversible, will be found in the first of the following problems.

### THIRD ADJUSTMENT.

*To make the line of collimation at right angles to the axis.*

Turn the telescope till it stands directly over two of the levelling-screws, and by means of them bring the bubble to stand at the middle of the tube. Then turn the telescope half round, that is, till it stands over the same screws, but pointing in the opposite direction, and if the bubble does not remain in the middle, correct one half of the deviation by the levelling-screws and the other half by the screw N. Now place the telescope over the other levelling-screws, and proceed in a similar manner. Continue the corrections till the bubble will remain in the middle of the tube during an entire revolution.

These adjustments having been carefully made, the instrument is ready for use. When on the ground it must, in each new position in which it is placed, be *levelled*. This is done by placing the telescope over two of the levelling-screws and by their means bringing the bubble of the level to the middle of the tube, then doing the same with the telescope over the other two, and again over the first two. Then, if the third adjustment has been accurately made, the bubble will stand in the middle of the tube in any position of the telescope.

### OF THE LEVELLING STAFF.

A *Levelling Staff* consists of a square or rectangular staff and a small circular or rectangular board,

called a *Vane*, which is so attached to the staff as to be moveable along it from end to end. It is used for measuring the height of the line of apparent level passing through the telescope of the levelling instrument, above the place where the staff is placed.

The face of the vane, represented in Fig. 121, is divided into four equal parts by two straight lines intersecting each other at right angles; one line being horizontal, and consequently the other vertical. Two opposite parts of the face are painted white and the other two red; thus the lines and their intersection are easily distinguished even at a considerable distance. A screw, the head of which is shown at *m*, serves to clamp the vane to the staff in any required position.

The staff is composed of two rectangular bars of wood, between six and seven feet long, placed side by side, and forming together a square staff, the breadth of each side of which is about an inch and a quarter. The bars are so connected that one, which is two or three inches the shorter of the two, may be made to slide along the other or principal bar, and thus, when necessary, increase the length of the staff. In order to this, the front or sliding bar has throughout its length, on the side next the other, a projection which is terminated by a brass plate a little wider than the projection and firmly attached to it; and the principal bar has a groove in it to receive the projection and plate of the former. The figure on the next page shows the upper portion of the levelling rod.

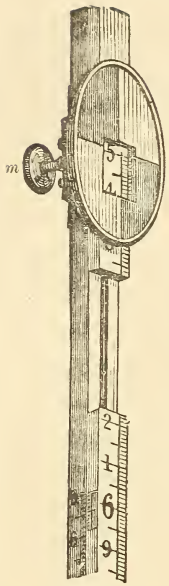


Fig. 121.

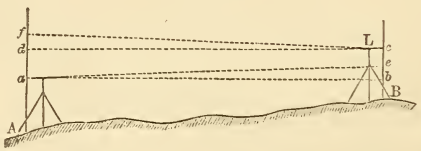
The graduation begins at the bottom, and is to feet and hundredths of a foot. The height to the horizontal mark on the upper vane is the required distance. This is obtained from the divisions on the rod, and a vernier that slides with the vane. To read heights greater than 6½ feet the vane is placed at its highest point, and the bar carrying it is made to slide upwards on the other. A clamp-screw fastens the vane in any required position. Two vanes sometimes slide on the same rod, so that difference of level may be determined.

## PROBLEM I.

*To test the adjustment of the level.*

Select a place where the ground is tolerably level for a distance of 15 or 20 chains, and at each extremity of the distance chosen, as at A and B, Fig. 122, drive

Fig. 122.




a short stake. Set up the level by the stake at A, placing it so that the eye-end of the telescope may be over or nearly over the stake, and level the instrument. Place the levelling-staff on the stake, raising or lowering the vane till its horizontal line is at exactly the same height as the centre of the eye-end of the telescope, and note the height. Now let an assistant take the staff and set it up vertically on the stake at B. Direct the telescope to the middle of the breadth of the staff, and then, by raising or lowering the hand as a signal, direct the staff-bearer to raise or

lower the vane, repeating the signal till its centre appears precisely in the direction of the line of collimation, or at least exactly of the same height. When this is the case, by a circular motion of the hand, direct the vane to be clamped, and again sight to it to ascertain that in clamping, its height has not been changed. Note the height of the vane, and subtract from it the correction of apparent level corresponding to the distance between the stakes, taken from the preceding table, or rather this correction diminished by a $\frac{1}{6}$ part, to allow for refraction. Then, if the instrument is accurately adjusted and the observations have been carefully made, the difference between the height of the vane at A, and its corrected height at B, will be the true difference of level of the tops of the two stakes; the higher being that at which the height of the vane is the less. Placing now the instrument at B, proceed in the same manner to find again the difference of level of the stakes. If this difference is the same as the former, the adjustment of the instrument is correct. But if there is any material difference in the two results, we infer that the axis of the level is not parallel to the line of collimation. To make it so, take half the difference of the results obtained, and let the vane, taken in its last position, be *elevated* or *depressed* by that quantity, according as the result obtained at the more elevated of the two stakes, is *less* or *greater* than that obtained at the other.* Then, by the screw N, bring the line of colli-

* Let *ab* be the line of apparent level through the instrument when placed at A, and *ae*, the line of apparent level indicated by the instrument. Then *cd* being the line of apparent level through the instrument when placed at B, the line *cf* making the angle *f*L*d* equal to *b*L*e*, and consequently *df* equal to *be*, will be the line of apparent level indicated by the instrument

mation to point exactly to the horizontal line of the vane, and with the telescope in that position, bring the bubble again to the middle of the tube by the screw *m*

*Note.* In this manner the axis of the level may be made parallel to the line of collimation when the telescope is not reversible, as is the case in some instruments.

EXAMPLE.

Let the height A*a*, be 5.295ft.;* B*e*, 2.063ft.; B*c*, 5.527ft.; and A*f*, 8.935ft.; and the horizontal distance from A to B, 20 chains.

The distance being 20 chains, the correction for apparent level, taken from the table, is 0.042ft. From this deducting $\frac{1}{6}$ of 0.042, we have 0.035ft. for the correction to be subtracted from the heights B*e* and A*f*. This gives, for the corrected heights, B*e* = 2.028ft., and A*f* = 8.900ft. Hence, taking 2.028 from 5.295, we have 3.227ft. for the true difference of level obtained with the instrument at A; the place B being higher than A. And taking 5.527 from 8.900, we have 3.373ft. for the difference of level obtained with the instrument at B. Half the difference of these results gives 0.053ft. for the value of *df*, the error due to the error in the

---

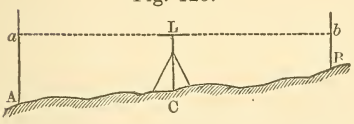
It is therefore evident that the difference of level of the two places obtained with the instrument at A, is less than the true difference by the quantity *be*, and the difference obtained with the instrument at B, is greater than the true difference, by the equal quantity *df*. Hence the difference of the two results must be twice *df*, the error in height produced by the error in the adjustment of the instrument.

* Although the station staff is only divided to hundredths of a foot, we may with tolerable precision estimate the thousandths; and when great accuracy is desired, it is better to do so.

adjustment of the instrument. Hence, as the difference of level given by the instrument at the more elevated place is greater than the other, the vane must be lowered 0.053 ft. from its last height. The adjustment may then be made as directed.

## PROBLEM II.

Fig. 123.




*To determine the difference of level of two places,* A *and* B, Fig. 123, *when they are visible from each other and do not differ in level more than* 8 *or* 10 *feet.*

Place the level in some position C, about equally distant from the places, either in the line joining them or on either side as may be most convenient, and level the instrument. Let the staff-bearer set up the levelling-staff at A, and having sighted to it and obtained the height of the vane at that station, let the staff be removed to B, and do the same. Then the difference of the two heights, without any corrections, will be the difference of level of the places; that place being the higher, at which the height of the vane is the less.

If the surface of the ground between the places is such that when the level is placed at equal or nearly equal distances from them, the line of apparent level of the instrument would pass below one place or too high above the other, as in Fig. 124, it may be placed in any position C, that will permit the sights to be taken to both places. Then, having measured the horizontal distances from A to C, and from C to B, we proceed as above, except that the observed heights A*a* and B*b* must be corrected by the differences between the true and apparent level, as taken from the table:

or, which is generally better, by the tabular quantities diminished by a $\frac{1}{6}$ part. It may, however, be observed that when neither of the distances AC nor BC, exceeds five or six chains, the corrections are so small that they may be generally omitted.

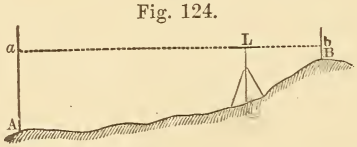
When a valley intervenes between the two places so that there is no suitable intermediate situation for the instrument, it may be placed at one of them; and then the difference of level may be determined as in the preceding problem.

EXAMPLES.

1. Let the observed height A*a*, Fig. 123, be 7.343 ft. and B*b*, 3.635 ft.; then the difference of these, 3.708 ft. is the difference of level of A and B, the place B being higher than A.

2. Let the observed height A*a*, Fig. 124, be 8.457 ft.;

Fig. 124.




B*b*, 1.525 ft.; the distance from A to C, 24.1 ch.; and the distance from C to B, 8.2 ch.

The correction for the distance 24.1 ch., taken from the table and diminished by a $\frac{1}{6}$ part, is 0.050 ft., and for the distance 8.2 ch., it is 0.006 ft. The corrected heights are therefore A*a* 8.407 ft., and B*b* 1.519 ft. Hence the difference of level is 6.888 ft.

## PROBLEM III.

*To find the difference of level of two places not visible from each other, or if visible, differing considerably in level.*

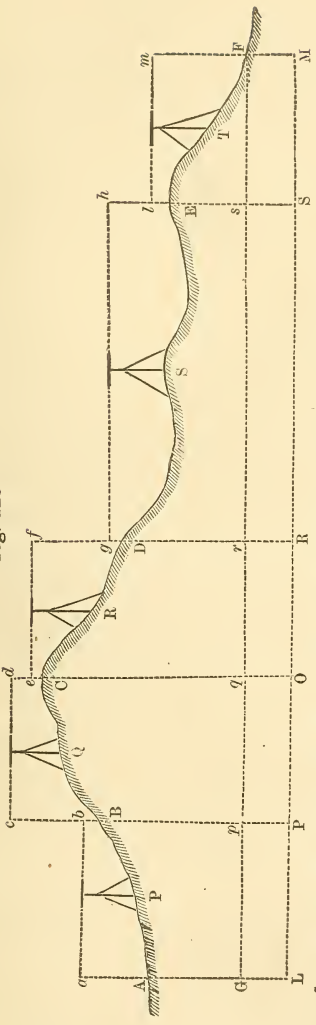


Fig. 125

Let A and F, Fig. 125, be the two places. Place the level in some position P, that will permit a sight to be taken to A, and also to some other place towards F, at about the same distance from the instrument. Having levelled the instrument, let the staff-bearer set up the staff at A, and when the sight has been taken, let him note the height of the vane as a first *back-sight*. Then let the staff be taken to some station B, about as far from the instrument as that is from the station A, and when the sight has been taken, let the height of the vane be noted as the first *fore-sight*. Next, the staff-bearer remaining at B, take the level to some suitable place Q, beyond B, and proceed to take a back-sight to B, and then a fore-sight to a new station C. In the same manner the operation must be continued from C to D, from D to E, and from E to F; the number of intermediate stations necessary to be taken, depending on the irregularities in the

ground and on the difference of level of the given places.

Take the difference between the sum of the back-sights and the sum of the fore-sights, and it will be the difference of level of E and F; the place F being *higher* or *lower* than A, according as the sum of the back-sights is *greater* or *less* than that of the fore-sights.*

*Note.*—It is not necessary that either the intermediate stations or the places of the instrument should be in the direct line between the given places; and frequently it will be found convenient to deviate considerably from that line. It may further be observed that although it is generally best to take each pair of the sights at equal or nearly equal distances, as the correction for apparent level is thus avoided, and also a slight deviation of the axis of the level from parallelism with the line of collimation will not then sensibly affect the accuracy of the result, yet sometimes in order to diminish the number of stations the sights are taken at unequal distances. When this is done, the distances must be measured and the sights be corrected as directed in such case, in the last problem.

---

* Let GF be a line of level through the place F; then AG is the difference of level of the two places A and F. Now we have,

Sum of back-sights,
$= Aa + Bc + Ce + Dg + El = Aa + bc + Bb + Ce + Dg + El$ Also,

Sum of fore-sights,
$= Bb + Cd + Df + Eh + Fm = Bb + Ce + ed + Dg + gf + El + lh + F$
$= Fm + lh + gf + ed + Bb + Ce + Dg + El = qd + Bb + Ce + Dg + El$
$= pc + Bb + Ce + Dg + El = AG + Aa + bc + Bb + Ce + Dg + El.$

The difference of these sums is AG, the difference of level of the places A and F.

EXAMPLE.

Let the back-sights and fore-sights, Fig. 125, be as in the following table.

| | Back-sights. | | Fore-sights. |
|---|---|---|---|
| 1. | 7.103 ft. | | 1.566 ft. |
| 2. | 9.227 | | 3.178 |
| 3. | 1.236 | | 9.415 |
| 4. | 1.610 | | 6.367 |
| 5. | 2.125 | | 9.910 |
| Sum | 21.301 | Sum | 30.436 |
| | | | 21.301 |
| | | Diff. | 9.135 |

Hence the difference of level of A and F is 9.135 ft., and as the sum of the back-sights is less than that of the fore-sights, the place F is lower than A.

# CHAPTER IX.

## TOPOGRAPHY.

Topography is a branch of surveying,* the object of which is to determine and designate on a map, the various undulations and inequalities in the surface of a particular place, tract of land or district of country. A map in which these inequalities, the courses of streams, and sometimes other circumstances, as the positions and extents of forests, marshes, &c., are designated, is called a *topographical map.*

In addition to the boundaries and content of a tract of land, it is frequently required that the various slopes and irregularities of the surface should be determined and designated, in order to give a more complete view of the ground, and to afford the means for an appropriate location of buildings or works of any kind that may be designed to be erected on it.

If we assume the surface of a tract of land to be intersected by a number of level surfaces or horizontal planes† at equal distances from one another, and transfer all the lines of level in which these planes meet the surface of the ground to an assumed horizontal

* The term surveying is here used in a more extended sense than as defined in the first chapter.

† In tracts of any moderate extent, the surfaces of level may, for the purpose for which they are here introduced, be regarded as horizontal planes.

plane passing through the lowest point, making them occupy positions on that plane, corresponding with their positions on their respective planes, the variations in the distances of the lines from one another, when thus transferred, will indicate the variations in the inclination of the ground. For as the difference of level from line to line is the same, it is evident that the horizontal distances of the lines, taken in any direction, will diminish as the inclination in that direction increases. Thus in ABCD, Fig. 129, which represents a small tract, of which the length AB is 1000 feet and the breadth AD, 800 feet, the lines 10, 20, 30, &c., represent lines of level in which horizontal planes at the distance of 10 feet from one another, intersect the surface of the ground. The lowest level passes through the point F, at which the stream EF leaves the tract. From F the ground rises more rapidly to the left than to the right, as is indicated by the lines of level being nearer to one another on that side than on the other. In passing from F towards the corner B of the tract, we may observe that the acclivity, which is gentle, increases till we come to the 30 feet level, then diminishes to the 40 feet level; it then again increases, and more rapidly, to the 60 feet level, and lastly slightly diminishes to the 70 feet, or highest level. In descending, the declivity continually diminishes to B. From the point *l* in the side AD, the ground descends moderately towards the corner A, the declivity diminishing till the surface becomes nearly level; and from the stream, towards A, the acclivity, which is slight, diminishes, so that between the two branches of the 30 feet line of level and the corner A, the ground is nearly level.

The distance which should be taken for the distance of the assumed planes from one another, must depend on the extent of the survey, the inequalities in the surface, and the degree of minuteness with which it is required that they should be designated. It may vary from 3 or 4 to 20 or 30 feet, according to circumstances.

The levelling required for a topographical survey may be performed either with a level or theodolite. Where there is considerable ascent or descent in the ground, the latter is the most convenient instrument; and although the results obtained with it are not in general as accurate as those that may be obtained with a good level, they are, when due care is taken, sufficiently so for the object in view. When the theodolite is used, the sight should be taken to a point at the same height above the ground at the station, as the axis about which the upper telescope revolves is above the ground at the place of the instrument. To do this, let an assistant place and clamp the vane of a levelling-staff at that height, or make a mark at the same height on a pole; and when he has taken the staff or pole to the station and set it up vertically, sight to the vane or mark. When this is done, the difference of level between the station and place of the instrument, expressed in 100th parts of the horizontal distance of the two, will be indicated on the vertical limb. It is however better, except when the horizontal distance is quite small, to obtain the difference of level from that distance and the angle of elevation or depression.

## PROBLEM I.

*Having given the back-sights and fore-sights taken to a number of consecutive stations, or to two or more connected series of stations, to determine the heights of the stations above a line of level or surface of level through the lowest.*

1. *When there is but one series of stations.* Assume for the height of the first station above some assumed line of level, any quantity taken at pleasure, observing however to make it sufficiently great for the assumed line of level to be lower than the lowest station, or at least as low. To the assumed height of the first station add the first back-sight, and from the sum subtract the first fore-sight, and the remainder will be the height of the second station above the assumed line of level. With this height and the second back-sight and second fore-sight proceed in like manner to find the height of the third station; and thus on to the last. Now subtract the least of the heights obtained, which must be that of the lowest station, from each of the others, and the remainders will be the heights of the other stations above the line of level passing through the lowest.

2. *When there are two series of stations connected by intervening sights between the first station of the first series and the first of the second series.* Assume for the height of the first station of the first series, a quantity sufficiently great for the assumed surface of level to be below all the stations, or at least as low as

the lowest, and proceed as directed above, to obtain the heights of the other stations of that series above the assumed surface of level. Then commencing again with the assumed height of the first station, proceed in like manner with the sights connecting that with the first station of the second series, and with the sights to the stations in this series, to find the heights of all these stations. When this is done, subtract the least of all the heights obtained from each of the others, and the remainders will be the required heights.

3. In like manner, whatever be the number of the series of stations, the heights of all the stations, above a surface of level passing through the lowest, may be obtained.

*Note.* If it is required to find the heights of the stations above a line or surface of level at a given distance below the lowest or any other given station, it is easily performed by applying to the heights of the stations above the assumed level, the difference between the height of that station above the assumed level and its height above the given level.

### EXAMPLES.

1. Taking to the nearest length of a foot, the back-sights and fore-sights given in the example to the last problem of the preceding chapter, it is required to find the heights of the stations A, B, C, &c. Fig. 125, above a line of level passing through the lowest.

Assume the height AL, of the station A, above an assumed line of level LM to be 15 feet. Then we have

Feet.

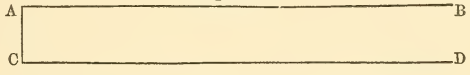
15, height of A, above LM,

15 +7.1—1.6=20.5 " B, "
20.5+9.2—3.2=26.5 " C, "
26.5+1.2—9.4=16.3 " D, "
16.3+1.6—6.4=11.5 " E, "
11.5+2.1—9.9= 5.7 " F, "

Subtracting 5.7 from each of the above heights, we have the heights above GF, the line of level through F. These are, for A, 9.3ft.; B, 14.8 ft.; C. 20.8 ft.; D, 10.6 ft.; E, 5.8 ft.; and F, 0 ft.

2. Let the back-sights and fore-sights, taken from A to B, Fig. 126, from A to C, and from C to D, be as

Fig. 126.




given below; to find the heights of the stations along AB, AC, and CD, above a surface of level through the lowest station.

AB

| | B-sts. | F-sts. |
|---|---|---|
| 1 | 8.7 ft. | 3.8 ft. |
| 2 | 1.1 | 9.9 |
| 3 | 0.7 | 9.3 |
| 4 | 2.2 | 8.0 |
| 5 | 1.9 | 7.1 |
| 6 | 7.4 | 4.2 |

AC

| | B-sts. | F-sts. |
|---|---|---|
| 1 | 8.5 ft. | 5.2 ft. |
| 2 | 0.6 | 9.1 |

CD

| | B-sts. | F-sts. |
|---|---|---|
| 1 | 3.1 ft. | 8.6 ft. |
| 2 | 2.6 | 7.7 |
| 3 | 1.5 | 9.3 |
| 4 | 4.2 | 7.9 |
| 5 | 3.8 | 7.3 |
| 6 | 9.3 | 2.2 |
| 7 | 2.8 | 7.4 |

By proceeding as directed in the rule, we obtain the following heights of the stations above the surface of level passing through the lowest.

| | AB | | AC | | CD |
|---|---|---|---|---|---|
| 1 | 30.8 ft. | 1 | 30.8 ft. | 1 | 25.6 ft. |
| 2 | 35.7 | 2 | 34.1 | 2 | 20.1 |
| 3 | 26.9 | 3 | 25.6 | 3 | 15.0 |
| 4 | 18.3 | | | 4 | 7.2 |
| 5 | 12.5 | | | 5 | 3.5 |
| 6 | 7.3 | | | 6 | 0.0 |
| 7 | 10.5 | | | 7 | 7.1 |
| | | | | 8 | 2.5 |

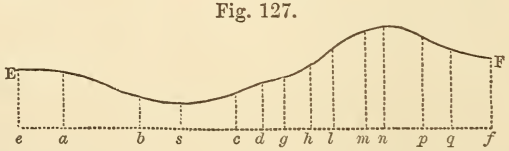
## PROBLEM II.

*To determine the inequalities in the surface of the ground along a line running in a given direction, and to draw an irregular or curved line to represent them.*

Let short stakes be driven at the beginning and end of the line, and at each point along it where there is any material change in the inclination of the ground; and let the horizontal distance of each stake from the beginning of the line, or the distances from stake to stake, be measured. Then level from stake to stake, using intermediate stations whenever the difference of level between any two is too great to permit sights to be taken to both from a single position of the instrument. Find, by the last problem, the heights of the stations, where the stakes are placed, above a line of level passing through the lowest, or above any assumed or given line of level.

Draw a straight line *ef*, Fig. 127, to represent the

Fig. 127.




line of level to which the heights of the stations are referred, and on it make *ea*, *eb*, *es*, *ec*, &c., equal to the distances of the stations from the beginning of the line. From the points *e*, *a*, *b*, *s*, &c., draw lines per-

pendicular to *ef*, and make them equal to the heights of the respective stations. Through the tops of these perpendiculars, draw the curved line EF, which will be the line required to be drawn.

The line EF is called a *Profile* of the ground in the direction of the given line.

*Note.* The heights of the perpendiculars are frequently taken from a scale three or four times as great as that used in laying off the horizontal distances. When this is done, the curved line, or profile, as it is still called, indicates more distinctly the lesser changes in the inclination of the ground.

### EXAMPLE.

The distances of the stations along a given line, measured from the beginning of the line, and their heights above a given line of level, determined by the last problem, from sights taken on the ground, being as below, the profile of the ground in the direction of the line, obtained by taking the heights from a scale three times as great as that from which the distances are taken, will be that of Fig. 127.

| Sta. | Dist. | Ht. | Sta. | Dist | Ht. |
|---|---|---|---|---|---|
| 1 | 0ft. | 41.1ft. | 8 | 610ft. | 41.9ft. |
| 2 | 90 | 38.3 | 9 | 655 | 55.0 |
| 3 | 230 | 22.0 | 10 | 725 | 69.2 |
| 4 | 336str. | 15.3 | 11 | 770 | 72.0 |
| 5 | 445 | 21.1 | 12 | 850 | 69.1 |
| 6 | 505 | 32.5 | 13 | 915 | 56.5 |
| 7 | 555 | 36.5 | 14 | 1000 | 50.1 |

At the fourth station a stream of water crosses the line, and is noted by the letters *str.* placed by the side of the distance.

## PROBLEM III.

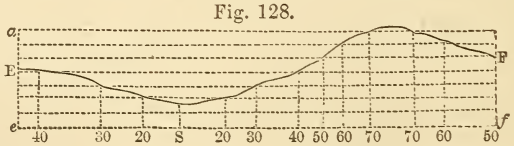
*To determine those points along a line running in a given direction, that are at given heights above a given line or surface of level.*

Proceed, as directed in the last problem, to find the heights above the given line or surface of level, of those points in the line where there is any material change in the inclination of the ground, and also their distances from the commencement of the line. Observe between which two of the heights obtained, any one of those given falls, and take their difference. Also take the difference between the given height and that one of the two which appertains to the point nearest the beginning of the line. Then, as the first difference : the second : : the horizontal distance between the points : to a fourth term, which added to the distance of the point nearest the beginning of the line, will give the distance of the required point.

Proceed in the same manner with the other given heights.

Or we may draw, by the last problem, a profile of the ground, as EF, Fig. 128, in reference to the given

Fig. 128.




line of level *ef*, and on *ea*, perpendicular to *ef*, set off the given heights, taking them from the scale used in setting off the heights of the stations. Then, through the points in the line *ea*, draw lines parallel to *ef*, as

in the figure; and from the points in which these meet the profile EF, draw lines parallel to *ea*. The distances from *e*, at which these last lines meet *ef*, will be the horizontal distances of the required points, from the beginning of the line.

EXAMPLE.

Let the data found on the ground be the same as in the example to the last problem, and let the given heights of the required points be 10, 20, 30, 40, &c. feet.

The distances and heights of the stations being the same as for the profile in Fig. 127, we shall obtain a similar profile EF, Fig. 128. Drawing the lines parallel to *ef* at distances from it, equal to 10, 20, 30, &c. feet, we find that the first line above *ef* does not meet the profile EF. Consequently there is no point in the latter so low as 10 feet above the given level. Each of the other parallels meets the line EF in two points. There are therefore two points in the line at the height of 20 feet above the given level; two at the height of 30 feet; and so on to 70 feet. The distances e, 40; *e*, 30; &c., on the line *ef*, are the distances of the required points from the beginning of the line.

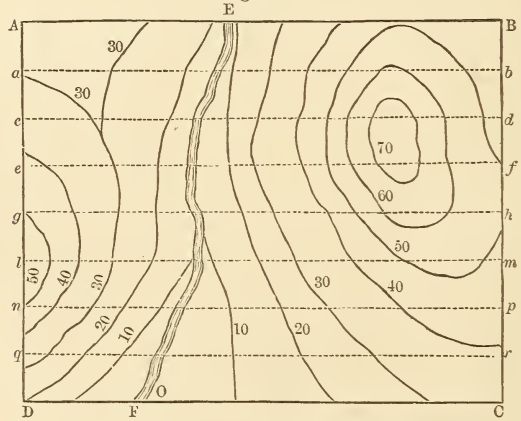
In illustration of the first method of finding the distances of the required points from the beginning of the line, as the given heights of the stations evidently show that there is no point in the line so low as 10 feet above the given level, let it be required to find a point at the height of 20 feet. On examining the given heights of the stations, we perceive that there must be two such points; one between the 3d and

4th stations, and one between the 4th and 5th. For the first of these we take the difference between 15.3 and 22.0, which is 6.7; the difference between 20.0 and 22.0, which is 2.0; and the difference between the distances 230 and 336, which is 106. Hence, as 6.7 : 2.0 : : 106 : 32. Adding therefore 32 to 230, we have 262 feet for the distance of this point.

## PROBLEM IV.

*To determine the undulations and inequalities of the surface in a tract of land* ABCD, Fig. 129, *and to draw a topographical map designating them.*

Fig. 129.




With a compass or theodolite, run a number of lines, *ab*, *cd*, *ef*, &c., across the tract, parallel to one of the sides, as AB; making them nearer together or farther apart, according to the inequalities in the ground and the degree of minuteness with which it is intended to designate them. Drive stakes at the beginnings and ends of the lines AB, *ab*, *cd*, &c., and at all the points along them where there is any material change in the inclination of the ground, and proceed to level from

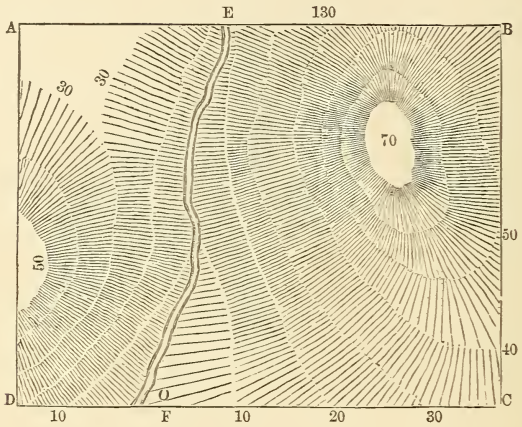
stake to stake along these lines and the line AC; also measure the distances of the stakes from the commencement of the lines, or from one another. Then, by problem I., find the heights of all the stations where the stakes are driven, above a surface of level passing through the lowest station.

Now, having drawn the lines *ab*, *cd*, *ef*, &c., in their proper positions on the map, determine, by the last problem, the points in these lines and the lines AB and DC, that correspond to heights in the lines on the ground, of 10, 20, 30, 40, &c., feet, above the surface of level passing through the lowest station, or to any other heights, increasing by equal differences, that may be deemed expedient. Through each set of points appertaining to the same height, draw a curve line. The curve lines thus drawn will represent lines of level of 10 feet, 20 feet, &c., or of the number of feet of the heights used in obtaining them, whatever that may be.

These lines serve to indicate the changes in the inclination of the ground. But it is usual, instead of drawing them distinctly with ink, to draw them with a pencil only, or faintly with Indian-ink, and then to shade the map by short straight lines, drawn perpendicularly from each curve of higher level, to that of the next lower; the lines being drawn closer together and rather heavier as the distance between the lines of level diminishes. For those parts of the ground that are level, or very nearly so, the shading is omitted. The greater or less darkness of the shading on the different parts of the map, therefore, indicates a greater or less inclination in the ground in those parts; and the omission of the shading in any parts, indicates that

in those places the surface is level, or very nearly so In Fig. 130, we have a map of the tract, shaded as described above.

Fig. 130.




*Note.*—It is not necessary that the lines on which the levels are taken should be run parallel to one another. They may be run making given angles with any given lines of the survey. Sometimes it is desired that the surface of a particular part of the tract should be designated with more minuteness than is important for other remote parts. In this case it will be found convenient to determine the position of some point near the middle of the part which it is desired particularly to designate, and then to run lines from this point, in directions making angles of 20° or 30° with one another.

It may be further remarked, that instead of taking the surface of level through the lowest point, as the plane of reference, we may, if preferred, take that through the highest. The former is however the one generally taken.

The operations to be performed in this problem, so far as observations on the ground and numbers are concerned, are merely repetitions of those which have been exemplified in the preceding problems. An example is not therefore necessary.

# CHAPTER X.

## THE COAST SURVEY.

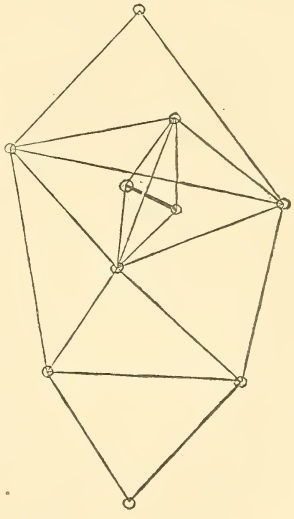
The Coast Survey is an establishment of the United States government to obtain the measurements necessary to a correct map of the country. Similar institutions are in existence in all civilized countries. Part of the work is astronomical, and part is trigonometric. The latter portion only falls within the scope of this treatise.

The method may be briefly described as follows: A base line is carefully measured, from the two ends of which angles are taken to prominent points. Triangles are thus formed, of which one side and the angles are known. The other sides may then be calculated, and will form the base of new triangles. Thus a network of triangles may be established over the country, from which the configuration of the coast line, location of towns, mountains, rivers, &c. may be accurately obtained.

As any error in the length of the base line would be carried through the whole system of triangles and multiplied many times, it is of the utmost importance that it be measured with the greatest possible accuracy. This is done by the use of two measuring bars, each 6 metres (about 20 feet) long, supported near each end by a tripod. One of these being placed in the line to be measured and levelled, the other is carefully placed against its end, aligned and levelled. The first bar is then moved forward, and the process repeated till the end of the line is reached. Care is taken that the

pressure of the advance bar does not disturb the position of the other. The bars are compensated for changes of temperature by constructing them of combinations of iron and brass rods, as in the case of the gridiron pendulum. Modern base lines are usually made about 10 kilometres (about 6 miles) in length, and so carefully are all chances for error eliminated, that it is possible to obtain accuracy within $\frac{1}{5}$ of a centimetre, or $\frac{1}{10}$ of an inch.

In triangulating from this base line it is considered desirable to obtain two nearly equilateral triangles, one on each side. The long diagonal of the trapezium thus formed may be calculated by two independent computations, thus obtaining a check to the accuracy of the work. The sides of the triangles are usually from 25 to 60 miles in length, and the stations are located on elevated peaks, easily distinguishable from the surrounding country. The instruments used for reading angles have circles of from 20 to 40 inches in diameter, thus permitting great accuracy in graduation. Each angle is read from 25 to 50 times, and on all parts of the divided limb. The possible error must not exceed one-half a second.



As the triangles are portions of the surface of the earth, they are spherical triangles, and the three angles should be slightly in excess of 180°. The amount of this *spherical excess*, as it is called, may be calculated if the length of the sides be known.

The triangles must also be reduced to the sea level. The stations being at different elevations, incorrect results would be obtained unless a common level were taken for reference. This reduction is easily performed by calculating the amount of convergence of two radii of the earth through the two ends of the line.

The bearings of various lines are then determined by ascertaining the direction of a true meridian by methods similar to those in chapter VI. The latitude of many of the stations is also carefully deduced by astronomical methods. From these the length of a degree of the meridian in different latitudes can be calculated, and the true shape and size of the earth known.

The large triangles thus formed are filled up by measurements of small triangles built on the sides of the large, and the stations may thus be brought as near together as desired, and all prominent points located. From these data maps of the whole country may be constructed.

# APPENDIX.

## CONTAINING INSTRUCTIONS FOR SURVEYING THE PUBLIC LANDS OF THE UNITED STATES.

BY GEORGE H. HOLLIDAY, A.M.

THE following statement of the mode of making the surveys of public lands of the United States, and the proper method of subdividing, is prepared to meet the wants of landowners and surveyors in those surveys. In most of the leading principles I take pleasure in acknowledging my indebtedness to a pamphlet of instructions prepared, I believe, by the late excellent Surveyor-General for the district of Illinois and Missouri, D. A. Spaulding, Esq., and furnished to applicants at that office, for information.

1. The surveys of the public lands of the United States are based on a line running east and west, called the base line.

2. From certain noted points, as the mouths of principal rivers, are lines run due north, called principal meridians.

3. Townships are bodies of land, six miles square, subdivided into thirty-six parts called sections. The following represents a township with the mode of numbering sections:

North.

| | | | | | |
|---|---|---|---|---|---|
| 6 | 5 | 4 | 3 | 2 | 1 |
| 7 | 8 | 9 | 10 | 11 | 12 |
| 18 | 17 | 16 | 15 | 14 | 13 |
| 19 | 20 | 21 | 22 | 23 | 24 |
| 30 | 29 | 28 | 27 | 26 | 25 |
| 31 | 32 | 33 | 34 | 35 | 36 |

4. Those lines of townships running north and south are called range lines. Rows or tiers of townships running north and south are called ranges. Tiers of townships east and west are called townships, and the lines dividing them are called township lines. Townships are numbered from the base line and the principal meridians, affording an easy method of describing lands. Thus, if the township above be considered township three north of the base line, in range four west of the third principal meridian, the one joining it on the north will be township four range four, the one to the west township three range five, which cannot be mistaken for any other. Sections and parts of sections are described with the same ease and certainty.

5. In running out townships as described hereafter, many errors arise which it would be inconvenient to carry to any considerable distance; to obviate these, generally at intervals of thirty miles, or five townships, a line is run due east and west from the principal meridians, called a standard or correction line. This line also serves to correct the difficulty otherwise arising from the convergence of the meridians.

6. A chain is a measure of four rods. It is more convenient in practice to use a half-chain of two rods, divided into fifty links; each link thus is the one-hundreth part of a chain, and is so expressed and used in taking field-notes and making calculations.

7. In running standard lines, corners are established every 40 chains, for quarter section and section corners; at the end of every sixth mile a township corner is made. These corners are starting points for surveys to the north of this line, and do not apply to any surveys to the south.

8. The mode of laying off a township is as follows:—The south line is a standard line or the north line of a township already surveyed. The east line is a principal meridian or the west line of the township to the east; it only remains to establish the west and north lines. The surveyor therefore proceeds to the south-west corner of the township already established, and runs due north, establishing, as he proceeds, corners at every 40 chains for quarter-section and section corners. These corners were formerly made in timber, by taking two trees as witnesses, by boxing into them, marking with the number of the section, and accurately taking their course and distance; now the regulations require four such witness-trees at section and township corners, and two at quarter-section corners. Formerly in prairie a small mound was thrown up, now one of considerable size is required, in which must be buried stones of size and shape to be described in the field-notes, or a certain quantity of charcoal. At the end of six miles the surveyor establishes his township corner, and the west line is finished. All the corners now established on this line belong to the township lying west, and not to the one now to be run out. The corners for this township are to be made as described hereafter. The surveyor next proceeds to the north-east corner of the township already established and runs west, setting at every 40 chains a temporary post, proceeding until he strikes the west line of the township, and notes down the distance he falls to the north or south of the township corner. If there is any excess or deficiency of six miles in this line, it is all thrown into the last 40 chains, or the west half of the north line of section 6; then if the line run is not a straight line from township corner to corner, he returns, and by offsets from the temporary posts already set he establishes all the corners in a straight line. All the corners on this line are intended for the township north, and not for the one now laid off. In case the township closes on a standard

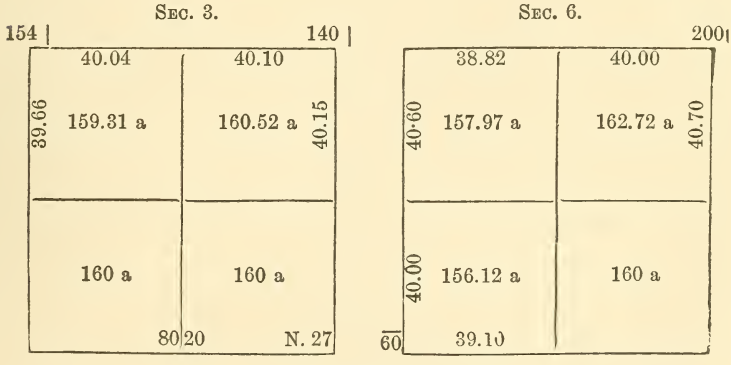
line, the north-west corner is made at the intersection of the west line with the standard line, whether it be more or less than six miles, and without regard to the township corner already established for the townships north, the surveyor giving in his notes the length of the lines and the distance between his corner and the one on the standard line.

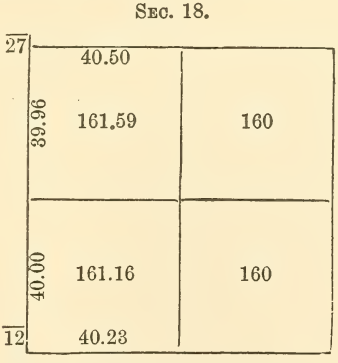
9. The township lines being established, the surveyor proceeds to the subdivision. He commences at the corner to sections 35 and 36 on the south line of the township and runs north, makes a corner at 40 chains, being the quarter-section corner between 35 and 36, and one at 80 chains, corner to 25, 26, 35, 36. Here he turns east, and at 40 chains sets a temporary post, and proceeds in the same line until he comes to the corner to 25 and 36 on range line; he notes the distance and the falling to the north or south of this corner, then returns, and by offsets from the line run establishes the quarter-section corner between sections 25 and 36, at an equal distance from each section corner, and in a straight line with them; then, taking a new start from the corner to 25, 26, 35, 36, proceeds in the same way to run out sections 25, 24, 13, and 12. Then from the corner to sections 1, 2, 11, 12, he runs north 40 chains, establishes the quarter-section corner between 1 and 2, then runs north until he strikes the north line of the township; here he establishes his corner, giving in his notes the length of the line and the distance from the corner already established for the township north. He then returns to the south line of the township, and commencing at corner to sections 34 and 35, runs out another tier of sections in the same manner as the first, and so on until he has completed four tiers of sections. He then returns to corner to sections 31 and 32, and proceeds in the same way with the tier of sections on the east; but at every section corner he also runs west, makes a quarter-section corner at 40 chains, and proceeds in the same line until he strikes the range line, where he makes his corner, giving the falling to the north or south of the corner already established for the township west.

10. It will be perceived that all the section corners are established on lines running north, and these, as well as the quarter-section corners on these lines, are established permanently in

the first instance, and all the quarter-section corners on east and west lines, are corrected to an average distance and in a straight line with the section corners, except those in the western tier of sections, which are established permanently in the first instance.

It will also be seen that all excess or deficiency of measurement is thus thrown into the north half of the sections on the north and the west half of the sections on the west of township; these sections are hence called fractional sections. A complete township with fractions would occupy too much space. Below are three fractional sections, one in the north, one in the northwest, and one in the west of townships; they are taken from the government surveys in township 11 north, range 8 west of third principal meridian:—




The numbers on the lines represent the length in chains and

hundredths; the numbers at offsets on range and township lines represent the difference in links with corners in other townships; the number in the south-east corner of sec. 3 shows the falling north in running the south line.

The general principles of making the public surveys are now before the reader; a knowledge of them is not only important but indispensable to every surveyor.

11. No corners are established in the interior of sections, and as the public lands are sold in tracts of 40, 80, 160, 320, or 640 acres, it devolves upon other surveyors to make the divisions. This at first view would seem a very simple affair; but the difficulties continually arising, and the frequent conflicting surveys by different surveyors, and even clashing surveys by the same, prove the necessity of some well-grounded system being adopted and universally acted upon to insure justice to all parties interested.

12. As the land was originally owned and surveyed by the United States, and conveyed by the corners thus established, it is of the first importance to find these corners before any divisions are attempted. This is frequently a difficult matter. Many witness-trees were killed by boxing; many others die, and even the roots disappear before the land is occupied. In prairie there are many hills that may be mistaken for mounds, the rooting of hogs destroys many more, and there is reason to believe that in many cases, particularly in prairies, corners never were established. In case of the loss of these corners they must be renewed in the same manner in which they should first have been established.

13. In case of loss of corners on a standard line or principal meridian, the line must be followed and measured from one known correct corner to another on this line. This is necessary in order to be assured that both the course and measurement is the same used by the first surveyors; many causes of disagreement in both are frequent, the causes of which it is not here necessary to explain. The missing corner should be placed in a straight line between the original corners, and at an equal or proportional distance.

14. Should any discrepancy, however, appear in either the

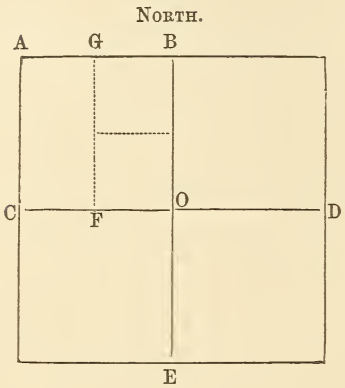
course or length of this line, it will be necessary to examine the section lines intersecting it. The corners on the standard line should be at the points of intersection of the sectional lines originating or terminating in it. The finding of these points as called for by the field-notes, frequently relieves the surveyor of a great deal of difficulty, and throws unexpected light on some of the most complicated cases of difficulty. This remark applies also to all other lines of townships.

15. In running a township corner, the better plan is to commence at the nearest known corner on the range line, and then run north or south past the township corner, until another corner is found, then place the missing corner at an equal or proportional distance as required by the field-notes, and in a straight line; should this, from any circumstances, appear not to have been the point where the original corner was established, take the range of the corners on the township lines to the east and west, and should these unite in one point on the north and south line, the presumption is that that is the true place for the corner, and it should be there established.

16. Section and quarter-section corners on the north and south lines are to be renewed in the same manner, by starting at some corner on these lines, and running past the missing corners until some reliable corner is found, then establishing the missing corner in a straight line and at its proper distance. If the surveyor, in the extension of these lines, strikes a township line, he is to proceed no further on the same course, but the corner here, if missing, must be renewed, as in sec. 13, before any further steps are taken. It is important to re-establish all section corners on north and south lines, because they were originally established in this mode, on the same course and at equal distances, while the lines of sections running east and west are of very unequal lengths and in almost all cases make an angle at section corners. Should any discrepancy, however, arise, these lines should be examined as in establishment of township corners.

17. When all the corners of the original survey are found or renewed, the surveyor is ready to make the subdivisions. and

not before. The following plan will represent the course to be pursued:




Suppose, for instance, a quarter-section is to be surveyed; say the north-west quarter. The corners at A, B, and C, are already established; it only remains to establish the corner at O.

Experience shows that the quarter-section corners on the north and south of sections, required to be made on corrected lines, are much more frequently out of place than those on the east and west, made on the first line run; consequently, as it is desirable to carry errors in the establishment of original corners no farther than possible, sections should be divided by a straight line between the quarter-section corners on the east and west These errors thus never pass the centre of the section. The surveyor should therefore commence at C and run east, set a random stake at 40 chains on his line, continue the same line until he intersects the section line on the east at D, and divides the distance and establishes his corner at O accordingly, without any regard to the corners at B and E.

18. Should a smaller division be necessary, as, for instance, the north-east quarter of the north-west quarter. On the line from C to D a temporary stake must be set at F, as well as O. These, corrected, will be the southern corners of the east half of the quarter-section. A line must be run from A to B, and at an equal distance the corner G established, then a line run

from G to F, and also from O to B, at equal distances from these points, corners are to be established, and the tract is surveyed. If in timber, it only remains to mark the south line, a measurement is unnecessary.

19. The peculiar value of this system consists in never running a line witnout a corner at the starting point and one at the termination. By this means both errors in course and distance are corrected, and if corners are not established in precisely the same place they originally occupied, justice, as far as practicable, is done to all interested parties.

20. Errors in the original surveys often cause a great deal of trouble; those most frequent deserve mention. The measurement was kept by *outs*, each out being the distance one set of pins carries the chain-carriers when the pins are changed to the fore chain-man. Eight outs make the 40 chains, or half-mile, the distance between the original corners. Sometimes the chain-carriers only made seven outs, by mistake, between corners, and sometimes nine. In all cases, however, where the original corners are found they are immovable, however much they may be out of the way. Sometimes a pin was lost, and, consequently, 50 links in every out were lost until the fact was discovered. When chain-carriers are fresh, they carry the chain much tighter than when they are tired; thus a great deal of want of uniformity may be accounted for.

21. Crooked lines are sometimes found. These are frequent in section lines running east and west; the quarter-section corner being made sometimes on a random line without correcting; sometimes the surveyor made his offset in the wrong direction; and, generally in the older surveys, great carelessness was manifested. Sometimes considerable crooks are found in township lines, so as to lead to the opinion that all the corners were made at random. In dealing with these great care must be exercised, and no opinions formed without the most decisive evidence.

22. In the taking of witness-trees there are often considerable discrepancies in the notes. In distance I have often found 10 links, sometimes a chain, more or less than the notes. Sometimes there is reason to believe the distance to one of the trees

was lost, and filled up at random in copying. In bearing there are fewer errors, but more important. In two instances I have found 30° mistaken for 50°. A very frequent error is made by letting the eye rest upon some leading division upon the face of the compass, and counting degrees in the wrong direction. In all cases where there are two trees these are easily rectified; there are four data for determining the corner—the distance and bearing to each tree—when three of these agree, the presumption is that it is correct.

23. The labours of the surveyor, to result in any lasting good, should be spread on record; this is the duty of the county surveyor. But not only does the office fall into the hands of men incompetent to perform the work, but in no instance with which I am acquainted are the records kept in a manner likely ever to settle disputes, or render satisfaction to any intelligent person consulting them. Below is the plan adopted by me, and which should be followed in all records of surveys. It is simple and satisfactory. It will be seen that in all sectional lines and their parallels I make the variations entirely by the scale, noting by that the degree I run on.

FEBRUARY 23, 1854.

For David Holmes, S. E. ¼, N. W. ¼. Sec. 21, T. 10, R. 9. Magnetic var. E. 8° 00′.

Chain-carriers, Riley Hicks and A. B. Carr, sworn.

| Chains & Courses. | | |
|---|---|---|
| | | Began at ¼ section corner, between secs. 20 and 21. Stake in original mound; thence |
| North. | | Index 8° 00′. |
| 13 | 00 | Entered brush and young timber, bears E. and W. |
| 27 | 43 | Branch 6 links wide, runs E. |
| 32 | 47 | Left brush, entered Hubbard's field. |
| 40 | 00 | Mound ploughed up, set stake; thence |
| 20 | 27 | Left field, entered hazel. |
| 30 | 00 | Entered edge of prairie, as in field-notes. |
| 40 | 00 | Mound ploughed up between secs. 16 and 17. |
| 80 | 00 | Mound to 8, 9, 16, 17, lost; thence |
| 26 | 00 | Enter barrens, N. E. and S. W. as in field-notes. |

| Chains & Courses. | | |
|---|---|---|
| 40 | 00 | Mound between 8 and 9 lost, land broken. |
| 47 | 00 | Joe's Creek, 10 links wide, bears S. W. |
| 50 | 00 | Former bed of creek, 5.00 chains farther north than called for by field-notes. |
| 79 | 85 | 14 links east of corner to 4, 5, 8, 9, set by me in survey of Jayne's land. Then corrected stake corner to sections 16, 17, 20, 21, thence |
| East. | | Index 8° 00′. |
| 20 | 00 | Set stake. |
| 40 | 44 | 6 links south of ¼ sec. corner between sections 16 and 21, S. W. witness standing, N. E. cut down, renewed by marking elm 9 inches diameter, N. 63° E. 49 links. |
| | | Then from this corner |
| South. | | Index 8° 00′. |
| 20 | 00 | Set stake. |
| 41 | 06 | 57 links east of corner centre of section, set by me in survey of Carr's land. Corrected this line back, and set stake above at average distance, and took witness black oak, 32 inches diameter. N. 57° E. 9 links. |
| | | Then from corrected stakes, N. W. corner of E. ½, N. W. ¼ this section. |
| South. | | Index 7° 40′. |
| 20 | 00 | Set stake. |
| 40 | 48 | 8 links west of corner set by me in survey of Carr's land corrected corner on above line and took witness hickory, 8 inches diameter. S. 5° W. 93 links. |

# MATHEMATICAL TABLES.


PAGE

TABLE I.—Logarithms from 1 to 10000..... 1

TABLE II.—Logarithmic Sines and Tangents..... 18

TABLE III.—Proportional Parts..... 63

TABLE IV.—Traverse Table..... 66

TABLE V.—Natural Sines and Cosines..... 79

# A TABLE

OF THE

# LOGARITHMS OF NUMBERS

FROM 1 to 10,000.

---

| N. | Log. | N. | Log. | N. | Log. | N. | Log. |
|---|---|---|---|---|---|---|---|
| 1 | 0.000000 | 26 | 1.414973 | 51 | 1.707570 | 76 | 1.880814 |
| 2 | 0.301030 | 27 | 1.431364 | 52 | 1.716003 | 77 | 1.886491 |
| 3 | 0.477121 | 28 | 1.447158 | 53 | 1.724276 | 78 | 1.892095 |
| 4 | 0.602060 | 29 | 1.462398 | 54 | 1.732394 | 79 | 1.897627 |
| 5 | 0.698970 | 30 | 1.477121 | 55 | 1.740363 | 80 | 1.903090 |
| 6 | 0.778151 | 31 | 1.491362 | 56 | 1.748188 | 81 | 1.908485 |
| 7 | 0.845098 | 32 | 1.505150 | 57 | 1.755875 | 82 | 1.913814 |
| 8 | 0.903090 | 33 | 1.518514 | 58 | 1.763428 | 83 | 1.919078 |
| 9 | 0.954243 | 34 | 1.531479 | 59 | 1.770852 | 84 | 1.924279 |
| 10 | 1.000000 | 35 | 1.544068 | 60 | 1.778151 | 85 | 1.929419 |
| 11 | 1.041393 | 36 | 1.556303 | 61 | 1.785330 | 86 | 1.934498 |
| 12 | 1.079181 | 37 | 1.568202 | 62 | 1.792392 | 87 | 1.939519 |
| 13 | 1.113943 | 38 | 1.579784 | 63 | 1.799341 | 88 | 1.944483 |
| 14 | 1.146128 | 39 | 1.591065 | 64 | 1.806180 | 89 | 1.949390 |
| 15 | 1.176091 | 40 | 1.602060 | 65 | 1.812913 | 90 | 1.954243 |
| 16 | 1.204120 | 41 | 1.612784 | 66 | 1.819544 | 91 | 1.959041 |
| 17 | 1.230449 | 42 | 1.623249 | 67 | 1.826075 | 92 | 1.963788 |
| 18 | 1.255273 | 43 | 1.633468 | 68 | 1.832509 | 93 | 1.968483 |
| 19 | 1.278754 | 44 | 1.643453 | 69 | 1.838849 | 94 | 1.973128 |
| 20 | 1.301030 | 45 | 1.653213 | 70 | 1.845098 | 95 | 1.977724 |
| 21 | 1.322219 | 46 | 1.662758 | 71 | 1.851258 | 96 | 1.982271 |
| 22 | 1.342423 | 47 | 1.672098 | 72 | 1.857332 | 97 | 1.986772 |
| 23 | 1.361728 | 48 | 1.681241 | 73 | 1.863323 | 98 | 1.991226 |
| 24 | 1.380211 | 49 | 1.690196 | 74 | 1.869232 | 99 | 1.995635 |
| 25 | 1.397940 | 50 | 1.698970 | 75 | 1.875061 | 100 | 2.000000 |

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 100 | 000000 | 0434 | 0868 | 1301 | 1734 | 2166 | 2598 | 3029 | 3461 | 3891 | 432 |
| 101 | 4321 | 4751 | 5181 | 5609 | 6038 | 6466 | 6894 | 7321 | 7748 | 8174 | 428 |
| 102 | 8600 | 9026 | 9451 | 9876 | 0300 | 0724 | 1147 | 1570 | 1993 | 2415 | 424 |
| 103 | 012837 | 3259 | 3680 | 4100 | 4521 | 4940 | 5360 | 5779 | 6197 | 6616 | 419 |
| 104 | 7033 | 7451 | 7868 | 8284 | 8700 | 9116 | 9532 | 9947 | 0361 | 0775 | 416 |
| 105 | 021189 | 1603 | 2016 | 2428 | 2841 | 3252 | 3664 | 4075 | 4486 | 4896 | 412 |
| 106 | 5306 | 5715 | 6125 | 6533 | 6942 | 7350 | 7757 | 8164 | 8571 | 8978 | 408 |
| 107 | 9384 | 9789 | 0195 | 0600 | 1004 | 1408 | 1812 | 2216 | 2619 | 3021 | 404 |
| 108 | 033424 | 3826 | 4227 | 4628 | 5029 | 5430 | 5830 | 6230 | 6629 | 7028 | 400 |
| 109 | 7426 | 7825 | 8223 | 8620 | 9017 | 9414 | 9811 | 0207 | 0602 | 0998 | 396 |
| 110 | 041393 | 1787 | 2182 | 2576 | 2969 | 3362 | 3755 | 4148 | 4540 | 4932 | 393 |
| 111 | 5323 | 5714 | 6105 | 6495 | 6885 | 7275 | 7664 | 8053 | 8442 | 8830 | 389 |
| 112 | 9218 | 9606 | 9993 | 0380 | 0766 | 1153 | 1538 | 1924 | 2309 | 2694 | 386 |
| 113 | 053078 | 3463 | 3846 | 4230 | 4613 | 4996 | 5378 | 5760 | 6142 | 6524 | 382 |
| 114 | 6905 | 7286 | 7666 | 8046 | 8426 | 8805 | 9185 | 9563 | 9942 | 0320 | 379 |
| 115 | 060698 | 1075 | 1452 | 1829 | 2206 | 2582 | 2958 | 3333 | 3709 | 4083 | 376 |
| 116 | 4458 | 4832 | 5206 | 5580 | 5953 | 6326 | 6699 | 7071 | 7443 | 7815 | 372 |
| 117 | 8186 | 8557 | 8928 | 9298 | 9668 | 0038 | 0407 | 0776 | 1145 | 1514 | 369 |
| 118 | 071882 | 2250 | 2617 | 2985 | 3352 | 3718 | 4085 | 4451 | 4816 | 5182 | 366 |
| 119 | 5547 | 5912 | 6276 | 6640 | 7004 | 7368 | 7731 | 8094 | 8457 | 8819 | 363 |
| 120 | 079181 | 9543 | 9904 | 0266 | 0626 | 0987 | 1347 | 1707 | 2067 | 2426 | 360 |
| 121 | 082785 | 3144 | 3503 | 3861 | 4219 | 4576 | 4934 | 5291 | 5647 | 6004 | 357 |
| 122 | 6360 | 6716 | 7071 | 7426 | 7781 | 8136 | 8490 | 8845 | 9198 | 9552 | 355 |
| 123 | 9905 | 0258 | 0611 | 0963 | 1315 | 1667 | 2018 | 2370 | 2721 | 3071 | 351 |
| 124 | 093422 | 3772 | 4122 | 4471 | 4820 | 5169 | 5518 | 5866 | 6215 | 6562 | 349 |
| 125 | 6910 | 7257 | 7604 | 7951 | 8298 | 8644 | 8990 | 9335 | 9681 | 0026 | 346 |
| 126 | 100371 | 0715 | 1059 | 1403 | 1747 | 2091 | 2434 | 2777 | 3119 | 3462 | 343 |
| 127 | 3804 | 4146 | 4487 | 4828 | 5169 | 5510 | 5851 | 6191 | 6531 | 6871 | 340 |
| 128 | 7210 | 7549 | 7888 | 8227 | 8565 | 8903 | 9241 | 9579 | 9916 | 0253 | 338 |
| 129 | 110590 | 0926 | 1263 | 1599 | 1934 | 2270 | 2605 | 2940 | 3275 | 3609 | 335 |
| 130 | 3943 | 4277 | 4611 | 4944 | 5278 | 5611 | 5943 | 6276 | 6608 | 6940 | 333 |
| 131 | 7271 | 7603 | 7934 | 8265 | 8595 | 8926 | 9256 | 9586 | 9915 | 0245 | 330 |
| 132 | 120574 | 0903 | 1231 | 1560 | 1888 | 2216 | 2544 | 2871 | 3198 | 3525 | 328 |
| 133 | 3852 | 4178 | 4504 | 4830 | 5156 | 5481 | 5806 | 6131 | 6456 | 6781 | 325 |
| 134 | 7105 | 7429 | 7753 | 8076 | 8399 | 8722 | 9045 | 9368 | 9690 | 0012 | 323 |
| 135 | 130334 | 0655 | 0977 | 1298 | 1619 | 1939 | 2260 | 2580 | 2900 | 3219 | 321 |
| 136 | 3539 | 3858 | 4177 | 4496 | 4814 | 5133 | 5451 | 5769 | 6086 | 6403 | 318 |
| 137 | 6721 | 7037 | 7354 | 7671 | 7987 | 8303 | 8618 | 8934 | 9249 | 9564 | 315 |
| 138 | 9879 | 0194 | 0508 | 0822 | 1136 | 1450 | 1763 | 2076 | 2389 | 2702 | 314 |
| 139 | 143015 | 3327 | 3639 | 3951 | 4263 | 4574 | 4885 | 5196 | 5507 | 5818 | 311 |
| 140 | 6128 | 6438 | 6748 | 7058 | 7367 | 7676 | 7985 | 8294 | 8603 | 8911 | 309 |
| 141 | 9219 | 9527 | 9835 | 0142 | 0449 | 0756 | 1063 | 1370 | 1676 | 1982 | 307 |
| 142 | 152288 | 2594 | 2900 | 3205 | 3510 | 3815 | 4120 | 4424 | 4728 | 5032 | 305 |
| 143 | 5336 | 5640 | 5943 | 6246 | 6549 | 6852 | 7154 | 7457 | 7759 | 8061 | 303 |
| 144 | 8362 | 8664 | 8965 | 9266 | 9567 | 9868 | 0168 | 0469 | 0769 | 1068 | 301 |
| 145 | 161368 | 1667 | 1967 | 2266 | 2564 | 2863 | 3161 | 3460 | 3758 | 4055 | 299 |
| 146 | 4353 | 4650 | 4947 | 5244 | 5541 | 5838 | 6134 | 6430 | 6726 | 7022 | 297 |
| 147 | 7317 | 7613 | 7908 | 8203 | 8497 | 8792 | 9086 | 9380 | 9674 | 9968 | 295 |
| 148 | 170262 | 0555 | 0848 | 1141 | 1434 | 1726 | 2019 | 2311 | 2603 | 2895 | 293 |
| 149 | 3186 | 3478 | 3769 | 4060 | 4351 | 4641 | 4932 | 5222 | 5512 | 5802 | 291 |
| 150 | 6091 | 6381 | 6670 | 6959 | 7248 | 7536 | 7825 | 8113 | 8401 | 8689 | 289 |
| 151 | 8977 | 9264 | 9552 | 9839 | 0126 | 0413 | 0699 | 0986 | 1272 | 1558 | 287 |
| 152 | 181844 | 2129 | 2415 | 2700 | 2985 | 3270 | 3555 | 3839 | 4123 | 4407 | 285 |
| 153 | 4691 | 4975 | 5259 | 5542 | 5825 | 6108 | 6391 | 6674 | 6956 | 7239 | 283 |
| 154 | 7521 | 7803 | 8084 | 8366 | 8647 | 8928 | 9209 | 9490 | 9771 | 0051 | 281 |
| 155 | 190332 | 0612 | 0892 | 1171 | 1451 | 1730 | 2010 | 2289 | 2567 | 2846 | 279 |
| 156 | 3125 | 3403 | 3681 | 3959 | 4237 | 4514 | 4792 | 5069 | 5346 | 5623 | 278 |
| 157 | 5900 | 6176 | 6453 | 6729 | 7005 | 7281 | 7556 | 7832 | 8107 | 8382 | 276 |
| 158 | 8657 | 8932 | 9206 | 9481 | 9755 | 0029 | 0303 | 0577 | 0850 | 1124 | 274 |
| 159 | 201397 | 1670 | 1943 | 2216 | 2488 | 2761 | 3033 | 3305 | 3577 | 3848 | 272 |
| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 160 | 204120 | 4391 | 4663 | 4934 | 5204 | 5475 | 5746 | 6016 | 6286 | 6556 | 271 |
| 161 | 6826 | 7096 | 7365 | 7634 | 7904 | 8173 | 8441 | 8710 | 8979 | 9247 | 269 |
| 162 | 9515 | 9783 | $^{0}$051 | $^{0}$319 | $^{0}$586 | $^{0}$853 | $^{1}$121 | $^{1}$388 | $^{1}$654 | $^{1}$921 | 267 |
| 163 | 212188 | 2454 | 2720 | 2986 | 3252 | 3518 | 3783 | 4049 | 4314 | 4579 | 266 |
| 164 | 4844 | 5109 | 5373 | 5638 | 5902 | 6166 | 6430 | 6694 | 6957 | 7221 | 264 |
| 165 | 7484 | 7747 | 8010 | 8273 | 8536 | 8798 | 9060 | 9323 | 9585 | 9846 | 262 |
| 166 | 220108 | 0370 | 0631 | 0892 | 1153 | 1414 | 1675 | 1936 | 2196 | 2456 | 261 |
| 167 | 2716 | 2976 | 3236 | 3496 | 3755 | 4015 | 4274 | 4533 | 4792 | 5051 | 259 |
| 168 | 5309 | 5568 | 5826 | 6084 | 6342 | 6600 | 6858 | 7115 | 7372 | 7630 | 258 |
| 169 | 7887 | 8144 | 8400 | 8657 | 8913 | 9170 | 9426 | 9682 | 9938 | $^{0}$193 | 256 |
| 170 | 230449 | 0704 | 0960 | 1215 | 1470 | 1724 | 1979 | 2234 | 2488 | 2742 | 254 |
| 171 | 2996 | 3250 | 3504 | 3757 | 4011 | 4264 | 4517 | 4770 | 5023 | 5276 | 253 |
| 172 | 5528 | 5781 | 6033 | 6285 | 6537 | 6789 | 7041 | 7292 | 7544 | 7795 | 252 |
| 173 | 8046 | 8297 | 8548 | 8799 | 9049 | 9299 | 9550 | 9800 | $^{0}$050 | $^{0}$300 | 250 |
| 174 | 240549 | 0799 | 1048 | 1297 | 1546 | 1795 | 2044 | 2293 | 2541 | 2790 | 249 |
| 175 | 3038 | 3286 | 3534 | 3782 | 4030 | 4277 | 4525 | 4772 | 5019 | 5266 | 248 |
| 176 | 5513 | 5759 | 6006 | 6252 | 6499 | 6745 | 6991 | 7237 | 7482 | 7728 | 246 |
| 177 | 7973 | 8219 | 8464 | 8709 | 8954 | 9198 | 9443 | 9687 | 9932 | $^{0}$176 | 245 |
| 178 | 250420 | 0664 | 0908 | 1151 | 1395 | 1638 | 1881 | 2125 | 2368 | 2610 | 243 |
| 179 | 2853 | 3096 | 3338 | 3580 | 3822 | 4064 | 4306 | 4548 | 4790 | 5031 | 242 |
| 180 | 5273 | 5514 | 5755 | 5996 | 6237 | 6477 | 6718 | 6958 | 7198 | 7439 | 241 |
| 181 | 7679 | 7918 | 8158 | 8398 | 8637 | 8877 | 9116 | 9355 | 9594 | 9833 | 239 |
| 182 | 260071 | 0310 | 0548 | 0787 | 1025 | 1263 | 1501 | 1739 | 1976 | 2214 | 238 |
| 183 | 2451 | 2688 | 2925 | 3162 | 3399 | 3636 | 3873 | 4109 | 4346 | 4582 | 237 |
| 184 | 4818 | 5054 | 5290 | 5525 | 5761 | 5996 | 6232 | 6467 | 6702 | 6937 | 235 |
| 185 | 7172 | 7406 | 7641 | 7875 | 8110 | 8344 | 8578 | 8812 | 9046 | 9279 | 234 |
| 186 | 9513 | 9746 | 9980 | $^{0}$213 | $^{0}$446 | $^{0}$679 | $^{0}$912 | $^{1}$144 | $^{1}$377 | $^{1}$609 | 233 |
| 187 | 271842 | 2074 | 2306 | 2538 | 2770 | 3001 | 3233 | 3464 | 3696 | 3927 | 232 |
| 188 | 4158 | 4389 | 4620 | 4850 | 5081 | 5311 | 5542 | 5772 | 6002 | 6232 | 230 |
| 189 | 6462 | 6692 | 6921 | 7151 | 7380 | 7609 | 7838 | 8067 | 8296 | 8525 | 229 |
| 190 | 8754 | 8982 | 9211 | 9439 | 9667 | 9895 | $^{0}$123 | $^{0}$351 | $^{0}$578 | $^{0}$806 | 228 |
| 191 | 281033 | 1261 | 1488 | 1715 | 1942 | 2169 | 2396 | 2622 | 2849 | 3075 | 227 |
| 192 | 3301 | 3527 | 3753 | 3979 | 4205 | 4431 | 4656 | 4882 | 5107 | 5332 | 226 |
| 193 | 5557 | 5782 | 6007 | 6232 | 6456 | 6681 | 6905 | 7130 | 7354 | 7578 | 225 |
| 194 | 7802 | 8026 | 8249 | 8473 | 8696 | 8920 | 9143 | 9366 | 9589 | 9812 | 223 |
| 195 | 290035 | 0257 | 0480 | 0702 | 0925 | 1147 | 1369 | 1591 | 1813 | 2034 | 222 |
| 196 | 2256 | 2478 | 2699 | 2920 | 3141 | 3363 | 3584 | 3804 | 4025 | 4246 | 221 |
| 197 | 4466 | 4687 | 4907 | 5127 | 5347 | 5567 | 5787 | 6007 | 6226 | 6446 | 220 |
| 198 | 6665 | 6884 | 7104 | 7323 | 7542 | 7761 | 7979 | 8198 | 8416 | 8635 | 219 |
| 199 | 8853 | 9071 | 9289 | 9507 | 9725 | 9943 | $^{0}$161 | $^{0}$378 | $^{0}$595 | $^{0}$813 | 218 |
| 200 | 301030 | 1247 | 1464 | 1681 | 1898 | 2114 | 2331 | 2547 | 2764 | 2980 | 217 |
| 201 | 3196 | 3412 | 3628 | 3844 | 4059 | 4275 | 4491 | 4706 | 4921 | 5136 | 216 |
| 202 | 5351 | 5566 | 5781 | 5996 | 6211 | 6425 | 6639 | 6854 | 7068 | 7282 | 215 |
| 203 | 7496 | 7710 | 7924 | 8137 | 8351 | 8564 | 8778 | 8991 | 9204 | 9417 | 213 |
| 204 | 9630 | 9843 | $^{0}$056 | $^{0}$268 | $^{0}$481 | $^{0}$693 | $^{0}$906 | $^{1}$118 | $^{1}$330 | $^{1}$542 | 212 |
| 205 | 311754 | 1966 | 2177 | 2389 | 2600 | 2812 | 3023 | 3234 | 3445 | 3656 | 211 |
| 206 | 3867 | 4078 | 4289 | 4499 | 4710 | 4920 | 5130 | 5340 | 5551 | 5760 | 210 |
| 207 | 5970 | 6180 | 6390 | 6599 | 6809 | 7018 | 7227 | 7436 | 7646 | 7854 | 209 |
| 208 | 8063 | 8272 | 8481 | 8689 | 8898 | 9106 | 9314 | 9522 | 9730 | 9938 | 208 |
| 209 | 320146 | 0354 | 0562 | 0769 | 0977 | 1184 | 1391 | 1598 | 1805 | 2012 | 207 |
| 210 | 2219 | 2426 | 2633 | 2839 | 3046 | 3252 | 3458 | 3665 | 3871 | 4077 | 206 |
| 211 | 4282 | 4488 | 4694 | 4899 | 5105 | 5310 | 5516 | 5721 | 5926 | 6131 | 205 |
| 212 | 6336 | 6541 | 6745 | 6950 | 7155 | 7359 | 7563 | 7767 | 7972 | 8176 | 204 |
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| 216 | 4454 | 4655 | 4856 | 5057 | 5257 | 5458 | 5658 | 5859 | 6059 | 6260 | 201 |
| 217 | 6460 | 6660 | 6860 | 7060 | 7260 | 7459 | 7659 | 7858 | 8058 | 8257 | 200 |
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| 219 | 340444 | 0642 | 0841 | 1039 | 1237 | 1435 | 1632 | 1830 | 2028 | 2225 | 198 |
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| 221 | 4392 | 4589 | 4785 | 4981 | 5178 | 5374 | 5570 | 5766 | 5962 | 6157 | 196 |
| 222 | 6353 | 6549 | 6744 | 6939 | 7135 | 7330 | 7525 | 7720 | 7915 | 8110 | 194 |
| 223 | 8305 | 8500 | 8694 | 8889 | 9083 | 9278 | 9472 | 9666 | 9860 | 0054 | 193 |
| 224 | 350248 | 0442 | 0636 | 0829 | 1023 | 1216 | 1410 | 1603 | 1796 | 1989 | 193 |
| 225 | 2183 | 2375 | 2568 | 2761 | 2954 | 3147 | 3339 | 3532 | 3724 | 3916 | 192 |
| 226 | 4108 | 4301 | 4493 | 4685 | 4876 | 5068 | 5260 | 5452 | 5643 | 5834 | 191 |
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| 228 | 7935 | 8125 | 8316 | 8506 | 8696 | 8886 | 9076 | 9266 | 9456 | 9646 | 189 |
| 229 | 9835 | 0025 | 0215 | 0404 | 0593 | 0783 | 0972 | 1161 | 1350 | 1539 | 189 |
| 230 | 361728 | 1917 | 2105 | 2294 | 2482 | 2671 | 2859 | 3048 | 3236 | 3424 | 188 |
| 231 | 3612 | 3800 | 3988 | 4176 | 4363 | 4551 | 4739 | 4926 | 5113 | 5301 | 188 |
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| 233 | 7356 | 7542 | 7729 | 7915 | 8101 | 8287 | 8473 | 8659 | 8845 | 9030 | 186 |
| 234 | 9216 | 9401 | 9587 | 9772 | 9958 | 0143 | 0328 | 0513 | 0698 | 0883 | 185 |
| 235 | 371068 | 1253 | 1437 | 1622 | 1806 | 1991 | 2175 | 2360 | 2544 | 2728 | 184 |
| 236 | 2912 | 3096 | 3280 | 3464 | 3647 | 3831 | 4015 | 4198 | 4382 | 4565 | 184 |
| 237 | 4748 | 4932 | 5115 | 5298 | 5481 | 5664 | 5846 | 6029 | 6212 | 6394 | 183 |
| 238 | 6577 | 6759 | 6942 | 7124 | 7306 | 7488 | 7670 | 7852 | 8034 | 8216 | 182 |
| 239 | 8398 | 8580 | 8761 | 8943 | 9124 | 9306 | 9487 | 9668 | 9849 | 0030 | 181 |
| 240 | 380211 | 0392 | 0573 | 0754 | 0934 | 1115 | 1296 | 1476 | 1656 | 1837 | 181 |
| 241 | 2017 | 2197 | 2377 | 2557 | 2737 | 2917 | 3097 | 3277 | 3456 | 3636 | 180 |
| 242 | 3815 | 3995 | 4174 | 4353 | 4533 | 4712 | 4891 | 5070 | 5249 | 5428 | 179 |
| 243 | 5606 | 5785 | 5964 | 6142 | 6321 | 6499 | 6677 | 6856 | 7034 | 7212 | 178 |
| 244 | 7390 | 7568 | 7746 | 7923 | 8101 | 8279 | 8456 | 8634 | 8811 | 8989 | 178 |
| 245 | 9166 | 9343 | 9520 | 9698 | 9875 | 0051 | 0228 | 0405 | 0582 | 0759 | 177 |
| 246 | 390935 | 1112 | 1288 | 1464 | 1641 | 1817 | 1993 | 2169 | 2345 | 2521 | 176 |
| 247 | 2697 | 2873 | 3048 | 3224 | 3400 | 3575 | 3751 | 3926 | 4101 | 4277 | 176 |
| 248 | 4452 | 4627 | 4802 | 4977 | 5152 | 5326 | 5501 | 5676 | 5850 | 6025 | 175 |
| 249 | 6199 | 6374 | 6548 | 6722 | 6896 | 7071 | 7245 | 7419 | 7592 | 7766 | 174 |
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| 251 | 9674 | 9847 | 0020 | 0192 | 0365 | 0538 | 0711 | 0883 | 1056 | 1228 | 173 |
| 252 | 401401 | 1573 | 1745 | 1917 | 2089 | 2261 | 2433 | 2605 | 2777 | 2949 | 172 |
| 253 | 3121 | 3292 | 3464 | 3635 | 3807 | 3978 | 4149 | 4320 | 4492 | 4663 | 171 |
| 254 | 4834 | 5005 | 5176 | 5346 | 5517 | 5688 | 5858 | 6029 | 6199 | 6370 | 171 |
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| 256 | 8240 | 8410 | 8579 | 8749 | 8918 | 9087 | 9257 | 9426 | 9595 | 9764 | 169 |
| 257 | 9933 | 0102 | 0271 | 0440 | 0609 | 0777 | 0946 | 1114 | 1283 | 1451 | 169 |
| 258 | 411620 | 1788 | 1956 | 2124 | 2293 | 2461 | 2629 | 2796 | 2964 | 3132 | 168 |
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| 261 | 6641 | 6807 | 6973 | 7139 | 7306 | 7472 | 7638 | 7804 | 7970 | 8135 | 166 |
| 262 | 8301 | 8467 | 8633 | 8798 | 8964 | 9129 | 9295 | 9460 | 9625 | 9791 | 165 |
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| 266 | 4882 | 5045 | 5208 | 5371 | 5534 | 5697 | 5860 | 6023 | 6186 | 6349 | 163 |
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| 268 | 8135 | 8297 | 8459 | 8621 | 8783 | 8944 | 9106 | 9268 | 9429 | 9591 | 162 |
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| 271 | 2969 | 3130 | 3290 | 3450 | 3610 | 3770 | 3930 | 4090 | 4249 | 4409 | 160 |
| 272 | 4569 | 4729 | 4888 | 5048 | 5207 | 5367 | 5526 | 5685 | 5844 | 6004 | 159 |
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| 274 | 7751 | 7909 | 8067 | 8226 | 8384 | 8542 | 8701 | 8859 | 9017 | 9175 | 158 |
| 275 | 9333 | 9491 | 9648 | 9806 | 9964 | 0122 | 0279 | 0437 | 0594 | 0752 | 158 |
| 276 | 440909 | 1066 | 1224 | 1381 | 1538 | 1695 | 1852 | 2009 | 2166 | 2323 | 157 |
| 277 | 2480 | 2637 | 2793 | 2950 | 3106 | 3263 | 3419 | 3576 | 3732 | 3889 | 157 |
| 278 | 4045 | 4201 | 4357 | 4513 | 4669 | 4825 | 4981 | 5137 | 5293 | 5449 | 156 |
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| 282 | 450249 | 0403 | 0557 | 0711 | 0865 | 1018 | 1172 | 1326 | 1479 | 1633 | 154 |
| 283 | 1786 | 1940 | 2093 | 2247 | 2400 | 2553 | 2706 | 2859 | 3012 | 3165 | 153 |
| 284 | 3318 | 3471 | 3624 | 3777 | 3930 | 4082 | 4235 | 4387 | 4540 | 4692 | 153 |
| 285 | 4845 | 4997 | 5150 | 5302 | 5454 | 5606 | 5758 | 5910 | 6062 | 6214 | 152 |
| 286 | 6366 | 6518 | 6670 | 6821 | 6973 | 7125 | 7276 | 7428 | 7579 | 7731 | 152 |
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| 289 | 460898 | 1048 | 1198 | 1348 | 1499 | 1649 | 1799 | 1948 | 2098 | 2248 | 150 |
| 290 | 2398 | 2548 | 2697 | 2847 | 2997 | 3146 | 3296 | 3445 | 3594 | 3744 | 150 |
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| 293 | 6868 | 7016 | 7164 | 7312 | 7460 | 7608 | 7756 | 7904 | 8052 | 8200 | 148 |
| 294 | 8347 | 8495 | 8643 | 8790 | 8938 | 9085 | 9233 | 9380 | 9527 | 9675 | 148 |
| 295 | 9822 | 9969 | 0116 | 0263 | 0410 | 0557 | 0704 | 0851 | 0998 | 1145 | 147 |
| 296 | 471292 | 1438 | 1585 | 1732 | 1878 | 2025 | 2171 | 2318 | 2464 | 2610 | 146 |
| 297 | 2756 | 2903 | 3049 | 3195 | 3341 | 3487 | 3633 | 3779 | 3925 | 4071 | 146 |
| 298 | 4216 | 4362 | 4508 | 4653 | 4799 | 4944 | 5090 | 5235 | 5381 | 5526 | 146 |
| 299 | 5671 | 5816 | 5962 | 6107 | 6252 | 6397 | 6542 | 6687 | 6832 | 6976 | 145 |
| 300 | 7121 | 7266 | 7411 | 7555 | 7700 | 7844 | 7989 | 8133 | 8278 | 8422 | 145 |
| 301 | 8566 | 8711 | 8855 | 8999 | 9143 | 9287 | 9431 | 9575 | 9719 | 9863 | 144 |
| 302 | 480007 | 0151 | 0294 | 0438 | 0582 | 0725 | 0869 | 1012 | 1156 | 1299 | 144 |
| 303 | 1443 | 1586 | 1729 | 1872 | 2016 | 2159 | 2302 | 2445 | 2588 | 2731 | 143 |
| 304 | 2874 | 3016 | 3159 | 3302 | 3445 | 3587 | 3730 | 3872 | 4015 | 4157 | 143 |
| 305 | 4300 | 4442 | 4585 | 4727 | 4869 | 5011 | 5153 | 5295 | 5437 | 5579 | 142 |
| 306 | 5721 | 5863 | 6005 | 6147 | 6289 | 6430 | 6572 | 6714 | 6855 | 6997 | 142 |
| 307 | 7138 | 7280 | 7421 | 7563 | 7704 | 7845 | 7986 | 8127 | 8269 | 8410 | 141 |
| 308 | 8551 | 8692 | 8833 | 8974 | 9114 | 9255 | 9396 | 9537 | 9677 | 9818 | 141 |
| 309 | 9958 | 0099 | 0239 | 0380 | 0520 | 0661 | 0801 | 0941 | 1081 | 1222 | 140 |
| 310 | 491362 | 1502 | 1642 | 1782 | 1922 | 2062 | 2201 | 2341 | 2481 | 2621 | 140 |
| 311 | 2760 | 2900 | 3040 | 3179 | 3319 | 3458 | 3597 | 3737 | 3876 | 4015 | 139 |
| 312 | 4155 | 4294 | 4433 | 4572 | 4711 | 4850 | 4989 | 5128 | 5267 | 5406 | 139 |
| 313 | 5544 | 5683 | 5822 | 5960 | 6099 | 6238 | 6376 | 6515 | 6653 | 6791 | 139 |
| 314 | 6930 | 7068 | 7206 | 7344 | 7483 | 7621 | 7759 | 7897 | 8035 | 8173 | 138 |
| 315 | 8311 | 8448 | 8586 | 8724 | 8862 | 8999 | 9137 | 9275 | 9412 | 9550 | 138 |
| 316 | 9687 | 9824 | 9962 | 0099 | 0236 | 0374 | 0511 | 0648 | 0785 | 0922 | 137 |
| 317 | 501059 | 1196 | 1333 | 1470 | 1607 | 1744 | 1880 | 2017 | 2154 | 2291 | 137 |
| 318 | 2427 | 2564 | 2700 | 2837 | 2973 | 3109 | 3246 | 3382 | 3518 | 3655 | 136 |
| 319 | 3791 | 3927 | 4063 | 4199 | 4335 | 4471 | 4607 | 4743 | 4878 | 5014 | 136 |
| 320 | 5150 | 5286 | 5421 | 5557 | 5693 | 5828 | 5964 | 6099 | 6234 | 6370 | 136 |
| 321 | 6505 | 6640 | 6776 | 6911 | 7046 | 7181 | 7316 | 7451 | 7586 | 7721 | 135 |
| 322 | 7856 | 7991 | 8126 | 8260 | 8395 | 8530 | 8664 | 8799 | 8934 | 9068 | 135 |
| 323 | 9203 | 9337 | 9471 | 9606 | 9740 | 9874 | 0009 | 0143 | 0277 | 0411 | 134 |
| 324 | 510545 | 0679 | 0813 | 0947 | 1081 | 1215 | 1349 | 1482 | 1616 | 1750 | 134 |
| 325 | 1883 | 2017 | 2151 | 2284 | 2418 | 2551 | 2684 | 2818 | 2951 | 3084 | 133 |
| 326 | 3218 | 3351 | 3484 | 3617 | 3750 | 3883 | 4016 | 4149 | 4282 | 4415 | 133 |
| 327 | 4548 | 4681 | 4813 | 4946 | 5079 | 5211 | 5344 | 5476 | 5609 | 5741 | 133 |
| 328 | 5874 | 6006 | 6139 | 6271 | 6403 | 6535 | 6668 | 6800 | 6932 | 7064 | 132 |
| 329 | 7196 | 7328 | 7460 | 7592 | 7724 | 7855 | 7987 | 8119 | 8251 | 8382 | 132 |
| 330 | 8514 | 8646 | 8777 | 8909 | 9040 | 9171 | 9303 | 9434 | 9566 | 9697 | 131 |
| 331 | 9828 | 9959 | 0090 | 0221 | 0353 | 0484 | 0615 | 0745 | 0876 | 1007 | 131 |
| 332 | 521138 | 1269 | 1400 | 1530 | 1661 | 1792 | 1922 | 2053 | 2183 | 2314 | 131 |
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| 334 | 3746 | 3876 | 4006 | 4136 | 4266 | 4396 | 4526 | 4656 | 4785 | 4915 | 130 |
| 335 | 5045 | 5174 | 5304 | 5434 | 5563 | 5693 | 5822 | 5951 | 6081 | 6210 | 129 |
| 336 | 6339 | 6469 | 6598 | 6727 | 6856 | 6985 | 7114 | 7243 | 7372 | 7501 | 129 |
| 337 | 7630 | 7759 | 7888 | 8016 | 8145 | 8274 | 8402 | 8531 | 8660 | 8788 | 129 |
| 338 | 8917 | 9045 | 9174 | 9302 | 9430 | 9559 | 9687 | 9815 | 9943 | 0072 | 128 |
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| 345 | 7819 | 7945 | 8071 | 8197 | 8322 | 8448 | 8574 | 8699 | 8825 | 8951 | 126 |
| 346 | 9076 | 9202 | 9327 | 9452 | 9578 | 9703 | 9829 | 9954 | 0079 | 0204 | 125 |
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| 351 | 5307 | 5431 | 5555 | 5678 | 5802 | 5925 | 6049 | 6172 | 6296 | 6419 | 124 |
| 352 | 6543 | 6666 | 6789 | 6913 | 7036 | 7159 | 7282 | 7405 | 7529 | 7652 | 123 |
| 353 | 7775 | 7898 | 8021 | 8144 | 8267 | 8389 | 8512 | 8635 | 8758 | 8881 | 123 |
| 354 | 9003 | 9126 | 9249 | 9371 | 9494 | 9616 | 9739 | 9861 | 9984 | 0106 | 123 |
| 355 | 550228 | 0351 | 0473 | 0595 | 0717 | 0840 | 0962 | 1084 | 1206 | 1328 | 122 |
| 356 | 1450 | 1572 | 1694 | 1816 | 1938 | 2060 | 2181 | 2303 | 2425 | 2547 | 122 |
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| 360 | 6303 | 6423 | 6544 | 6664 | 6785 | 6905 | 7026 | 7146 | 7267 | 7387 | 120 |
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| 362 | 8709 | 8829 | 8948 | 9068 | 9188 | 9308 | 9428 | 9548 | 9667 | 9787 | 120 |
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| 365 | 2293 | 2412 | 2531 | 2650 | 2769 | 2887 | 3006 | 3125 | 3244 | 3362 | 119 |
| 366 | 3481 | 3600 | 3718 | 3837 | 3955 | 4074 | 4192 | 4311 | 4429 | 4548 | 119 |
| 367 | 4666 | 4784 | 4903 | 5021 | 5139 | 5257 | 5376 | 5494 | 5612 | 5730 | 118 |
| 368 | 5848 | 5966 | 6084 | 6202 | 6320 | 6437 | 6555 | 6673 | 6791 | 6909 | 118 |
| 369 | 7026 | 7144 | 7262 | 7379 | 7497 | 7614 | 7732 | 7849 | 7967 | 8084 | 118 |
| 370 | 8202 | 8319 | 8436 | 8554 | 8671 | 8788 | 8905 | 9023 | 9140 | 9257 | 117 |
| 371 | 9374 | 9491 | 9608 | 9725 | 9842 | 9959 | 0076 | 0193 | 0309 | 0426 | 117 |
| 372 | 570543 | 0660 | 0776 | 0893 | 1010 | 1126 | 1243 | 1359 | 1476 | 1592 | 117 |
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| 374 | 2872 | 2988 | 3104 | 3220 | 3336 | 3452 | 3568 | 3684 | 3800 | 3915 | 116 |
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| 385 | 5461 | 5574 | 5686 | 5799 | 5912 | 6024 | 6137 | 6250 | 6362 | 6475 | 113 |
| 386 | 6587 | 6700 | 6812 | 6925 | 7037 | 7149 | 7262 | 7374 | 7486 | 7599 | 112 |
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| 422 | 5312 | 5415 | 5518 | 5621 | 5724 | 5827 | 5929 | 6032 | 6135 | 6238 | 103 |
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| 424 | 7366 | 7468 | 7571 | 7673 | 7775 | 7878 | 7980 | 8082 | 8185 | 8287 | 102 |
| 425 | 8389 | 8491 | 8593 | 8695 | 8797 | 8900 | 9002 | 9104 | 9206 | 9308 | 102 |
| 426 | 9410 | 9512 | 9613 | 9715 | 9817 | 9919 | 0021 | 0123 | 0224 | 0326 | 102 |
| 427 | 630428 | 0530 | 0631 | 0733 | 0835 | 0936 | 1038 | 1139 | 1241 | 1342 | 102 |
| 428 | 1444 | 1545 | 1647 | 1748 | 1849 | 1951 | 2052 | 2153 | 2255 | 2356 | 101 |
| 429 | 2457 | 2559 | 2660 | 2761 | 2862 | 2963 | 3064 | 3165 | 3266 | 3367 | 101 |
| 430 | 3468 | 3569 | 3670 | 3771 | 3872 | 3973 | 4074 | 4175 | 4276 | 4376 | 100 |
| 431 | 4477 | 4578 | 4679 | 4779 | 4880 | 4981 | 5081 | 5182 | 5283 | 5383 | 100 |
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| 434 | 7490 | 7590 | 7690 | 7790 | 7890 | 7990 | 8090 | 8190 | 8290 | 8389 | 99 |
| 435 | 8489 | 8589 | 8689 | 8789 | 8888 | 8988 | 9088 | 9188 | 9287 | 9387 | 99 |
| 436 | 9486 | 9586 | 9686 | 9785 | 9885 | 9984 | 0084 | 0183 | 0283 | 0382 | 99 |
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| 438 | 1474 | 1573 | 1672 | 1771 | 1871 | 1970 | 2069 | 2168 | 2267 | 2366 | 99 |
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| 440 | 3453 | 3551 | 3650 | 3749 | 3847 | 3946 | 4044 | 4143 | 4242 | 4340 | 98 |
| 441 | 4439 | 4537 | 4636 | 4734 | 4832 | 4931 | 5029 | 5127 | 5226 | 5324 | 98 |
| 442 | 5422 | 5521 | 5619 | 5717 | 5815 | 5913 | 6011 | 6110 | 6208 | 6306 | 98 |
| 443 | 6404 | 6502 | 6600 | 6698 | 6796 | 6894 | 6992 | 7089 | 7187 | 7285 | 98 |
| 444 | 7383 | 7481 | 7579 | 7676 | 7774 | 7872 | 7969 | 8067 | 8165 | 8262 | 98 |
| 445 | 8360 | 8458 | 8555 | 8653 | 8750 | 8848 | 8945 | 9043 | 9140 | 9237 | 97 |
| 446 | 9335 | 9432 | 9530 | 9627 | 9724 | 9821 | 9919 | 0016 | 0113 | 0210 | 97 |
| 447 | 650308 | 0405 | 0502 | 0599 | 0696 | 0793 | 0890 | 0987 | 1084 | 1181 | 97 |
| 448 | 1278 | 1375 | 1472 | 1569 | 1666 | 1762 | 1859 | 1956 | 2053 | 2150 | 97 |
| 449 | 2246 | 2343 | 2440 | 2536 | 2633 | 2730 | 2826 | 2923 | 3019 | 3116 | 97 |
| 450 | 3213 | 3309 | 3405 | 3502 | 3598 | 3695 | 3791 | 3888 | 3984 | 4080 | 96 |
| 451 | 4177 | 4273 | 4369 | 4465 | 4562 | 4658 | 4754 | 4850 | 4946 | 5042 | 96 |
| 452 | 5138 | 5235 | 5331 | 5427 | 5523 | 5619 | 5715 | 5810 | 5906 | 6002 | 96 |
| 453 | 6098 | 6194 | 6290 | 6386 | 6482 | 6577 | 6673 | 6769 | 6864 | 6960 | 96 |
| 454 | 7056 | 7152 | 7247 | 7343 | 7438 | 7534 | 7629 | 7725 | 7820 | 7916 | 96 |
| 455 | 8011 | 8107 | 8202 | 8298 | 8393 | 8488 | 8584 | 8679 | 8774 | 8870 | 95 |
| 456 | 8965 | 9060 | 9155 | 9250 | 9346 | 9441 | 9536 | 9631 | 9726 | 9821 | 95 |
| 457 | 9916 | 0011 | 0106 | 0201 | 0296 | 0391 | 0486 | 0581 | 0676 | 0771 | 95 |
| 458 | 660865 | 0960 | 1055 | 1150 | 1245 | 1339 | 1434 | 1529 | 1623 | 1718 | 95 |
| 459 | 1813 | 1907 | 2002 | 2096 | 2191 | 2286 | 2380 | 2475 | 2569 | 2663 | 95 |
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| 462 | 4642 | 4736 | 4830 | 4924 | 5018 | 5112 | 5206 | 5299 | 5393 | 5487 | 94 |
| 463 | 5581 | 5675 | 5769 | 5862 | 5956 | 6050 | 6143 | 6237 | 6331 | 6424 | 94 |
| 464 | 6518 | 6612 | 6705 | 6799 | 6892 | 6986 | 7079 | 7173 | 7266 | 7360 | 94 |
| 465 | 7453 | 7546 | 7640 | 7733 | 7826 | 7920 | 8013 | 8106 | 8199 | 8293 | 93 |
| 466 | 8386 | 8479 | 8572 | 8665 | 8759 | 8852 | 8945 | 9038 | 9131 | 9224 | 93 |
| 467 | 9317 | 9410 | 9503 | 9596 | 9689 | 9782 | 9875 | 9967 | 0060 | 0153 | 93 |
| 468 | 670246 | 0339 | 0431 | 0524 | 0617 | 0710 | 0802 | 0895 | 0988 | 1080 | 93 |
| 469 | 1173 | 1265 | 1358 | 1451 | 1543 | 1636 | 1728 | 1821 | 1913 | 2005 | 93 |
| 470 | 2098 | 2190 | 2283 | 2375 | 2467 | 2560 | 2652 | 2744 | 2836 | 2929 | 92 |
| 471 | 3021 | 3113 | 3205 | 3297 | 3390 | 3482 | 3574 | 3666 | 3758 | 3850 | 92 |
| 472 | 3942 | 4034 | 4126 | 4218 | 4310 | 4402 | 4494 | 4586 | 4677 | 4769 | 92 |
| 473 | 4861 | 4953 | 5045 | 5137 | 5228 | 5320 | 5412 | 5503 | 5595 | 5687 | 92 |
| 474 | 5778 | 5870 | 5962 | 6053 | 6145 | 6236 | 6328 | 6419 | 6511 | 6602 | 92 |
| 475 | 6694 | 6785 | 6876 | 6968 | 7059 | 7151 | 7242 | 7333 | 7424 | 7516 | 91 |
| 476 | 7607 | 7698 | 7789 | 7881 | 7972 | 8063 | 8154 | 8245 | 8336 | 8427 | 91 |
| 477 | 8518 | 8609 | 8700 | 8791 | 8882 | 8973 | 9064 | 9155 | 9246 | 9337 | 91 |
| 478 | 9428 | 9519 | 9610 | 9700 | 9791 | 9882 | 9973 | 0063 | 0154 | 0245 | 91 |
| 479 | 680336 | 0426 | 0517 | 0607 | 0698 | 0789 | 0879 | 0970 | 1060 | 1151 | 91 |
| 480 | 1241 | 1332 | 1422 | 1513 | 1603 | 1693 | 1784 | 1874 | 1964 | 2055 | 90 |
| 481 | 2145 | 2235 | 2326 | 2416 | 2506 | 2596 | 2686 | 2777 | 2867 | 2957 | 90 |
| 482 | 3047 | 3137 | 3227 | 3317 | 3407 | 3497 | 3587 | 3677 | 3767 | 3857 | 90 |
| 483 | 3947 | 4037 | 4127 | 4217 | 4307 | 4396 | 4486 | 4576 | 4666 | 4756 | 90 |
| 484 | 4845 | 4935 | 5025 | 5114 | 5204 | 5294 | 5383 | 5473 | 5563 | 5652 | 90 |
| 485 | 5742 | 5831 | 5921 | 6010 | 6100 | 6189 | 6279 | 6368 | 6458 | 6547 | 89 |
| 486 | 6636 | 6726 | 6815 | 6904 | 6994 | 7083 | 7172 | 7261 | 7351 | 7440 | 89 |
| 487 | 7529 | 7618 | 7707 | 7796 | 7886 | 7975 | 8064 | 8153 | 8242 | 8331 | 89 |
| 488 | 8420 | 8509 | 8598 | 8687 | 8776 | 8865 | 8953 | 9042 | 9131 | 9220 | 89 |
| 489 | 9309 | 9398 | 9486 | 9575 | 9664 | 9753 | 9841 | 9930 | 0019 | 0107 | 89 |
| 490 | 690196 | 0285 | 0373 | 0462 | 0550 | 0639 | 0728 | 0816 | 0905 | 0993 | 89 |
| 491 | 1081 | 1170 | 1258 | 1347 | 1435 | 1524 | 1612 | 1700 | 1789 | 1877 | 88 |
| 492 | 1965 | 2053 | 2142 | 2230 | 2318 | 2406 | 2494 | 2583 | 2671 | 2759 | 88 |
| 493 | 2847 | 2935 | 3023 | 3111 | 3199 | 3287 | 3375 | 3463 | 3551 | 3639 | 88 |
| 494 | 3727 | 3815 | 3903 | 3991 | 4078 | 4166 | 4254 | 4342 | 4430 | 4517 | 88 |
| 495 | 4605 | 4693 | 4781 | 4868 | 4956 | 5044 | 5131 | 5219 | 5307 | 5394 | 88 |
| 496 | 5482 | 5569 | 5657 | 5744 | 5832 | 5919 | 6007 | 6094 | 6182 | 6269 | 87 |
| 497 | 6356 | 6444 | 6531 | 6618 | 6706 | 6793 | 6880 | 6968 | 7055 | 7142 | 87 |
| 498 | 7229 | 7317 | 7404 | 7491 | 7578 | 7665 | 7752 | 7839 | 7926 | 8014 | 87 |
| 499 | 8101 | 8188 | 8275 | 8362 | 8449 | 8535 | 8622 | 8709 | 8796 | 8883 | 87 |
| 500 | 8970 | 9057 | 9144 | 9231 | 9317 | 9404 | 9491 | 9578 | 9664 | 9751 | 87 |
| 501 | 9838 | 9924 | 0011 | 0098 | 0184 | 0271 | 0358 | 0444 | 0531 | 0617 | 87 |
| 502 | 700704 | 0790 | 0877 | 0963 | 1050 | 1136 | 1222 | 1309 | 1395 | 1482 | 86 |
| 503 | 1568 | 1654 | 1741 | 1827 | 1913 | 1999 | 2086 | 2172 | 2258 | 2344 | 86 |
| 504 | 2431 | 2517 | 2603 | 2689 | 2775 | 2861 | 2947 | 3033 | 3119 | 3205 | 86 |
| 505 | 3291 | 3377 | 3463 | 3549 | 3635 | 3721 | 3807 | 3893 | 3979 | 4065 | 86 |
| 506 | 4151 | 4236 | 4322 | 4408 | 4494 | 4579 | 4665 | 4751 | 4837 | 4922 | 86 |
| 507 | 5008 | 5094 | 5179 | 5265 | 5350 | 5436 | 5522 | 5607 | 5693 | 5778 | 86 |
| 508 | 5864 | 5949 | 6035 | 6120 | 6206 | 6291 | 6376 | 6462 | 6547 | 6632 | 85 |
| 509 | 6718 | 6803 | 6888 | 6974 | 7059 | 7144 | 7229 | 7315 | 7400 | 7485 | 85 |
| 510 | 7570 | 7655 | 7740 | 7826 | 7911 | 7996 | 8081 | 8166 | 8251 | 8336 | 85 |
| 511 | 8421 | 8506 | 8591 | 8676 | 8761 | 8846 | 8931 | 9015 | 9100 | 9185 | 85 |
| 512 | 9270 | 9355 | 9440 | 9524 | 9609 | 9694 | 9779 | 9863 | 9948 | 0033 | 85 |
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| 517 | 3491 | 3575 | 3659 | 3742 | 3826 | 3910 | 3994 | 4078 | 4162 | 4246 | 84 |
| 518 | 4330 | 4414 | 4497 | 4581 | 4665 | 4749 | 4833 | 4916 | 5000 | 5084 | 84 |
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| 522 | 7671 | 7754 | 7837 | 7920 | 8003 | 8086 | 8169 | 8253 | 8336 | 8419 | 83 |
| 523 | 8502 | 8585 | 8668 | 8751 | 8834 | 8917 | 9000 | 9083 | 9165 | 9248 | 83 |
| 524 | 9331 | 9414 | 9497 | 9580 | 9663 | 9745 | 9828 | 9911 | 9994 | 0077 | 83 |
| 525 | 720159 | 0242 | 0325 | 0407 | 0490 | 0573 | 0655 | 0738 | 0821 | 0903 | 83 |
| 526 | 0986 | 1068 | 1151 | 1233 | 1316 | 1398 | 1481 | 1563 | 1646 | 1728 | 82 |
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| 530 | 4276 | 4358 | 4440 | 4522 | 4604 | 4685 | 4767 | 4849 | 4931 | 5013 | 82 |
| 531 | 5095 | 5176 | 5258 | 5340 | 5422 | 5503 | 5585 | 5667 | 5748 | 5830 | 82 |
| 532 | 5912 | 5993 | 6075 | 6156 | 6238 | 6320 | 6401 | 6483 | 6564 | 6646 | 82 |
| 533 | 6727 | 6809 | 6890 | 6972 | 7053 | 7134 | 7216 | 7297 | 7379 | 7460 | 81 |
| 534 | 7541 | 7623 | 7704 | 7785 | 7866 | 7948 | 8029 | 8110 | 8191 | 8273 | 81 |
| 535 | 8354 | 8435 | 8516 | 8597 | 8678 | 8759 | 8841 | 8922 | 9003 | 9084 | 81 |
| 536 | 9165 | 9246 | 9327 | 9408 | 9489 | 9570 | 9651 | 9732 | 9813 | 9893 | 81 |
| 537 | 9974 | 0055 | 0136 | 0217 | 0298 | 0378 | 0459 | 0540 | 0621 | 0702 | 81 |
| 538 | 730782 | 0863 | 0944 | 1024 | 1105 | 1186 | 1266 | 1347 | 1428 | 1508 | 81 |
| 539 | 1589 | 1669 | 1750 | 1830 | 1911 | 1991 | 2072 | 2152 | 2233 | 2313 | 81 |
| 540 | 2394 | 2474 | 2555 | 2635 | 2715 | 2796 | 2876 | 2956 | 3037 | 3117 | 80 |
| 541 | 3197 | 3278 | 3358 | 3438 | 3518 | 3598 | 3679 | 3759 | 3839 | 3919 | 80 |
| 542 | 3999 | 4079 | 4160 | 4240 | 4320 | 4400 | 4480 | 4560 | 4640 | 4720 | 80 |
| 543 | 4800 | 4880 | 4960 | 5040 | 5120 | 5200 | 5279 | 5359 | 5439 | 5519 | 80 |
| 544 | 5599 | 5679 | 5759 | 5838 | 5918 | 5998 | 6078 | 6157 | 6237 | 6317 | 80 |
| 545 | 6397 | 6476 | 6556 | 6635 | 6715 | 6795 | 6874 | 6954 | 7034 | 7113 | 80 |
| 546 | 7193 | 7272 | 7352 | 7431 | 7511 | 7590 | 7670 | 7749 | 7829 | 7908 | 79 |
| 547 | 7987 | 8067 | 8146 | 8225 | 8305 | 8384 | 8463 | 8543 | 8622 | 8701 | 79 |
| 548 | 8781 | 8860 | 8939 | 9018 | 9097 | 9177 | 9256 | 9335 | 9414 | 9493 | 79 |
| 549 | 9572 | 9651 | 9731 | 9810 | 9889 | 9968 | 0047 | 0126 | 0205 | 0284 | 79 |
| 550 | 740363 | 0442 | 0521 | 0600 | 0678 | 0757 | 0836 | 0915 | 0994 | 1073 | 79 |
| 551 | 1152 | 1230 | 1309 | 1388 | 1467 | 1546 | 1624 | 1703 | 1782 | 1860 | 79 |
| 552 | 1939 | 2018 | 2096 | 2175 | 2254 | 2332 | 2411 | 2489 | 2568 | 2647 | 79 |
| 553 | 2725 | 2804 | 2882 | 2961 | 3039 | 3118 | 3196 | 3275 | 3353 | 3431 | 78 |
| 554 | 3510 | 3588 | 3667 | 3745 | 3823 | 3902 | 3980 | 4058 | 4136 | 4215 | 78 |
| 555 | 4293 | 4371 | 4449 | 4528 | 4606 | 4684 | 4762 | 4840 | 4919 | 4997 | 78 |
| 556 | 5075 | 5153 | 5231 | 5309 | 5387 | 5465 | 5543 | 5621 | 5699 | 5777 | 78 |
| 557 | 5855 | 5933 | 6011 | 6089 | 6167 | 6245 | 6323 | 6401 | 6479 | 6556 | 78 |
| 558 | 6634 | 6712 | 6790 | 6868 | 6945 | 7023 | 7101 | 7179 | 7256 | 7334 | 78 |
| 559 | 7412 | 7489 | 7567 | 7645 | 7722 | 7800 | 7878 | 7955 | 8033 | 8110 | 78 |
| 560 | 8188 | 8266 | 8343 | 8421 | 8498 | 8576 | 8653 | 8731 | 8808 | 8885 | 77 |
| 561 | 8963 | 9040 | 9118 | 9195 | 9272 | 9350 | 9427 | 9504 | 9582 | 9659 | 77 |
| 562 | 9736 | 9814 | 9891 | 9968 | 0045 | 0123 | 0200 | 0277 | 0354 | 0431 | 77 |
| 563 | 750508 | 0586 | 0663 | 0740 | 0817 | 0894 | 0971 | 1048 | 1125 | 1202 | 77 |
| 564 | 1279 | 1356 | 1433 | 1510 | 1587 | 1664 | 1741 | 1818 | 1895 | 1972 | 77 |
| 565 | 2048 | 2125 | 2202 | 2279 | 2356 | 2433 | 2509 | 2586 | 2663 | 2740 | 77 |
| 566 | 2816 | 2893 | 2970 | 3047 | 3123 | 3200 | 3277 | 3353 | 3430 | 3506 | 77 |
| 567 | 3583 | 3660 | 3736 | 3813 | 3889 | 3966 | 4042 | 4119 | 4195 | 4272 | 77 |
| 568 | 4348 | 4425 | 4501 | 4578 | 4654 | 4730 | 4807 | 4883 | 4960 | 5036 | 76 |
| 569 | 5112 | 5189 | 5265 | 5341 | 5417 | 5494 | 5570 | 5646 | 5722 | 5799 | 76 |
| 570 | 5875 | 5951 | 6027 | 6103 | 6180 | 6256 | 6332 | 6408 | 6484 | 6560 | 76 |
| 571 | 6636 | 6712 | 6788 | 6864 | 6940 | 7016 | 7092 | 7168 | 7244 | 7320 | 76 |
| 572 | 7396 | 7472 | 7548 | 7624 | 7700 | 7775 | 7851 | 7927 | 8003 | 8079 | 76 |
| 573 | 8155 | 8230 | 8306 | 8382 | 8458 | 8533 | 8609 | 8685 | 8761 | 8836 | 76 |
| 574 | 8912 | 8988 | 9063 | 9139 | 9214 | 9290 | 9366 | 9441 | 9517 | 9592 | 76 |
| 575 | 9668 | 9743 | 9819 | 9894 | 9970 | 0045 | 0121 | 0196 | 0272 | 0347 | 75 |
| 576 | 760422 | 0498 | 0573 | 0649 | 0724 | 0799 | 0875 | 0950 | 1025 | 1101 | 75 |
| 577 | 1176 | 1251 | 1326 | 1402 | 1477 | 1552 | 1627 | 1702 | 1778 | 1853 | 75 |
| 578 | 1928 | 2003 | 2078 | 2153 | 2228 | 2303 | 2378 | 2453 | 2529 | 2604 | 75 |
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| 583 | 5669 | 5743 | 5818 | 5892 | 5966 | 6041 | 6115 | 6190 | 6264 | 6338 | 74 |
| 584 | 6413 | 6487 | 6562 | 6636 | 6710 | 6785 | 6859 | 6933 | 7007 | 7082 | 74 |
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| 586 | 7898 | 7972 | 8046 | 8120 | 8194 | 8268 | 8342 | 8416 | 8490 | 8564 | 74 |
| 587 | 8638 | 8712 | 8786 | 8860 | 8934 | 9008 | 9082 | 9156 | 9230 | 9303 | 74 |
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| 589 | 770115 | 0189 | 0263 | 0336 | 0410 | 0484 | 0557 | 0631 | 0705 | 0778 | 74 |
| 590 | 0852 | 0926 | 0999 | 1073 | 1146 | 1220 | 1293 | 1367 | 1440 | 1514 | 74 |
| 591 | 1587 | 1661 | 1734 | 1808 | 1881 | 1955 | 2028 | 2102 | 2175 | 2248 | 73 |
| 592 | 2322 | 2395 | 2468 | 2542 | 2615 | 2688 | 2762 | 2835 | 2908 | 2981 | 73 |
| 593 | 3055 | 3128 | 3201 | 3274 | 3348 | 3421 | 3494 | 3567 | 3640 | 3713 | 73 |
| 594 | 3786 | 3860 | 3933 | 4006 | 4079 | 4152 | 4225 | 4298 | 4371 | 4444 | 73 |
| 595 | 4517 | 4590 | 4663 | 4736 | 4809 | 4882 | 4955 | 5028 | 5100 | 5173 | 73 |
| 596 | 5246 | 5319 | 5392 | 5465 | 5538 | 5610 | 5683 | 5756 | 5829 | 5902 | 73 |
| 597 | 5974 | 6047 | 6120 | 6193 | 6265 | 6338 | 6411 | 6483 | 6556 | 6629 | 73 |
| 598 | 6701 | 6774 | 6846 | 6919 | 6992 | 7064 | 7137 | 7209 | 7282 | 7354 | 73 |
| 599 | 7427 | 7499 | 7572 | 7644 | 7717 | 7789 | 7862 | 7934 | 8006 | 8079 | 72 |
| 600 | 8151 | 8224 | 8296 | 8368 | 8441 | 8513 | 8585 | 8658 | 8730 | 8802 | 72 |
| 601 | 8874 | 8947 | 9019 | 9091 | 9163 | 9236 | 9308 | 9380 | 9452 | 9524 | 72 |
| 602 | 9596 | 9669 | 9741 | 9813 | 9885 | 9957 | 0029 | 0101 | 0173 | 0245 | 72 |
| 603 | 780317 | 0389 | 0461 | 0533 | 0605 | 0677 | 0749 | 0821 | 0893 | 0965 | 72 |
| 604 | 1037 | 1109 | 1181 | 1253 | 1324 | 1396 | 1468 | 1540 | 1612 | 1684 | 72 |
| 605 | 1755 | 1827 | 1899 | 1971 | 2042 | 2114 | 2186 | 2258 | 2329 | 2401 | 72 |
| 606 | 2473 | 2544 | 2616 | 2688 | 2759 | 2831 | 2902 | 2974 | 3046 | 3117 | 72 |
| 607 | 3189 | 3260 | 3332 | 3403 | 3475 | 3546 | 3618 | 3689 | 3761 | 3832 | 71 |
| 608 | 3904 | 3975 | 4046 | 4118 | 4189 | 4261 | 4332 | 4403 | 4475 | 4546 | 71 |
| 609 | 4617 | 4689 | 4760 | 4831 | 4902 | 4974 | 5045 | 5116 | 5187 | 5259 | 71 |
| 610 | 5330 | 5401 | 5472 | 5543 | 5615 | 5686 | 5757 | 5828 | 5899 | 5970 | 71 |
| 611 | 6041 | 6112 | 6183 | 6254 | 6325 | 6396 | 6467 | 6538 | 6609 | 6680 | 71 |
| 612 | 6751 | 6822 | 6893 | 6964 | 7035 | 7106 | 7177 | 7248 | 7319 | 7390 | 71 |
| 613 | 7460 | 7531 | 7602 | 7673 | 7744 | 7815 | 7885 | 7956 | 8027 | 8098 | 71 |
| 614 | 8168 | 8239 | 8310 | 8381 | 8451 | 8522 | 8593 | 8663 | 8734 | 8804 | 71 |
| 615 | 8875 | 8946 | 9016 | 9087 | 9157 | 9228 | 9299 | 9369 | 9440 | 9510 | 71 |
| 616 | 9581 | 9651 | 9722 | 9792 | 9863 | 9933 | 0004 | 0074 | 0144 | 0215 | 70 |
| 617 | 790285 | 0356 | 0426 | 0496 | 0567 | 0637 | 0707 | 0778 | 0848 | 0918 | 70 |
| 618 | 0988 | 1059 | 1129 | 1199 | 1269 | 1340 | 1410 | 1480 | 1550 | 1620 | 70 |
| 619 | 1691 | 1761 | 1831 | 1901 | 1971 | 2041 | 2111 | 2181 | 2252 | 2322 | 70 |
| 620 | 2392 | 2462 | 2532 | 2602 | 2672 | 2742 | 2812 | 2882 | 2952 | 3022 | 70 |
| 621 | 3092 | 3162 | 3231 | 3301 | 3371 | 3441 | 3511 | 3581 | 3651 | 3721 | 70 |
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| 623 | 4488 | 4558 | 4627 | 4697 | 4767 | 4836 | 4906 | 4976 | 5045 | 5115 | 70 |
| 624 | 5185 | 5254 | 5324 | 5393 | 5463 | 5532 | 5602 | 5672 | 5741 | 5811 | 70 |
| 625 | 5880 | 5949 | 6019 | 6088 | 6158 | 6227 | 6297 | 6366 | 6436 | 6505 | 69 |
| 626 | 6574 | 6644 | 6713 | 6782 | 6852 | 6921 | 6990 | 7060 | 7129 | 7198 | 69 |
| 627 | 7268 | 7337 | 7406 | 7475 | 7545 | 7614 | 7683 | 7752 | 7821 | 7890 | 69 |
| 628 | 7960 | 8029 | 8098 | 8167 | 8236 | 8305 | 8374 | 8443 | 8513 | 8582 | 69 |
| 629 | 8651 | 8720 | 8789 | 8858 | 8927 | 8996 | 9065 | 9134 | 9203 | 9272 | 69 |
| 630 | 9341 | 9409 | 9478 | 9547 | 9616 | 9685 | 9754 | 9823 | 9892 | 9961 | 69 |
| 631 | 800029 | 0098 | 0167 | 0236 | 0305 | 0373 | 0442 | 0511 | 0580 | 0648 | 69 |
| 632 | 0717 | 0786 | 0854 | 0923 | 0992 | 1061 | 1129 | 1198 | 1266 | 1335 | 69 |
| 633 | 1404 | 1472 | 1541 | 1609 | 1678 | 1747 | 1815 | 1884 | 1952 | 2021 | 69 |
| 634 | 2089 | 2158 | 2226 | 2295 | 2363 | 2432 | 2500 | 2568 | 2637 | 2705 | 69 |
| 635 | 2774 | 2842 | 2910 | 2979 | 3047 | 3116 | 3184 | 3252 | 3321 | 3389 | 68 |
| 636 | 3457 | 3525 | 3594 | 3662 | 3730 | 3798 | 3867 | 3935 | 4003 | 4071 | 68 |
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| 638 | 4821 | 4889 | 4957 | 5025 | 5093 | 5161 | 5229 | 5297 | 5365 | 5433 | 68 |
| 639 | 5501 | 5569 | 5637 | 5705 | 5773 | 5841 | 5908 | 5976 | 6044 | 6112 | 68 |
| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |

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| 645 | 9560 | 9627 | 9694 | 9762 | 9829 | 9896 | 9964 | 0031 | 0098 | 0165 | 67 |
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| 657 | 7565 | 7631 | 7698 | 7764 | 7830 | 7896 | 7962 | 8028 | 8094 | 8160 | 66 |
| 658 | 8226 | 8292 | 8358 | 8424 | 8490 | 8556 | 8622 | 8688 | 8754 | 8820 | 66 |
| 659 | 8885 | 8951 | 9017 | 9083 | 9149 | 9215 | 9281 | 9346 | 9412 | 9478 | 66 |
| 660 | 9544 | 9610 | 9676 | 9741 | 9807 | 9873 | 9939 | 0004 | 0070 | 0136 | 66 |
| 661 | 820201 | 0267 | 0333 | 0399 | 0464 | 0530 | 0595 | 0661 | 0727 | 0792 | 66 |
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| 663 | 1514 | 1579 | 1645 | 1710 | 1775 | 1841 | 1906 | 1972 | 2037 | 2103 | 65 |
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| 667 | 4126 | 4191 | 4256 | 4321 | 4386 | 4451 | 4516 | 4581 | 4646 | 4711 | 65 |
| 668 | 4776 | 4841 | 4906 | 4971 | 5036 | 5101 | 5166 | 5231 | 5296 | 5361 | 65 |
| 669 | 5426 | 5491 | 5556 | 5621 | 5686 | 5751 | 5815 | 5880 | 5945 | 6010 | 65 |
| 670 | 6075 | 6140 | 6204 | 6269 | 6334 | 6399 | 6464 | 6528 | 6593 | 6658 | 65 |
| 671 | 6723 | 6787 | 6852 | 6917 | 6981 | 7046 | 7111 | 7175 | 7240 | 7305 | 65 |
| 672 | 7369 | 7434 | 7499 | 7563 | 7628 | 7692 | 7757 | 7821 | 7886 | 7951 | 65 |
| 673 | 8015 | 8080 | 8144 | 8209 | 8273 | 8338 | 8402 | 8467 | 8531 | 8595 | 64 |
| 674 | 8660 | 8724 | 8789 | 8853 | 8918 | 8982 | 9046 | 9111 | 9175 | 9239 | 64 |
| 675 | 9304 | 9368 | 9432 | 9497 | 9561 | 9625 | 9690 | 9754 | 9818 | 9882 | 64 |
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| 684 | 5056 | 5120 | 5183 | 5247 | 5310 | 5373 | 5437 | 5500 | 5564 | 5627 | 63 |
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| 687 | 6957 | 7020 | 7083 | 7146 | 7210 | 7273 | 7336 | 7399 | 7462 | 7525 | 63 |
| 688 | 7588 | 7652 | 7715 | 7778 | 7841 | 7904 | 7967 | 8030 | 8093 | 8156 | 63 |
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| 693 | 0733 | 0796 | 0859 | 0921 | 0984 | 1046 | 1109 | 1172 | 1234 | 1297 | 63 |
| 694 | 1359 | 1422 | 1485 | 1547 | 1610 | 1672 | 1735 | 1797 | 1860 | 1922 | 63 |
| 695 | 1985 | 2047 | 2110 | 2172 | 2235 | 2297 | 2360 | 2422 | 2484 | 2547 | 62 |
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| 698 | 3855 | 3918 | 3980 | 4042 | 4104 | 4166 | 4229 | 4291 | 4353 | 4415 | 62 |
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| 703 | 6955 | 7017 | 7079 | 7141 | 7202 | 7264 | 7326 | 7388 | 7449 | 7511 | 62 |
| 704 | 7573 | 7634 | 7696 | 7758 | 7819 | 7881 | 7943 | 8004 | 8066 | 8128 | 62 |
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| 706 | 8805 | 8866 | 8928 | 8989 | 9051 | 9112 | 9174 | 9235 | 9297 | 9358 | 61 |
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| 724 | 9739 | 9799 | 9859 | 9918 | 9978 | 0038 | 0098 | 0158 | 0218 | 0278 | 60 |
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| 733 | 5104 | 5163 | 5222 | 5282 | 5341 | 5400 | 5459 | 5519 | 5578 | 5637 | 59 |
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| 735 | 6287 | 6346 | 6405 | 6465 | 6524 | 6583 | 6642 | 6701 | 6760 | 6819 | 59 |
| 736 | 6878 | 6937 | 6996 | 7055 | 7114 | 7173 | 7232 | 7291 | 7350 | 7409 | 59 |
| 737 | 7467 | 7526 | 7585 | 7644 | 7703 | 7762 | 7821 | 7880 | 7939 | 7998 | 59 |
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| 740 | 9232 | 9290 | 9349 | 9408 | 9466 | 9525 | 9584 | 9642 | 9701 | 9760 | 59 |
| 741 | 9818 | 9877 | 9935 | 9994 | 0053 | 0111 | 0170 | 0228 | 0287 | 0345 | 59 |
| 742 | 870404 | 0462 | 0521 | 0579 | 0638 | 0696 | 0755 | 0813 | 0872 | 0930 | 58 |
| 743 | 0989 | 1047 | 1106 | 1164 | 1223 | 1281 | 1339 | 1398 | 1456 | 1515 | 58 |
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| 747 | 3321 | 3379 | 3437 | 3495 | 3553 | 3611 | 3669 | 3727 | 3785 | 3844 | 58 |
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| 749 | 4482 | 4540 | 4598 | 4656 | 4714 | 4772 | 4830 | 4888 | 4945 | 5003 | 58 |
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| 755 | 7947 | 8004 | 8062 | 8119 | 8177 | 8234 | 8292 | 8349 | 8407 | 8464 | 57 |
| 756 | 8522 | 8579 | 8637 | 8694 | 8752 | 8809 | 8866 | 8924 | 8981 | 9039 | 57 |
| 757 | 9096 | 9153 | 9211 | 9268 | 9325 | 9383 | 9440 | 9497 | 9555 | 9612 | 57 |
| 758 | 9669 | 9726 | 9784 | 9841 | 9898 | 9956 | 0013 | 0070 | 0127 | 0185 | 57 |
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| 770 | 6491 | 6547 | 6604 | 6660 | 6716 | 6773 | 6829 | 6885 | 6942 | 6998 | 56 |
| 771 | 7054 | 7111 | 7167 | 7223 | 7280 | 7336 | 7392 | 7449 | 7505 | 7561 | 56 |
| 772 | 7617 | 7674 | 7730 | 7786 | 7842 | 7898 | 7955 | 8011 | 8067 | 8123 | 56 |
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| 774 | 8741 | 8797 | 8853 | 8909 | 8965 | 9021 | 9077 | 9134 | 9190 | 9246 | 56 |
| 775 | 9302 | 9358 | 9414 | 9470 | 9526 | 9582 | 9638 | 9694 | 9750 | 9806 | 56 |
| 776 | 9862 | 9918 | 9974 | 0030 | 0086 | 0141 | 0197 | 0253 | 0309 | 0365 | 56 |
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| 781 | 2651 | 2707 | 2762 | 2818 | 2873 | 2929 | 2985 | 3040 | 3096 | 3151 | 56 |
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| 787 | 5975 | 6030 | 6085 | 6140 | 6195 | 6251 | 6306 | 6361 | 6416 | 6471 | 55 |
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| 790 | 7627 | 7682 | 7737 | 7792 | 7847 | 7902 | 7957 | 8012 | 8067 | 8122 | 55 |
| 791 | 8176 | 8231 | 8286 | 8341 | 8396 | 8451 | 8506 | 8561 | 8615 | 8670 | 55 |
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| 796 | 0913 | 0968 | 1022 | 1077 | 1131 | 1186 | 1240 | 1295 | 1349 | 1404 | 55 |
| 797 | 1458 | 1513 | 1567 | 1622 | 1676 | 1731 | 1785 | 1840 | 1894 | 1948 | 54 |
| 798 | 2003 | 2057 | 2112 | 2166 | 2221 | 2275 | 2329 | 2384 | 2438 | 2492 | 54 |
| 799 | 2547 | 2601 | 2655 | 2710 | 2764 | 2818 | 2873 | 2927 | 2981 | 3036 | 54 |
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| 801 | 3633 | 3687 | 3741 | 3795 | 3849 | 3904 | 3958 | 4012 | 4066 | 4120 | 54 |
| 802 | 4174 | 4229 | 4283 | 4337 | 4391 | 4445 | 4499 | 4553 | 4607 | 4661 | 54 |
| 803 | 4716 | 4770 | 4824 | 4878 | 4932 | 4986 | 5040 | 5094 | 5148 | 5202 | 54 |
| 804 | 5256 | 5310 | 5364 | 5418 | 5472 | 5526 | 5580 | 5634 | 5688 | 5742 | 54 |
| 805 | 5796 | 5850 | 5904 | 5958 | 6012 | 6066 | 6119 | 6173 | 6227 | 6281 | 54 |
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| 812 | 9556 | 9610 | 9663 | 9716 | 9770 | 9823 | 9877 | 9930 | 9984 | 0037 | 53 |
| 813 | 910091 | 0144 | 0197 | 0251 | 0304 | 0358 | 0411 | 0464 | 0518 | 0571 | 53 |
| 814 | 0624 | 0678 | 0731 | 0784 | 0838 | 0891 | 0944 | 0998 | 1051 | 1104 | 53 |
| 815 | 1158 | 1211 | 1264 | 1317 | 1371 | 1424 | 1477 | 1530 | 1584 | 1637 | 53 |
| 816 | 1690 | 1743 | 1797 | 1850 | 1903 | 1956 | 2009 | 2063 | 2116 | 2169 | 53 |
| 817 | 2222 | 2275 | 2328 | 2381 | 2435 | 2488 | 2541 | 2594 | 2647 | 2700 | 53 |
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| 832 | 920123 | 0176 | 0228 | 0280 | 0332 | 0384 | 0436 | 0489 | 0541 | 0593 | 52 |
| 833 | 0645 | 0697 | 0749 | 0801 | 0853 | 0906 | 0958 | 1010 | 1062 | 1114 | 52 |
| 834 | 1166 | 1218 | 1270 | 1322 | 1374 | 1426 | 1478 | 1530 | 1582 | 1634 | 52 |
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| 842 | 5312 | 5364 | 5415 | 5467 | 5518 | 5570 | 5621 | 5673 | 5725 | 5776 | 52 |
| 843 | 5828 | 5879 | 5931 | 5982 | 6034 | 6085 | 6137 | 6188 | 6240 | 6291 | 51 |
| 844 | 6342 | 6394 | 6445 | 6497 | 6548 | 6600 | 6651 | 6702 | 6754 | 6805 | 51 |
| 845 | 6857 | 6908 | 6959 | 7011 | 7062 | 7114 | 7165 | 7216 | 7268 | 7319 | 51 |
| 846 | 7370 | 7422 | 7473 | 7524 | 7576 | 7627 | 7678 | 7730 | 7781 | 7832 | 51 |
| 847 | 7883 | 7935 | 7986 | 8037 | 8088 | 8140 | 8191 | 8242 | 8293 | 8345 | 51 |
| 848 | 8396 | 8447 | 8498 | 8549 | 8601 | 8652 | 8703 | 8754 | 8805 | 8857 | 51 |
| 849 | 8908 | 8959 | 9010 | 9061 | 9112 | 9163 | 9215 | 9266 | 9317 | 9368 | 51 |
| 850 | 9419 | 9470 | 9521 | 9572 | 9623 | 9674 | 9725 | 9776 | 9827 | 9879 | 51 |
| 851 | 9930 | 9981 | 0032 | 0083 | 0134 | 0185 | 0236 | 0287 | 0338 | 0389 | 51 |
| 852 | 930440 | 0491 | 0542 | 0592 | 0643 | 0694 | 0745 | 0796 | 0847 | 0898 | 51 |
| 853 | 0949 | 1000 | 1051 | 1102 | 1153 | 1204 | 1254 | 1305 | 1356 | 1407 | 51 |
| 854 | 1458 | 1509 | 1560 | 1610 | 1661 | 1712 | 1763 | 1814 | 1865 | 1915 | 51 |
| 855 | 1966 | 2017 | 2068 | 2118 | 2169 | 2220 | 2271 | 2322 | 2372 | 2423 | 51 |
| 856 | 2474 | 2524 | 2575 | 2626 | 2677 | 2727 | 2778 | 2829 | 2879 | 2930 | 51 |
| 857 | 2981 | 3031 | 3082 | 3133 | 3183 | 3234 | 3285 | 3335 | 3386 | 3437 | 51 |
| 858 | 3487 | 3538 | 3589 | 3639 | 3690 | 3740 | 3791 | 3841 | 3892 | 3943 | 51 |
| 859 | 3993 | 4044 | 4094 | 4145 | 4195 | 4246 | 4296 | 4347 | 4397 | 4448 | 51 |
| 860 | 4498 | 4549 | 4599 | 4650 | 4700 | 4751 | 4801 | 4852 | 4902 | 4953 | 50 |
| 861 | 5003 | 5054 | 5104 | 5154 | 5205 | 5255 | 5306 | 5356 | 5406 | 5457 | 50 |
| 862 | 5507 | 5558 | 5608 | 5658 | 5709 | 5759 | 5809 | 5860 | 5910 | 5960 | 50 |
| 863 | 6011 | 6061 | 6111 | 6162 | 6212 | 6262 | 6313 | 6363 | 6413 | 6463 | 50 |
| 864 | 6514 | 6564 | 6614 | 6665 | 6715 | 6765 | 6815 | 6865 | 6916 | 6966 | 50 |
| 865 | 7016 | 7066 | 7117 | 7167 | 7217 | 7267 | 7317 | 7367 | 7418 | 7468 | 50 |
| 866 | 7518 | 7568 | 7618 | 7668 | 7718 | 7769 | 7819 | 7869 | 7919 | 7969 | 50 |
| 867 | 8019 | 8069 | 8119 | 8169 | 8219 | 8269 | 8320 | 8370 | 8420 | 8470 | 50 |
| 868 | 8520 | 8570 | 8620 | 8670 | 8720 | 8770 | 8820 | 8870 | 8920 | 8970 | 50 |
| 869 | 9020 | 9070 | 9120 | 9170 | 9220 | 9270 | 9320 | 9369 | 9419 | 9469 | 50 |
| 870 | 9519 | 9569 | 9619 | 9669 | 9719 | 9769 | 9819 | 9869 | 9918 | 9968 | 50 |
| 871 | 940018 | 0068 | 0118 | 0168 | 0218 | 0267 | 0317 | 0367 | 0417 | 0467 | 50 |
| 872 | 0516 | 0566 | 0616 | 0666 | 0716 | 0765 | 0815 | 0865 | 0915 | 0964 | 50 |
| 873 | 1014 | 1064 | 1114 | 1163 | 1213 | 1263 | 1313 | 1362 | 1412 | 1462 | 50 |
| 874 | 1511 | 1561 | 1611 | 1660 | 1710 | 1760 | 1809 | 1859 | 1909 | 1958 | 50 |
| 875 | 2008 | 2058 | 2107 | 2157 | 2207 | 2256 | 2306 | 2355 | 2405 | 2455 | 50 |
| 876 | 2504 | 2554 | 2603 | 2653 | 2702 | 2752 | 2801 | 2851 | 2901 | 2950 | 50 |
| 877 | 3000 | 3049 | 3099 | 3148 | 3198 | 3247 | 3297 | 3346 | 3396 | 3445 | 49 |
| 878 | 3495 | 3544 | 3593 | 3643 | 3692 | 3742 | 3791 | 3841 | 3890 | 3939 | 49 |
| 879 | 3989 | 4038 | 4088 | 4137 | 4186 | 4236 | 4285 | 4335 | 4384 | 4433 | 49 |
| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 880 | 944483 | 4532 | 4581 | 4631 | 4680 | 4729 | 4779 | 4828 | 4877 | 4927 | 49 |
| 881 | 4976 | 5025 | 5074 | 5124 | 5173 | 5222 | 5272 | 5321 | 5370 | 5419 | 49 |
| 882 | 5469 | 5518 | 5567 | 5616 | 5665 | 5715 | 5764 | 5813 | 5862 | 5912 | 49 |
| 883 | 5961 | 6010 | 6059 | 6108 | 6157 | 6207 | 6256 | 6305 | 6354 | 6403 | 49 |
| 884 | 6452 | 6501 | 6551 | 6600 | 6649 | 6698 | 6747 | 6796 | 6845 | 6894 | 49 |
| 885 | 6943 | 6992 | 7041 | 7090 | 7140 | 7189 | 7238 | 7287 | 7336 | 7385 | 49 |
| 886 | 7434 | 7483 | 7532 | 7581 | 7630 | 7679 | 7728 | 7777 | 7826 | 7875 | 49 |
| 887 | 7924 | 7973 | 8022 | 8070 | 8119 | 8168 | 8217 | 8266 | 8315 | 8364 | 49 |
| 888 | 8413 | 8462 | 8511 | 8560 | 8609 | 8657 | 8706 | 8755 | 8804 | 8853 | 49 |
| 889 | 8902 | 8951 | 8999 | 9048 | 9097 | 9146 | 9195 | 9244 | 9292 | 9341 | 49 |
| 890 | 9390 | 9439 | 9488 | 9536 | 9585 | 9634 | 9683 | 9731 | 9780 | 9829 | 49 |
| 891 | 9878 | 9926 | 9975 | 0024 | 0073 | 0121 | 0170 | 0219 | 0267 | 0316 | 49 |
| 892 | 950365 | 0414 | 0462 | 0511 | 0560 | 0608 | 0657 | 0706 | 0754 | 0803 | 49 |
| 893 | 0851 | 0900 | 0949 | 0997 | 1046 | 1095 | 1143 | 1192 | 1240 | 1289 | 49 |
| 894 | 1338 | 1386 | 1435 | 1483 | 1532 | 1580 | 1629 | 1677 | 1726 | 1775 | 49 |
| 895 | 1823 | 1872 | 1920 | 1969 | 2017 | 2066 | 2114 | 2163 | 2211 | 2260 | 48 |
| 896 | 2308 | 2356 | 2405 | 2453 | 2502 | 2550 | 2599 | 2647 | 2696 | 2744 | 48 |
| 897 | 2792 | 2841 | 2889 | 2938 | 2986 | 3034 | 3083 | 3131 | 3180 | 3228 | 48 |
| 898 | 3276 | 3325 | 3373 | 3421 | 3470 | 3518 | 3566 | 3615 | 3663 | 3711 | 48 |
| 899 | 3760 | 3808 | 3856 | 3905 | 3953 | 4001 | 4049 | 4098 | 4146 | 4194 | 48 |
| 900 | 4243 | 4291 | 4339 | 4387 | 4435 | 4484 | 4532 | 4580 | 4628 | 4677 | 48 |
| 901 | 4725 | 4773 | 4821 | 4869 | 4918 | 4966 | 5014 | 5062 | 5110 | 5158 | 48 |
| 902 | 5207 | 5255 | 5303 | 5351 | 5399 | 5447 | 5495 | 5543 | 5592 | 5640 | 48 |
| 903 | 5688 | 5736 | 5784 | 5832 | 5880 | 5928 | 5976 | 6024 | 6072 | 6120 | 48 |
| 904 | 6168 | 6216 | 6265 | 6313 | 6361 | 6409 | 6457 | 6505 | 6553 | 6601 | 48 |
| 905 | 6649 | 6697 | 6745 | 6793 | 6840 | 6888 | 6936 | 6984 | 7032 | 7080 | 48 |
| 906 | 7128 | 7176 | 7224 | 7272 | 7320 | 7368 | 7416 | 7464 | 7512 | 7559 | 48 |
| 907 | 7607 | 7655 | 7703 | 7751 | 7799 | 7847 | 7894 | 7942 | 7990 | 8038 | 48 |
| 908 | 8086 | 8134 | 8181 | 8229 | 8277 | 8325 | 8373 | 8421 | 8468 | 8516 | 48 |
| 909 | 8564 | 8612 | 8659 | 8707 | 8755 | 8803 | 8850 | 8898 | 8946 | 8994 | 48 |
| 910 | 9041 | 9089 | 9137 | 9185 | 9232 | 9280 | 9328 | 9375 | 9423 | 9471 | 48 |
| 911 | 9518 | 9566 | 9614 | 9661 | 9709 | 9757 | 9804 | 9852 | 9900 | 9947 | 48 |
| 912 | 9995 | 0042 | 0090 | 0138 | 0185 | 0233 | 0280 | 0328 | 0376 | 0423 | 48 |
| 913 | 960471 | 0518 | 0566 | 0613 | 0661 | 0709 | 0756 | 0804 | 0851 | 0899 | 48 |
| 914 | 0946 | 0994 | 1041 | 1089 | 1136 | 1184 | 1231 | 1279 | 1326 | 1374 | 47 |
| 915 | 1421 | 1469 | 1516 | 1563 | 1611 | 1658 | 1706 | 1753 | 1801 | 1848 | 47 |
| 916 | 1895 | 1943 | 1990 | 2038 | 2085 | 2132 | 2180 | 2227 | 2275 | 2322 | 47 |
| 917 | 2369 | 2417 | 2464 | 2511 | 2559 | 2606 | 2653 | 2701 | 2748 | 2795 | 47 |
| 918 | 2843 | 2890 | 2937 | 2985 | 3032 | 3079 | 3126 | 3174 | 3221 | 3268 | 47 |
| 919 | 3316 | 3363 | 3410 | 3457 | 3504 | 3552 | 3599 | 3646 | 3693 | 3741 | 47 |
| 920 | 3788 | 3835 | 3882 | 3929 | 3977 | 4024 | 4071 | 4118 | 4165 | 4212 | 47 |
| 921 | 4260 | 4307 | 4354 | 4401 | 4448 | 4495 | 4542 | 4590 | 4637 | 4684 | 47 |
| 922 | 4731 | 4778 | 4825 | 4872 | 4919 | 4966 | 5013 | 5061 | 5108 | 5155 | 47 |
| 923 | 5202 | 5249 | 5296 | 5343 | 5390 | 5437 | 5484 | 5531 | 5578 | 5625 | 47 |
| 924 | 5672 | 5719 | 5766 | 5813 | 5860 | 5907 | 5954 | 6001 | 6048 | 6095 | 47 |
| 925 | 6142 | 6189 | 6236 | 6283 | 6329 | 6376 | 6423 | 6470 | 6517 | 6564 | 47 |
| 626 | 6611 | 6658 | 6705 | 6752 | 6799 | 6845 | 6892 | 6939 | 6986 | 7033 | 47 |
| 627 | 7080 | 7127 | 7173 | 7220 | 7267 | 7314 | 7361 | 7408 | 7454 | 7501 | 47 |
| 928 | 7548 | 7595 | 7642 | 7688 | 7735 | 7782 | 7829 | 7875 | 7922 | 7969 | 47 |
| 929 | 8016 | 8062 | 8109 | 8156 | 8203 | 8249 | 8296 | 8343 | 8390 | 8436 | 47 |
| 930 | 8483 | 8530 | 8576 | 8623 | 8670 | 8716 | 8763 | 8810 | 8856 | 8903 | 47 |
| 931 | 8950 | 8996 | 9043 | 9090 | 9136 | 9183 | 9229 | 9276 | 9323 | 9369 | 47 |
| 932 | 9416 | 9463 | 9509 | 9556 | 9602 | 9649 | 9695 | 9742 | 9789 | 9835 | 47 |
| 933 | 9882 | 9928 | 9975 | 0021 | 0068 | 0114 | 0161 | 0207 | 0254 | 0300 | 47 |
| 934 | 970347 | 0393 | 0440 | 0486 | 0533 | 0579 | 0626 | 0672 | 0719 | 0765 | 46 |
| 935 | 0812 | 0858 | 0904 | 0951 | 0997 | 1044 | 1090 | 1137 | 1183 | 1229 | 46 |
| 936 | 1276 | 1322 | 1369 | 1415 | 1461 | 1508 | 1554 | 1601 | 1647 | 1693 | 46 |
| 937 | 1740 | 1786 | 1832 | 1879 | 1925 | 1971 | 2018 | 2064 | 2110 | 2157 | 46 |
| 938 | 2203 | 2249 | 2295 | 2342 | 2388 | 2434 | 2481 | 2527 | 2573 | 2619 | 46 |
| 939 | 2666 | 2712 | 2758 | 2804 | 2851 | 2897 | 2943 | 2989 | 3035 | 3082 | 46 |
| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 940 | 973128 | 3174 | 3220 | 3266 | 3313 | 3359 | 3405 | 3451 | 3497 | 3543 | 46 |
| 941 | 3590 | 3636 | 3682 | 3728 | 3774 | 3820 | 3866 | 3913 | 3959 | 4005 | 46 |
| 942 | 4051 | 4097 | 4143 | 4189 | 4235 | 4281 | 4327 | 4374 | 4420 | 4466 | 46 |
| 943 | 4512 | 4558 | 4604 | 4650 | 4696 | 4742 | 4788 | 4834 | 4880 | 4926 | 46 |
| 944 | 4972 | 5018 | 5064 | 5110 | 5156 | 5202 | 5248 | 5294 | 5340 | 5386 | 46 |
| 945 | 5432 | 5478 | 5524 | 5570 | 5616 | 5662 | 5707 | 5753 | 5799 | 5845 | 46 |
| 946 | 5891 | 5937 | 5983 | 6029 | 6075 | 6121 | 6167 | 6212 | 6258 | 6304 | 46 |
| 947 | 6350 | 6396 | 6442 | 6488 | 6533 | 6579 | 6625 | 6671 | 6717 | 6763 | 46 |
| 948 | 6808 | 6854 | 6900 | 6946 | 6992 | 7037 | 7083 | 7129 | 7175 | 7220 | 46 |
| 949 | 7266 | 7312 | 7358 | 7403 | 7449 | 7495 | 7541 | 7586 | 7632 | 7678 | 46 |
| 950 | 7724 | 7769 | 7815 | 7861 | 7906 | 7952 | 7998 | 8043 | 8089 | 8135 | 46 |
| 951 | 8181 | 8226 | 8272 | 8317 | 8363 | 8409 | 8454 | 8500 | 8546 | 8591 | 46 |
| 952 | 8637 | 8683 | 8728 | 8774 | 8819 | 8865 | 8911 | 8956 | 9002 | 9047 | 46 |
| 953 | 9093 | 9138 | 9184 | 9230 | 9275 | 9321 | 9366 | 9412 | 9457 | 9503 | 46 |
| 954 | 9548 | 9594 | 9639 | 9685 | 9730 | 9776 | 9821 | 9867 | 9912 | 9958 | 46 |
| 955 | 980003 | 0049 | 0094 | 0140 | 0185 | 0231 | 0276 | 0322 | 0367 | 0412 | 45 |
| 956 | 0458 | 0503 | 0549 | 0594 | 0640 | 0685 | 0730 | 0776 | 0821 | 0867 | 45 |
| 957 | 0912 | 0957 | 1003 | 1048 | 1093 | 1139 | 1184 | 1229 | 1275 | 1320 | 45 |
| 958 | 1366 | 1411 | 1456 | 1501 | 1547 | 1592 | 1637 | 1683 | 1728 | 1773 | 45 |
| 959 | 1819 | 1864 | 1909 | 1954 | 2000 | 2045 | 2090 | 2135 | 2181 | 2226 | 45 |
| 960 | 2271 | 2316 | 2362 | 2407 | 2452 | 2497 | 2543 | 2588 | 2633 | 2678 | 45 |
| 961 | 2723 | 2769 | 2814 | 2859 | 2904 | 2949 | 2994 | 3040 | 3085 | 3130 | 45 |
| 962 | 3175 | 3220 | 3265 | 3310 | 3356 | 3401 | 3446 | 3491 | 3536 | 3581 | 45 |
| 963 | 3626 | 3671 | 3716 | 3762 | 3807 | 3852 | 3897 | 3942 | 3987 | 4032 | 45 |
| 964 | 4077 | 4122 | 4167 | 4212 | 4257 | 4302 | 4347 | 4392 | 4437 | 4482 | 45 |
| 965 | 4527 | 4572 | 4617 | 4662 | 4707 | 4752 | 4797 | 4842 | 4887 | 4932 | 45 |
| 966 | 4977 | 5022 | 5067 | 5112 | 5157 | 5202 | 5247 | 5292 | 5337 | 5382 | 45 |
| 967 | 5426 | 5471 | 5516 | 5561 | 5606 | 5651 | 5696 | 5741 | 5786 | 5830 | 45 |
| 968 | 5875 | 5920 | 5965 | 6010 | 6055 | 6100 | 6144 | 6189 | 6234 | 6279 | 45 |
| 969 | 6324 | 6369 | 6413 | 6458 | 6503 | 6548 | 6593 | 6637 | 6682 | 6727 | 45 |
| 970 | 6772 | 6817 | 6861 | 6906 | 6951 | 6996 | 7040 | 7085 | 7130 | 7175 | 45 |
| 971 | 7219 | 7264 | 7309 | 7353 | 7398 | 7443 | 7488 | 7532 | 7577 | 7622 | 45 |
| 972 | 7666 | 7711 | 7756 | 7800 | 7845 | 7890 | 7934 | 7979 | 8024 | 8068 | 45 |
| 973 | 8113 | 8157 | 8202 | 8247 | 8291 | 8336 | 8381 | 8425 | 8470 | 8514 | 45 |
| 974 | 8559 | 8604 | 8648 | 8693 | 8737 | 8782 | 8826 | 8871 | 8916 | 8960 | 45 |
| 975 | 9005 | 9049 | 9094 | 9138 | 9183 | 9227 | 9272 | 9316 | 9361 | 9405 | 45 |
| 976 | 9450 | 9494 | 9539 | 9583 | 9628 | 9672 | 9717 | 9761 | 9806 | 9850 | 44 |
| 977 | 9895 | 9939 | 9983 | 0028 | 0072 | 0117 | 0161 | 0206 | 0250 | 0294 | 44 |
| 978 | 990339 | 0383 | 0428 | 0472 | 0516 | 0561 | 0605 | 0650 | 0694 | 0738 | 44 |
| 979 | 0783 | 0827 | 0871 | 0916 | 0960 | 1004 | 1049 | 1093 | 1137 | 1182 | 44 |
| 980 | 1226 | 1270 | 1315 | 1359 | 1403 | 1448 | 1492 | 1536 | 1580 | 1625 | 44 |
| 981 | 1669 | 1713 | 1758 | 1802 | 1846 | 1890 | 1935 | 1979 | 2023 | 2067 | 44 |
| 982 | 2111 | 2156 | 2200 | 2244 | 2288 | 2333 | 2377 | 2421 | 2465 | 2509 | 44 |
| 983 | 2554 | 2598 | 2642 | 2686 | 2730 | 2774 | 2819 | 2863 | 2907 | 2951 | 44 |
| 984 | 2995 | 3039 | 3083 | 3127 | 3172 | 3216 | 3260 | 3304 | 3348 | 3392 | 44 |
| 985 | 3436 | 3480 | 3524 | 3568 | 3613 | 3657 | 3701 | 3745 | 3789 | 3833 | 44 |
| 986 | 3877 | 3921 | 3965 | 4009 | 4053 | 4097 | 4141 | 4185 | 4229 | 4273 | 44 |
| 987 | 4317 | 4361 | 4405 | 4449 | 4493 | 4537 | 4581 | 4625 | 4669 | 4713 | 44 |
| 988 | 4757 | 4801 | 4845 | 4889 | 4933 | 4977 | 5021 | 5065 | 5108 | 5152 | 44 |
| 989 | 5196 | 5240 | 5284 | 5328 | 5372 | 5416 | 5460 | 5504 | 5547 | 5591 | 44 |
| 990 | 5635 | 5679 | 5723 | 5767 | 5811 | 5854 | 5898 | 5942 | 5986 | 6030 | 44 |
| 991 | 6074 | 6117 | 6161 | 6205 | 6249 | 6293 | 6337 | 6380 | 6424 | 6468 | 44 |
| 992 | 6512 | 6555 | 6599 | 6643 | 6687 | 6731 | 6774 | 6818 | 6862 | 6906 | 44 |
| 993 | 6949 | 6993 | 7037 | 7080 | 7124 | 7168 | 7212 | 7255 | 7299 | 7343 | 44 |
| 994 | 7386 | 7430 | 7474 | 7517 | 7561 | 7605 | 7648 | 7692 | 7736 | 7779 | 44 |
| 995 | 7823 | 7867 | 7910 | 7954 | 7998 | 8041 | 8085 | 8129 | 8172 | 8216 | 44 |
| 996 | 8259 | 8303 | 8347 | 8390 | 8434 | 8477 | 8521 | 8564 | 8608 | 8652 | 44 |
| 997 | 8695 | 8739 | 8782 | 8826 | 8869 | 8913 | 8956 | 9000 | 9043 | 9087 | 44 |
| 998 | 9131 | 9174 | 9218 | 9261 | 9305 | 9348 | 9392 | 9435 | 9479 | 9522 | 44 |
| 999 | 9565 | 9609 | 9652 | 9696 | 9739 | 9783 | 9826 | 9870 | 9913 | 9957 | 43 |
| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |

# TABLE

OF

# LOGARITHMIC SINES

AND

# TANGENTS.

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | Inf. neg. | | 10.000000 | .00 | Inf. neg. | | Infinite. | 60 |
| 1 | 6.463726 | 5017.17 | 0 | | 6.463726 | 5017.17 | 13.536274 | 59 |
| 2 | 764756 | 2934.85 | 0 | | 764756 | 2934.85 | 235244 | 58 |
| 3 | 6.940847 | 2082.31 | 0 | | 6.940847 | 2082.31 | 13.059153 | 57 |
| 4 | 7.065786 | 1615.17 | 0 | | 7.065786 | 1615.17 | 12.934214 | 56 |
| 5 | 162696 | 1319.68 | 0 | .00 | 162696 | 1319.69 | 837304 | 55 |
| 6 | 241877 | 1115.78 | 9.999999 | .01 | 241878 | 1115.78 | 758122 | 54 |
| 7 | 308824 | 966.53 | 99 | | 308825 | 966.54 | 691175 | 53 |
| 8 | 366816 | 852.54 | 99 | | 366817 | 852.54 | 633183 | 52 |
| 9 | 417968 | 762.62 | 99 | | 417970 | 762.63 | 582030 | 51 |
| 10 | 463726 | 689.88 | 98 | | 463727 | 689.88 | 536273 | 50 |
| 11 | 7.505118 | 629.81 | 9.999998 | | 7.505120 | 629.81 | 12.494880 | 49 |
| 12 | 542906 | 579.36 | 97 | | 542909 | 579.37 | 457091 | 48 |
| 13 | 577668 | 536.41 | 97 | | 577672 | 536.42 | 422328 | 47 |
| 14 | 609853 | 499.38 | 96 | | 609857 | 499.39 | 390143 | 46 |
| 15 | 639816 | 467.14 | 96 | | 639820 | 467.15 | 360180 | 45 |
| 16 | 667845 | 438.81 | 95 | | 667849 | 438.82 | 332151 | 44 |
| 17 | 694173 | 413.72 | 95 | | 694179 | 413.73 | 305821 | 43 |
| 18 | 718997 | 391.35 | 94 | | 719003 | 391.36 | 280997 | 42 |
| 19 | 742477 | 371.27 | 93 | | 742484 | 371.28 | 257516 | 41 |
| 20 | 764754 | 353.15 | 93 | | 764761 | 353.16 | 235239 | 40 |
| 21 | 7.785943 | 336.72 | 9.999992 | | 7.785951 | 336.73 | 12.214049 | 39 |
| 22 | 806146 | 321.75 | 91 | | 806155 | 321.76 | 193845 | 38 |
| 23 | 825451 | 308.05 | 90 | .01 | 825460 | 308.07 | 174540 | 37 |
| 24 | 843934 | 295.47 | 89 | .02 | 843944 | 295.49 | 156056 | 36 |
| 25 | 861662 | 283.88 | 88 | | 861674 | 283.90 | 138326 | 35 |
| 26 | 878695 | 273.17 | 88 | | 878708 | 273.18 | 121292 | 34 |
| 27 | 895085 | 263.23 | 87 | | 895099 | 263.25 | 104901 | 33 |
| 28 | 910879 | 253.99 | 86 | | 910894 | 254.01 | 089106 | 32 |
| 29 | 926119 | 245.38 | 85 | | 926134 | 245.40 | 073866 | 31 |
| 30 | 940842 | 237.33 | 83 | | 940858 | 237.35 | 059142 | 30 |
| 31 | 7.955082 | 229.80 | 9.999982 | | 7.955100 | 229.82 | 12.044900 | 29 |
| 32 | 968870 | 222.73 | 81 | | 968889 | 222.75 | 031111 | 28 |
| 33 | 982233 | 216.08 | 80 | | 982253 | 216.10 | 017747 | 27 |
| 34 | 7.995198 | 209.81 | 79 | | 7.995219 | 209.83 | 12.004781 | 26 |
| 35 | 8.007787 | 203.90 | 77 | | 8.007809 | 203.92 | 11.992191 | 25 |
| 36 | 020021 | 198.31 | 76 | | 020045 | 198.33 | 979955 | 24 |
| 37 | 031919 | 193.02 | 75 | | 031945 | 193.05 | 968055 | 23 |
| 38 | 043501 | 188.01 | 73 | | 043527 | 188.03 | 956473 | 22 |
| 39 | 054781 | 183.25 | 72 | | 054809 | 183.27 | 945191 | 21 |
| 40 | 065776 | 178.72 | 71 | | 065806 | 178.75 | 934194 | 20 |
| 41 | 8.076500 | 174.41 | 9.999969 | | 8.076531 | 174.44 | 11.923469 | 19 |
| 42 | 086965 | 170.31 | 68 | | 086997 | 170.34 | 913003 | 18 |
| 43 | 097183 | 166.39 | 66 | .02 | 097217 | 166.42 | 902783 | 17 |
| 44 | 107167 | 162.65 | 64 | .03 | 107202 | 162.68 | 892798 | 16 |
| 45 | 116926 | 159.08 | 63 | | 116963 | 159.11 | 883037 | 15 |
| 46 | 126471 | 155.66 | 61 | | 126510 | 155.68 | 873490 | 14 |
| 47 | 135810 | 152.38 | 59 | | 135851 | 152.41 | 864149 | 13 |
| 48 | 144953 | 149.24 | 58 | | 144996 | 149.27 | 855004 | 12 |
| 49 | 153907 | 146.22 | 56 | | 153952 | 146.25 | 846048 | 11 |
| 50 | 162681 | 143.33 | 54 | | 162727 | 143.36 | 837273 | 10 |
| 51 | 8.171280 | 140.54 | 9.999952 | | 8.171328 | 140.57 | 11.828672 | 9 |
| 52 | 179713 | 137.86 | 50 | | 179763 | 137.90 | 820237 | 8 |
| 53 | 187985 | 135.29 | 48 | | 188036 | 135.32 | 811964 | 7 |
| 54 | 196102 | 132.80 | 46 | | 196156 | 132.84 | 803844 | 6 |
| 55 | 204070 | 130.41 | 44 | .03 | 204126 | 130.44 | 795874 | 5 |
| 56 | 211895 | 128.10 | 42 | .04 | 211953 | 128.14 | 788047 | 4 |
| 57 | 219581 | 125.87 | 40 | | 219641 | 125.91 | 780359 | 3 |
| 58 | 227134 | 123.72 | 38 | | 227195 | 123.76 | 772805 | 2 |
| 59 | 234557 | 121.64 | 36 | .04 | 234621 | 121.68 | 765379 | 1 |
| 60 | 8.241855 | | 9.999934 | | 8.241921 | | 11.758079 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 89° |

## 1° SINES AND TANGENTS.

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 8.241855 | 119.63 | 9.999934 | .04 | 8.241921 | 119.67 | 11.758079 | 60 |
| 1 | 249033 | 117.68 | 932 | | 249102 | 117.72 | 750898 | 59 |
| 2 | 256094 | 115.80 | 929 | | 256165 | 115.84 | 743835 | 58 |
| 3 | 263042 | 113.98 | 927 | | 263115 | 114.02 | 736885 | 57 |
| 4 | 269881 | 112.21 | 925 | | 269956 | 112.25 | 730044 | 56 |
| 5 | 276614 | 110.50 | 922 | | 276691 | 110.54 | 723309 | 55 |
| 6 | 283243 | 108.83 | 920 | | 283323 | 108.87 | 716677 | 54 |
| 7 | 289773 | 107.22 | 917 | | 289856 | 107.26 | 710144 | 53 |
| 8 | 296207 | 105.65 | 915 | | 296292 | 105.70 | 703708 | 52 |
| 9 | 302546 | 104.13 | 912 | | 302634 | 104.18 | 697366 | 51 |
| 10 | 308794 | 102.66 | 910 | | 308884 | 102.70 | 691116 | 50 |
| 11 | 8.314954 | 101.22 | 9.999907 | | 8.315046 | 101.26 | 11.684954 | 49 |
| 12 | 321027 | 99.82 | 905 | | 321122 | 99.87 | 678878 | 48 |
| 13 | 327016 | 98.47 | 902 | .04 | 327114 | 98.51 | 672886 | 47 |
| 14 | 332924 | 97.14 | 899 | .05 | 333025 | 97.19 | 666975 | 46 |
| 15 | 338753 | 95.86 | 897 | | 338856 | 95.90 | 661144 | 45 |
| 16 | 344504 | 94.60 | 894 | | 344610 | 94.65 | 655390 | 44 |
| 17 | 350181 | 93.38 | 891 | | 350289 | 93.43 | 649711 | 43 |
| 18 | 355783 | 92.19 | 888 | | 355895 | 92.24 | 644105 | 42 |
| 19 | 361315 | 91.03 | 885 | | 361430 | 91.08 | 638570 | 41 |
| 20 | 366777 | 89.90 | 882 | | 366895 | 89.95 | 633105 | 40 |
| 21 | 8.372171 | 88.80 | 9.999879 | | 8.372292 | 88.85 | 11.627708 | 39 |
| 22 | 377499 | 87.72 | 876 | | 377622 | 87.77 | 622378 | 38 |
| 23 | 382762 | 86.67 | 873 | | 382889 | 86.72 | 617111 | 37 |
| 24 | 387962 | 85.64 | 870 | | 388092 | 85.70 | 611908 | 36 |
| 25 | 393101 | 84.64 | 867 | | 393234 | 84.69 | 606766 | 35 |
| 26 | 398179 | 83.66 | 864 | | 398315 | 83.71 | 601685 | 34 |
| 27 | 403199 | 82.71 | 861 | | 403338 | 82.76 | 596662 | 33 |
| 28 | 408161 | 81.77 | 858 | | 408304 | 81.82 | 591696 | 32 |
| 29 | 413068 | 80.86 | 854 | .05 | 413213 | 80.91 | 586787 | 31 |
| 30 | 417919 | 79.96 | 851 | .06 | 418068 | 80.02 | 581932 | 30 |
| 31 | 8.422717 | 79.09 | 9.999848 | | 8.422869 | 79.14 | 11.577131 | 29 |
| 32 | 427462 | 78.23 | 844 | | 427618 | 78.29 | 572382 | 28 |
| 33 | 432156 | 77.40 | 841 | | 432315 | 77.45 | 567685 | 27 |
| 34 | 436800 | 76.57 | 838 | | 436962 | 76.63 | 563038 | 26 |
| 35 | 441394 | 75.77 | 834 | | 441560 | 75.83 | 558440 | 25 |
| 36 | 445941 | 74.99 | 831 | | 446110 | 75.05 | 553890 | 24 |
| 37 | 450440 | 74.22 | 827 | | 450613 | 74.28 | 549387 | 23 |
| 38 | 454893 | 73.46 | 824 | | 455070 | 73.52 | 544930 | 22 |
| 39 | 459301 | 72.73 | 820 | | 459481 | 72.79 | 540519 | 21 |
| 40 | 463665 | 72.00 | 816 | | 463849 | 72.06 | 536151 | 20 |
| 41 | 8.467985 | 71.29 | 9.999813 | | 8.468173 | 71.35 | 11.531827 | 19 |
| 42 | 472263 | 70.60 | 809 | | 472454 | 70.66 | 527546 | 18 |
| 43 | 476498 | 69.91 | 805 | | 476693 | 69.98 | 523307 | 17 |
| 44 | 480693 | 69.24 | 801 | .06 | 480892 | 69.31 | 519108 | 16 |
| 45 | 484848 | 68.59 | 797 | .07 | 485050 | 68.65 | 514950 | 15 |
| 46 | 488963 | 67.94 | 794 | | 489170 | 68.01 | 510830 | 14 |
| 47 | 493040 | 67.31 | 790 | | 493250 | 67.38 | 506750 | 13 |
| 48 | 497078 | 66.69 | 786 | | 497293 | 66.76 | 502707 | 12 |
| 49 | 501080 | 66.08 | 782 | | 501298 | 66.15 | 498702 | 11 |
| 50 | 505045 | 65.48 | 778 | | 505267 | 65.55 | 494733 | 10 |
| 51 | 8.508974 | 64.89 | 9.999774 | | 8.509200 | 64.96 | 11.490800 | 9 |
| 52 | 512867 | 64.32 | 769 | | 513098 | 64.39 | 486902 | 8 |
| 53 | 516726 | 63.75 | 765 | | 516961 | 63.82 | 483039 | 7 |
| 54 | 520551 | 63.19 | 761 | | 520790 | 63.26 | 479210 | 6 |
| 55 | 524343 | 62.64 | 757 | | 524586 | 62.72 | 475414 | 5 |
| 56 | 528102 | 62.11 | 753 | | 528349 | 62.18 | 471651 | 4 |
| 57 | 531828 | 61.58 | 748 | | 532080 | 61.65 | 467920 | 3 |
| 58 | 535523 | 61.06 | 744 | | 535779 | 61.13 | 464221 | 2 |
| 59 | 539186 | 60.55 | 740 | .07 | 539447 | 60.62 | 460553 | 1 |
| 60 | 8.542819 | | 9.999735 | | 8.543084 | | 11.456916 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 88° |

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 8.542819 | 60.04 | 9.999735 | .07 | 8.543084 | 60.12 | 11.456916 | 60 |
| 1 | 46422 | 59.55 | 731 | .07 | 46691 | 59.62 | 53309 | 59 |
| 2 | 49995 | 59.06 | 726 | .07 | 50268 | 59.14 | 49732 | 58 |
| 3 | 53539 | 58.58 | 722 | .08 | 53817 | 58.66 | 46183 | 57 |
| 4 | 57054 | 58.11 | 717 | | 57336 | 58.19 | 42664 | 56 |
| 5 | 60540 | 57.65 | 713 | | 60828 | 57.73 | 39172 | 55 |
| 6 | 63999 | 57.19 | 708 | | 64291 | 57.27 | 35709 | 54 |
| 7 | 67431 | 56.74 | 704 | | 67727 | 56.82 | 32273 | 53 |
| 8 | 70836 | 56.30 | 699 | | 71137 | 56.38 | 28863 | 52 |
| 9 | 74214 | 55.87 | 694 | | 74520 | 55.95 | 25480 | 51 |
| 10 | 77566 | 55.44 | 689 | | 77877 | 55.52 | 22123 | 50 |
| 11 | 8.580892 | 55.02 | 9.999685 | | 8.581208 | 55.10 | 11.418792 | 49 |
| 12 | 84193 | 54.60 | 680 | | 84514 | 54.68 | 15486 | 48 |
| 13 | 87469 | 54.19 | 675 | | 87795 | 54.27 | 12205 | 47 |
| 14 | 90721 | 53.79 | 670 | | 91051 | 53.87 | 08949 | 46 |
| 15 | 93948 | 53.39 | 665 | | 94283 | 53.47 | 05717 | 45 |
| 16 | 8.597152 | 53.00 | 660 | | 8.597492 | 53.08 | 11.402508 | 44 |
| 17 | 8.600332 | 52.61 | 655 | | 8.600677 | 52.70 | 11.399323 | 43 |
| 18 | 03489 | 52.23 | 650 | .08 | 03839 | 52.32 | 96161 | 42 |
| 19 | 06623 | 51.86 | 645 | .09 | 06978 | 51.94 | 93022 | 41 |
| 20 | 09734 | 51.49 | 640 | | 10094 | 51.58 | 89906 | 40 |
| 21 | 8.612823 | 51.12 | 9.999635 | | 8.613189 | 51.21 | 11.386811 | 39 |
| 22 | 15891 | 50.76 | 629 | | 16262 | 50.85 | 83738 | 38 |
| 23 | 18937 | 50.41 | 624 | | 19313 | 50.50 | 80687 | 37 |
| 24 | 21962 | 50.06 | 619 | | 22343 | 50.15 | 77657 | 36 |
| 25 | 24965 | 49.72 | 614 | | 25352 | 49.81 | 74648 | 35 |
| 26 | 27948 | 49.38 | 608 | | 28340 | 49.47 | 71660 | 34 |
| 27 | 30911 | 49.04 | 603 | | 31308 | 49.13 | 68692 | 33 |
| 28 | 33854 | 48.71 | 597 | | 34256 | 48.80 | 65744 | 32 |
| 29 | 36776 | 48.39 | 592 | | 37184 | 48.48 | 62816 | 31 |
| 30 | 39680 | 48.06 | 586 | | 40093 | 48.16 | 59907 | 30 |
| 31 | 8.642563 | 47.75 | 9.999581 | | 8.642983 | 47.84 | 11.357017 | 29 |
| 32 | 45428 | 47.43 | 575 | | 45853 | 47.53 | 54147 | 28 |
| 33 | 48274 | 47.12 | 570 | | 48704 | 47.22 | 51296 | 27 |
| 34 | 51102 | 46.82 | 564 | .09 | 51537 | 46.91 | 48463 | 26 |
| 35 | 53911 | 46.52 | 558 | .10 | 54352 | 46.61 | 45648 | 25 |
| 36 | 56702 | 46.22 | 553 | | 57149 | 46.31 | 42851 | 24 |
| 37 | 59475 | 45.92 | 547 | | 59928 | 46.02 | 40072 | 23 |
| 38 | 62230 | 45.63 | 541 | | 62689 | 45.73 | 37311 | 22 |
| 39 | 64968 | 45.35 | 535 | | 65433 | 45.44 | 34567 | 21 |
| 40 | 67689 | 45.06 | 529 | | 68160 | 45.16 | 31840 | 20 |
| 41 | 8.670393 | 44.79 | 9.999524 | | 8.670870 | 44.88 | 11.329130 | 19 |
| 42 | 73080 | 44.51 | 518 | | 73563 | 44.61 | 26437 | 18 |
| 43 | 75751 | 44.24 | 512 | | 76239 | 44.34 | 23761 | 17 |
| 44 | 78405 | 43.97 | 506 | | 78900 | 44.07 | 21100 | 16 |
| 45 | 81043 | 43.70 | 500 | | 81544 | 43.80 | 18456 | 15 |
| 46 | 83665 | 43.44 | 493 | | 84172 | 43.54 | 15828 | 14 |
| 47 | 86272 | 43.18 | 487 | | 86784 | 43.28 | 13216 | 13 |
| 48 | 88863 | 42.92 | 481 | | 89381 | 43.03 | 10619 | 12 |
| 49 | 91438 | 42.67 | 475 | | 91963 | 42.77 | 08037 | 11 |
| 50 | 93998 | 42.42 | 469 | .10 | 94529 | 42.52 | 05471 | 10 |
| 51 | 96543 | 42.17 | 9.999463 | .11 | 97081 | 42.28 | 02919 | 9 |
| 52 | 8.699073 | 41.92 | 456 | | 8.699617 | 42.03 | 11.300383 | 8 |
| 53 | 8.701589 | 41.68 | 450 | | 8.702139 | 41.79 | 11.297861 | 7 |
| 54 | 04090 | 41.44 | 443 | | 04646 | 41.55 | 95354 | 6 |
| 55 | 06577 | 41.21 | 437 | | 07140 | 41.32 | 92860 | 5 |
| 56 | 09049 | 40.97 | 431 | | 09618 | 41.08 | 90382 | 4 |
| 57 | 11507 | 40.74 | 424 | | 12083 | 40.85 | 87917 | 3 |
| 58 | 13952 | 40.51 | 418 | | 14535 | 40.62 | 85465 | 2 |
| 59 | 16383 | 40.29 | 411 | .11 | 16972 | 40.40 | 83028 | 1 |
| 60 | 8.718800 | | 9.999404 | | 8.719396 | | 11.280604 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 87° |

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 8.718800 | 40.06 | 9.999404 | .11 | 8.719396 | 40.17 | 11.280604 | 60 |
| 1 | 21204 | 39.84 | 9398 | | 21806 | 39.95 | 78194 | 59 |
| 2 | 23595 | 39.62 | 9391 | | 24203 | 39.74 | 75797 | 58 |
| 3 | 25972 | 39.41 | 9384 | | 26588 | 39.52 | 73412 | 57 |
| 4 | 28337 | 39.19 | 9378 | | 28959 | 39.31 | 71041 | 56 |
| 5 | 30688 | 38.98 | 9371 | .11 | 31317 | 39.09 | 68683 | 55 |
| 6 | 33027 | 38.77 | 9364 | .12 | 33663 | 38.89 | 66337 | 54 |
| 7 | 35354 | 38.57 | 9357 | | 35996 | 38.68 | 64004 | 53 |
| 8 | 37667 | 38.36 | 9350 | | 38317 | 38.48 | 61683 | 52 |
| 9 | 39969 | 38.16 | 9343 | | 40626 | 38.27 | 59374 | 51 |
| 10 | 42259 | 37.96 | 9336 | | 42922 | 38.07 | 57078 | 50 |
| 11 | 8.744536 | 37.76 | 9.999329 | | 8.745207 | 37.87 | 11.254793 | 49 |
| 12 | 46802 | 37.56 | 9322 | | 47479 | 37.68 | 52521 | 48 |
| 13 | 49055 | 37.37 | 9315 | | 49740 | 37.49 | 50260 | 47 |
| 14 | 51297 | 37.17 | 9308 | | 51989 | 37.29 | 48011 | 46 |
| 15 | 53528 | 36.98 | 9301 | | 54227 | 37.10 | 45773 | 45 |
| 16 | 55747 | 36.79 | 9294 | | 56453 | 36.92 | 43547 | 44 |
| 17 | 57955 | 36.61 | 9286 | | 58668 | 36.73 | 41332 | 43 |
| 18 | 60151 | 36.42 | 9279 | | 60872 | 36.55 | 39128 | 42 |
| 19 | 62337 | 36.24 | 9272 | | 63065 | 36.36 | 36935 | 41 |
| 20 | 64511 | 36.06 | 9265 | | 65246 | 36.18 | 34754 | 40 |
| 21 | 8.766675 | 35.88 | 9.999257 | .12 | 8.767417 | 36.00 | 11.232583 | 39 |
| 22 | 68828 | 35.70 | 9250 | .13 | 69578 | 35.83 | 30422 | 38 |
| 23 | 70970 | 35.53 | 9242 | | 71727 | 35.65 | 28273 | 37 |
| 24 | 73101 | 35.35 | 9235 | | 73866 | 35.48 | 26134 | 36 |
| 25 | 75223 | 35.18 | 9227 | | 75995 | 35.31 | 24005 | 35 |
| 26 | 77333 | 35.01 | 9220 | | 78114 | 35.14 | 21886 | 34 |
| 27 | 79434 | 34.84 | 9212 | | 80222 | 34.97 | 19778 | 33 |
| 28 | 81524 | 34.67 | 9205 | | 82320 | 34.80 | 17680 | 32 |
| 29 | 83605 | 34.51 | 9197 | | 84408 | 34.64 | 15592 | 31 |
| 30 | 85675 | 34.34 | 9189 | | 86486 | 34.47 | 13514 | 30 |
| 31 | 8.787736 | 34.18 | 9.999181 | | 8.788554 | 34.31 | 11.211446 | 29 |
| 32 | 89787 | 34.02 | 9174 | | 90613 | 34.15 | 09387 | 28 |
| 33 | 91828 | 33.86 | 9166 | | 92662 | 33.99 | 07338 | 27 |
| 34 | 93859 | 33.70 | 9158 | | 94701 | 33.83 | 05299 | 26 |
| 35 | 95881 | 33.54 | 9150 | | 96731 | 33.68 | 03269 | 25 |
| 36 | 97894 | 33.39 | 9142 | | 8.798752 | 33.52 | 11.201248 | 24 |
| 37 | 8.799897 | 33.23 | 9134 | | 8.800763 | 33.37 | 11.199237 | 23 |
| 38 | 8.801892 | 33.08 | 9126 | | 02765 | 33.22 | 97235 | 22 |
| 39 | 03876 | 32.93 | 9118 | | 04758 | 33.07 | 95242 | 21 |
| 40 | 05852 | 32.78 | 9110 | | 06742 | 32.92 | 93258 | 20 |
| 41 | 8.807819 | 32.63 | 9.999102 | .13 | 8.808717 | 32.77 | 11.191283 | 19 |
| 42 | 09777 | 32.49 | 9094 | .14 | 10683 | 32.62 | 89317 | 18 |
| 43 | 11726 | 32.34 | 9086 | | 12641 | 32.48 | 87359 | 17 |
| 44 | 13667 | 32.19 | 9077 | | 14589 | 32.33 | 85411 | 16 |
| 45 | 15599 | 32.05 | 9069 | | 16529 | 32.19 | 83471 | 15 |
| 46 | 17522 | 31.91 | 9061 | | 18461 | 32.05 | 81539 | 14 |
| 47 | 19436 | 31.77 | 9053 | | 20384 | 31.91 | 79616 | 13 |
| 48 | 21343 | 31.63 | 9044 | | 22298 | 31.77 | 77702 | 12 |
| 49 | 23240 | 31.49 | 9036 | | 24205 | 31.63 | 75795 | 11 |
| 50 | 25130 | 31.35 | 9027 | | 26103 | 31.50 | 73897 | 10 |
| 51 | 8.827011 | 31.22 | 9.999019 | | 8.827992 | 31.36 | 11.172008 | 9 |
| 52 | 28884 | 31.08 | 9010 | | 29874 | 31.23 | 70126 | 8 |
| 53 | 30749 | 30.95 | 9002 | | 31748 | 31.09 | 68252 | 7 |
| 54 | 32607 | 30.82 | 8993 | | 33613 | 30.96 | 66387 | 6 |
| 55 | 34456 | 30.69 | 8984 | | 35471 | 30.83 | 64529 | 5 |
| 56 | 36297 | 30.56 | 8976 | .14 | 37321 | 30.70 | 62679 | 4 |
| 57 | 38130 | 30.43 | 8967 | .15 | 39163 | 30.57 | 60837 | 3 |
| 58 | 39956 | 30.30 | 8958 | .15 | 40998 | 30.45 | 59002 | 2 |
| 59 | 41774 | 30.17 | 8950 | .15 | 42825 | 30.32 | 57175 | 1 |
| 60 | 8.843585 | | 9.998941 | | 8.844644 | | 11.155356 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 86° |

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 8.843585 | 30.05 | 9.998941 | .15 | 8.844644 | 30.19 | 11.155356 | 60 |
| 1 | 45387 | 29.92 | 932 | | 46455 | 30.07 | 53545 | 59 |
| 2 | 47183 | 29.80 | 923 | | 48260 | 29.95 | 51740 | 58 |
| 3 | 48971 | 29.67 | 914 | | 50057 | 29.82 | 49943 | 57 |
| 4 | 50751 | 29.55 | 905 | | 51846 | 29.70 | 48154 | 56 |
| 5 | 52525 | 29.43 | 896 | | 53628 | 29.58 | 46372 | 55 |
| 6 | 54291 | 29.31 | 887 | | 55403 | 29.46 | 44597 | 54 |
| 7 | 56049 | 29.19 | 878 | | 57171 | 29.35 | 42829 | 53 |
| 8 | 57801 | 29.08 | 869 | | 58932 | 29.23 | 41068 | 52 |
| 9 | 59546 | 28.96 | 860 | | 60686 | 29.11 | 39314 | 51 |
| 10 | 61283 | 28.84 | 851 | | 62433 | 29.00 | 37567 | 50 |
| 11 | 8.863014 | 28.73 | 9.998841 | | 8.864173 | 28.88 | 11.135827 | 49 |
| 12 | 64738 | 28.61 | 832 | .15 | 65906 | 28.77 | 34094 | 48 |
| 13 | 66455 | 28.50 | 823 | .16 | 67632 | 28.66 | 32368 | 47 |
| 14 | 68165 | 28.39 | 813 | | 69351 | 28.54 | 30649 | 46 |
| 15 | 69868 | 28.28 | 804 | | 71064 | 28.43 | 28936 | 45 |
| 16 | 71565 | 28.17 | 795 | | 72770 | 28.32 | 27230 | 44 |
| 17 | 73255 | 28.06 | 785 | | 74469 | 28.21 | 25531 | 43 |
| 18 | 74938 | 27.95 | 776 | | 76162 | 28.11 | 23838 | 42 |
| 19 | 76615 | 27.84 | 766 | | 77849 | 28.00 | 22151 | 41 |
| 20 | 78285 | 27.73 | 757 | | 79529 | 27.89 | 20471 | 40 |
| 21 | 8.879949 | 27.63 | 9.998747 | | 8.881202 | 27.79 | 11.118798 | 39 |
| 22 | 81607 | 27.52 | 738 | | 82869 | 27.68 | 17131 | 38 |
| 23 | 83258 | 27.42 | 728 | | 84530 | 27.58 | 15470 | 37 |
| 24 | 84903 | 27.31 | 718 | | 86185 | 27.47 | 13815 | 36 |
| 25 | 86542 | 27.21 | 708 | | 87833 | 27.37 | 12167 | 35 |
| 26 | 88174 | 27.11 | 699 | | 89476 | 27.27 | 10524 | 34 |
| 27 | 89801 | 27.00 | 689 | | 91112 | 27.17 | 08888 | 33 |
| 28 | 91421 | 26.90 | 679 | .16 | 92742 | 27.07 | 07258 | 32 |
| 29 | 93035 | 26.80 | 669 | .17 | 94366 | 26.97 | 05634 | 31 |
| 30 | 94643 | 26.70 | 659 | | 95984 | 26.87 | 04016 | 30 |
| 31 | 96245 | 26.60 | 9.998649 | | 97596 | 26.77 | 02404 | 29 |
| 32 | 97842 | 26.51 | 639 | | 8.899203 | 26.67 | 11.100797 | 28 |
| 33 | 8.899432 | 26.41 | 629 | | 8.900803 | 26.58 | 11.099197 | 27 |
| 34 | 8.901017 | 26.31 | 619 | | 02398 | 26.48 | 97602 | 26 |
| 35 | 02596 | 26.22 | 609 | | 03987 | 26.38 | 96013 | 25 |
| 36 | 04169 | 26.12 | 599 | | 05570 | 26.29 | 94430 | 24 |
| 37 | 05736 | 26.03 | 589 | | 07147 | 26.20 | 92853 | 23 |
| 38 | 07297 | 25.93 | 578 | | 08719 | 26.10 | 91281 | 22 |
| 39 | 08853 | 25.84 | 568 | | 10285 | 26.01 | 89715 | 21 |
| 40 | 10404 | 25.75 | 558 | | 11846 | 25.92 | 88154 | 20 |
| 41 | 8.911949 | 25.66 | 9.998548 | | 8.913401 | 25.83 | 11.086599 | 19 |
| 42 | 13488 | 25.56 | 537 | | 14951 | 25.74 | 85049 | 18 |
| 43 | 15022 | 25.47 | 527 | .17 | 16495 | 25.65 | 83505 | 17 |
| 44 | 16550 | 25.38 | 516 | .18 | 18034 | 25.56 | 81966 | 16 |
| 45 | 18073 | 25.29 | 506 | | 19568 | 25.47 | 80432 | 15 |
| 46 | 19591 | 25.20 | 495 | | 21096 | 25.38 | 78904 | 14 |
| 47 | 21103 | 25.12 | 485 | | 22619 | 25.30 | 77381 | 13 |
| 48 | 22610 | 25.03 | 474 | | 24136 | 25.21 | 75864 | 12 |
| 49 | 24112 | 24.94 | 464 | | 25649 | 25.12 | 74351 | 11 |
| 50 | 25609 | 24.86 | 453 | | 27156 | 25.03 | 72844 | 10 |
| 51 | 8.927100 | 24.77 | 9.998442 | | 8.928658 | 24.95 | 11.071342 | 9 |
| 52 | 28587 | 24.69 | 431 | | 30155 | 24.86 | 69845 | 8 |
| 53 | 30068 | 24.60 | 421 | | 31647 | 24.78 | 68353 | 7 |
| 54 | 31544 | 24.52 | 410 | | 33134 | 24.70 | 66866 | 6 |
| 55 | 33015 | 24.43 | 399 | | 34616 | 24.61 | 65384 | 5 |
| 56 | 34481 | 24.35 | 388 | | 36093 | 24.53 | 63907 | 4 |
| 57 | 35942 | 24.27 | 377 | | 37565 | 24.45 | 62435 | 3 |
| 58 | 37398 | 24.19 | 366 | | 39032 | 24.37 | 60968 | 2 |
| 59 | 38850 | 24.11 | 355 | .18 | 40494 | 24.29 | 59506 | 1 |
| 60 | 8.940296 | | 9.998344 | | 8.941952 | | 11.058048 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 85° |

## 5° SINES AND TANGENTS.

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 8.940296 | 24.03 | 9.998344 | .19 | 8.941952 | 24.21 | 11.058048 | 60 |
| 1 | 41738 | 23.94 | 333 | | 43404 | 24.13 | 56596 | 59 |
| 2 | 43174 | 23.87 | 322 | | 44852 | 24.05 | 55148 | 58 |
| 3 | 44606 | 23.79 | 311 | | 46295 | 23.97 | 53705 | 57 |
| 4 | 46034 | 23.71 | 300 | | 47734 | 23.90 | 52266 | 56 |
| 5 | 47456 | 23.63 | 289 | | 49168 | 23.82 | 50832 | 55 |
| 6 | 48874 | 23.55 | 277 | | 50597 | 23.74 | 49403 | 54 |
| 7 | 50287 | 23.48 | 266 | | 52021 | 23.66 | 47979 | 53 |
| 8 | 51696 | 23.40 | 255 | | 53441 | 23.59 | 46559 | 52 |
| 9 | 53100 | 23.32 | 243 | | 54856 | 23.51 | 45144 | 51 |
| 10 | 54499 | 23.25 | 232 | | 56267 | 23.44 | 43733 | 50 |
| 11 | 8.955894 | 23.17 | 9.998220 | | 8.957674 | 23.37 | 11.042326 | 49 |
| 12 | 57284 | 23.10 | 209 | | 59075 | 23.29 | 40925 | 48 |
| 13 | 58670 | 23.02 | 197 | | 60473 | 23.22 | 39527 | 47 |
| 14 | 60052 | 22.95 | 186 | | 61866 | 23.14 | 38134 | 46 |
| 15 | 61429 | 22.88 | 174 | | 63255 | 23.07 | 36745 | 45 |
| 16 | 62801 | 22.80 | 163 | | 64639 | 23.00 | 35361 | 44 |
| 17 | 64170 | 22.73 | 151 | .19 | 66019 | 22.93 | 33981 | 43 |
| 18 | 65534 | 22.66 | 139 | .20 | 67394 | 22.86 | 32606 | 42 |
| 19 | 66893 | 22.59 | 128 | | 68766 | 22.79 | 31234 | 41 |
| 20 | 68249 | 22.52 | 116 | | 70133 | 22.71 | 29867 | 40 |
| 21 | 8.969600 | 22.45 | 9.998104 | | 8.971496 | 22.65 | 11.028504 | 39 |
| 22 | 70947 | 22.38 | 092 | | 72855 | 22.57 | 27145 | 38 |
| 23 | 72289 | 22.31 | 080 | | 74209 | 22.51 | 25791 | 37 |
| 24 | 73628 | 22.24 | 068 | | 75560 | 22.44 | 24440 | 36 |
| 25 | 74962 | 22.17 | 056 | | 76906 | 22.37 | 23094 | 35 |
| 26 | 76293 | 22.10 | 044 | | 78248 | 22.30 | 21752 | 34 |
| 27 | 77619 | 22.03 | 032 | | 79586 | 22.23 | 20414 | 33 |
| 28 | 78941 | 21.97 | 020 | | 80921 | 22.17 | 19079 | 32 |
| 29 | 80259 | 21.90 | 9.998008 | | 82251 | 22.10 | 17749 | 31 |
| 30 | 81573 | 21.83 | 9.997996 | | 83577 | 22.04 | 16423 | 30 |
| 31 | 8.982883 | 21.77 | 984 | | 8.984899 | 21.97 | 11.015101 | 29 |
| 32 | 84189 | 21.70 | 972 | | 86217 | 21.91 | 13783 | 28 |
| 33 | 85491 | 21.63 | 959 | | 87532 | 21.84 | 12468 | 27 |
| 34 | 86789 | 21.57 | 947 | .20 | 88842 | 21.78 | 11158 | 26 |
| 35 | 88083 | 21.50 | 935 | .21 | 90149 | 21.71 | 09851 | 25 |
| 36 | 89374 | 21.44 | 922 | | 91451 | 21.65 | 08549 | 24 |
| 37 | 90660 | 21.38 | 910 | | 92750 | 21.58 | 07250 | 23 |
| 38 | 91943 | 21.31 | 897 | | 94045 | 21.52 | 05955 | 22 |
| 39 | 93222 | 21.25 | 885 | | 95337 | 21.46 | 04663 | 21 |
| 40 | 94497 | 21.19 | 872 | | 96624 | 21.40 | 03376 | 20 |
| 41 | 8.995768 | 21.12 | 9.997860 | | 97908 | 21.34 | 02092 | 19 |
| 42 | 97036 | 21.06 | 847 | | 8.999188 | 21.27 | 11.000812 | 18 |
| 43 | 98299 | 21.00 | 835 | | 9.000465 | 21.21 | 10.999535 | 17 |
| 44 | 8.999560 | 20.94 | 822 | | 01738 | 21.15 | 98262 | 16 |
| 45 | 9.000816 | 20.88 | 809 | | 03007 | 21.09 | 96993 | 15 |
| 46 | 02069 | 20.82 | 797 | | 04272 | 21.03 | 95728 | 14 |
| 47 | 03318 | 20.76 | 784 | | 05534 | 20.97 | 94466 | 13 |
| 48 | 04563 | 20.70 | 771 | | 06792 | 20.91 | 93208 | 12 |
| 49 | 05805 | 20.64 | 758 | | 08047 | 20.85 | 91953 | 11 |
| 50 | 07044 | 20.58 | 745 | | 09298 | 20.80 | 90702 | 10 |
| 51 | 9.008278 | 20.52 | 9.997732 | | 9.010546 | 20.74 | 10.989454 | 9 |
| 52 | 09510 | 20.46 | 719 | | 11790 | 20.68 | 88210 | 8 |
| 53 | 10737 | 20.40 | 706 | .21 | 13031 | 20.62 | 86969 | 7 |
| 54 | 11962 | 20.34 | 693 | .22 | 14268 | 20.56 | 85732 | 6 |
| 55 | 13182 | 20.29 | 680 | | 15502 | 20.51 | 84498 | 5 |
| 56 | 14400 | 20.23 | 667 | | 16732 | 20.45 | 83268 | 4 |
| 57 | 15613 | 20.17 | 654 | | 17959 | 20.40 | 82041 | 3 |
| 58 | 16824 | 20.12 | 641 | | 19183 | 20.34 | 80817 | 2 |
| 59 | 18031 | 20.06 | 628 | .22 | 20403 | 20.28 | 79597 | 1 |
| 60 | 9.019235 | | 9.997614 | | 9.021620 | | 10.978380 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 84° |

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.019235 | 20.00 | 9.997614 | .22 | 9.021620 | 20.23 | 10.978380 | 60 |
| 1 | 20435 | 19.95 | 601 | | 22834 | 20.17 | 77166 | 59 |
| 2 | 21632 | 19.89 | 588 | | 24044 | 20.11 | 75956 | 58 |
| 3 | 22825 | 19.84 | 574 | | 25251 | 20.06 | 74749 | 57 |
| 4 | 24016 | 19.78 | 561 | | 26455 | 20.01 | 73545 | 56 |
| 5 | 25203 | 19.73 | 547 | .22 | 27655 | 19.95 | 72345 | 55 |
| 6 | 26386 | 19.67 | 534 | .23 | 28852 | 19.90 | 71148 | 54 |
| 7 | 27567 | 19.62 | 520 | | 30046 | 19.85 | 69954 | 53 |
| 8 | 28744 | 19.57 | 507 | | 31237 | 19.79 | 68763 | 52 |
| 9 | 29918 | 19.51 | 493 | | 32425 | 19.74 | 67575 | 51 |
| 10 | 31089 | 19.46 | 480 | | 33609 | 19.69 | 66391 | 50 |
| 11 | 9.032257 | 19.41 | 9.997466 | | 9.034791 | 19.64 | 10.965209 | 49 |
| 12 | 33421 | 19.36 | 452 | | 35969 | 19.58 | 64031 | 48 |
| 13 | 34582 | 19.30 | 439 | | 37144 | 19.53 | 62856 | 47 |
| 14 | 35741 | 19.25 | 425 | | 38316 | 19.48 | 61684 | 46 |
| 15 | 36896 | 19.20 | 411 | | 39485 | 19.43 | 60515 | 45 |
| 16 | 38048 | 19.15 | 397 | | 40651 | 19.38 | 59349 | 44 |
| 17 | 39197 | 19.10 | 383 | | 41813 | 19.33 | 58187 | 43 |
| 18 | 40342 | 19.05 | 369 | | 42973 | 19.28 | 57027 | 42 |
| 19 | 41485 | 18.99 | 355 | | 44130 | 19.23 | 55870 | 41 |
| 20 | 42625 | 18.95 | 341 | .23 | 45284 | 19.18 | 54716 | 40 |
| 21 | 9.043762 | 18.89 | 9.997327 | .24 | 9.046434 | 19.13 | 10.953566 | 39 |
| 22 | 44895 | 18.84 | 313 | | 47582 | 19.08 | 52418 | 38 |
| 23 | 46026 | 18.79 | 299 | | 48727 | 19.03 | 51273 | 37 |
| 24 | 47154 | 18.75 | 285 | | 49869 | 18.98 | 50131 | 36 |
| 25 | 48279 | 18.70 | 271 | | 51008 | 18.93 | 48992 | 35 |
| 26 | 49400 | 18.65 | 257 | | 52144 | 18.89 | 47856 | 34 |
| 27 | 50519 | 18.60 | 242 | | 53277 | 18.84 | 46723 | 33 |
| 28 | 51635 | 18.55 | 228 | | 54407 | 18.79 | 45593 | 32 |
| 29 | 52749 | 18.50 | 214 | | 55535 | 18.74 | 44465 | 31 |
| 30 | 53859 | 18.45 | 199 | | 56659 | 18.70 | 43341 | 30 |
| 31 | 9.054966 | 18.41 | 9.997185 | | 9.057781 | 18.65 | 10.942219 | 29 |
| 32 | 56071 | 18.36 | 170 | | 58900 | 18.60 | 41100 | 28 |
| 33 | 57172 | 18.31 | 156 | | 60016 | 18.55 | 39984 | 27 |
| 34 | 58271 | 18.27 | 141 | | 61130 | 18.51 | 38870 | 26 |
| 35 | 59367 | 18.22 | 127 | | 62240 | 18.46 | 37760 | 25 |
| 36 | 60460 | 18.17 | 112 | | 63348 | 18.42 | 36652 | 24 |
| 37 | 61551 | 18.13 | 098 | .24 | 64453 | 18.37 | 35547 | 23 |
| 38 | 62639 | 18.08 | 083 | .25 | 65556 | 18.33 | 34444 | 22 |
| 39 | 63724 | 18.04 | 068 | | 66655 | 18.28 | 33345 | 21 |
| 40 | 64806 | 17.99 | 053 | | 67752 | 18.24 | 32248 | 20 |
| 41 | 9.065885 | 17.94 | 9.997039 | | 9.068846 | 18.19 | 10.931154 | 19 |
| 42 | 66962 | 17.90 | 024 | | 69938 | 18.15 | 30062 | 18 |
| 43 | 68036 | 17.86 | 9.997009 | | 71027 | 18.10 | 28973 | 17 |
| 44 | 69107 | 17.81 | 9.996994 | | 72113 | 18.06 | 27887 | 16 |
| 45 | 70176 | 17.77 | 979 | | 73197 | 18.02 | 26803 | 15 |
| 46 | 71242 | 17.72 | 964 | | 74278 | 17.97 | 25722 | 14 |
| 47 | 72305 | 17.68 | 949 | | 75356 | 17.93 | 24644 | 13 |
| 48 | 73366 | 17.63 | 934 | | 76432 | 17.89 | 23568 | 12 |
| 49 | 74424 | 17.59 | 919 | | 77505 | 17.84 | 22495 | 11 |
| 50 | 75480 | 17.55 | 904 | | 78576 | 17.80 | 21424 | 10 |
| 51 | 9.076533 | 17.50 | 9.996889 | | 9.079644 | 17.76 | 10.920356 | 9 |
| 52 | 77583 | 17.46 | 874 | | 80710 | 17.72 | 19290 | 8 |
| 53 | 78631 | 17.42 | 858 | | 81773 | 17.67 | 18227 | 7 |
| 54 | 79676 | 17.38 | 843 | | 82833 | 17.63 | 17167 | 6 |
| 55 | 80719 | 17.33 | 828 | .25 | 83891 | 17.59 | 16109 | 5 |
| 56 | 81759 | 17.29 | 812 | .26 | 84947 | 17.55 | 15053 | 4 |
| 57 | 82797 | 17.25 | 797 | | 86000 | 17.51 | 14000 | 3 |
| 58 | 83832 | 17.21 | 782 | | 87050 | 17.47 | 12950 | 2 |
| 59 | 84864 | 17.17 | 766 | .26 | 88098 | 17.43 | 11902 | 1 |
| 60 | 9.085894 | | 9.996751 | | 9.089144 | | 10.910856 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 83° |

## 7° SINES AND TANGENTS.

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.085894 | 17.13 | 9.996751 | .26 | 9.089144 | 17.38 | 10.910856 | 60 |
| 1 | 86922 | 17.09 | 735 | | 90187 | 17.35 | 09813 | 59 |
| 2 | 87947 | 17.04 | 720 | | 91228 | 17.30 | 08772 | 58 |
| 3 | 88970 | 17.00 | 704 | | 92266 | 17.27 | 07734 | 57 |
| 4 | 89990 | 16.96 | 688 | | 93302 | 17.22 | 06698 | 56 |
| 5 | 91008 | 16.92 | 673 | | 94335 | 17.19 | 05664 | 55 |
| 6 | 92024 | 16.88 | 657 | | 95367 | 17.15 | 04633 | 54 |
| 7 | 93037 | 16.84 | 641 | | 96395 | 17.11 | 03605 | 53 |
| 8 | 94047 | 16.80 | 625 | | 97422 | 17.07 | 02578 | 52 |
| 9 | 95056 | 16.76 | 610 | | 98446 | 17.03 | 01554 | 51 |
| 10 | 96062 | 16.73 | 594 | .26 | 99468 | 16.99 | 10.900532 | 50 |
| 11 | 97065 | 16.68 | 9.996578 | .27 | 9.100487 | 16.95 | 10.899513 | 49 |
| 12 | 98066 | 16.65 | 562 | | 01504 | 16.91 | 98496 | 48 |
| 13 | 9.099065 | 16.61 | 546 | | 02519 | 16.87 | 97481 | 47 |
| 14 | 9.100062 | 16.57 | 530 | | 03532 | 16.84 | 96468 | 46 |
| 15 | 01056 | 16.53 | 514 | | 04542 | 16.80 | 95458 | 45 |
| 16 | 02048 | 16.49 | 498 | | 05550 | 16.76 | 94450 | 44 |
| 17 | 03037 | 16.45 | 482 | | 06556 | 16.72 | 93444 | 43 |
| 18 | 04025 | 16.42 | 465 | | 07559 | 16.69 | 92441 | 42 |
| 19 | 05010 | 16.38 | 449 | | 08560 | 16.65 | 91440 | 41 |
| 20 | 05992 | 16.34 | 433 | | 09559 | 16.61 | 90441 | 40 |
| 21 | 9.106973 | 16.30 | 9.996417 | | 9.110556 | 16.58 | 10.889444 | 39 |
| 22 | 07951 | 16.27 | 400 | | 11551 | 16.54 | 88449 | 38 |
| 23 | 08927 | 16.23 | 384 | | 12543 | 16.50 | 87457 | 37 |
| 24 | 09901 | 16.19 | 368 | | 13533 | 16.47 | 86467 | 36 |
| 25 | 10873 | 16.16 | 351 | | 14521 | 16.43 | 85479 | 35 |
| 26 | 11842 | 16.12 | 335 | | 15507 | 16.39 | 84493 | 34 |
| 27 | 12809 | 16.08 | 318 | .27 | 16491 | 16.36 | 83509 | 33 |
| 28 | 13774 | 16.05 | 302 | .28 | 17472 | 16.32 | 82528 | 32 |
| 29 | 14737 | 16.01 | 285 | | 18452 | 16.29 | 81548 | 31 |
| 30 | 15698 | 15.97 | 269 | | 19429 | 16.25 | 80571 | 30 |
| 31 | 9.116656 | 15.94 | 9.996252 | | 9.120404 | 16.22 | 10.879596 | 29 |
| 32 | 17613 | 15.90 | 235 | | 21377 | 16.18 | 78623 | 28 |
| 33 | 18567 | 15.87 | 219 | | 22348 | 16.15 | 77652 | 27 |
| 34 | 19519 | 15.83 | 202 | | 23317 | 16.11 | 76683 | 26 |
| 35 | 20469 | 15.80 | 185 | | 24284 | 16.08 | 75716 | 25 |
| 36 | 21417 | 15.76 | 168 | | 25249 | 16.04 | 74751 | 24 |
| 37 | 22362 | 15.73 | 151 | | 26211 | 16.01 | 73789 | 23 |
| 38 | 23306 | 15.69 | 134 | | 27172 | 15.97 | 72828 | 22 |
| 39 | 24248 | 15.66 | 117 | | 28130 | 15.94 | 71870 | 21 |
| 40 | 25187 | 15.62 | 100 | .28 | 29087 | 15.91 | 70913 | 20 |
| 41 | 9.126125 | 15.59 | 9.996083 | .29 | 9.130041 | 15.87 | 10.869959 | 19 |
| 42 | 27060 | 15.56 | 066 | | 30994 | 15.84 | 69006 | 18 |
| 43 | 27993 | 15.52 | 049 | | 31944 | 15.81 | 68056 | 17 |
| 44 | 28925 | 15.49 | 032 | | 32893 | 15.77 | 67107 | 16 |
| 45 | 29854 | 15.45 | 9.996015 | | 33839 | 15.74 | 66161 | 15 |
| 46 | 30781 | 15.42 | 9.995998 | | 34784 | 15.71 | 65216 | 14 |
| 47 | 31706 | 15.39 | 980 | | 35726 | 15.67 | 64274 | 13 |
| 48 | 32630 | 15.35 | 963 | | 36667 | 15.64 | 63333 | 12 |
| 49 | 33551 | 15.32 | 946 | | 37605 | 15.61 | 62395 | 11 |
| 50 | 34470 | 15.29 | 928 | | 38542 | 15.58 | 61458 | 10 |
| 51 | 9.135387 | 15.25 | 9.995911 | | 9.139476 | 15.55 | 10.860524 | 9 |
| 52 | 36303 | 15.22 | 894 | | 40409 | 15.51 | 59591 | 8 |
| 53 | 37216 | 15.19 | 876 | | 41340 | 15.48 | 58660 | 7 |
| 54 | 38128 | 15.16 | 859 | | 42269 | 15.45 | 57731 | 6 |
| 55 | 39037 | 15.12 | 841 | | 43196 | 15.42 | 56804 | 5 |
| 56 | 39944 | 15.09 | 823 | | 44121 | 15.39 | 55879 | 4 |
| 57 | 40850 | 15.06 | 806 | | 45044 | 15.35 | 54956 | 3 |
| 58 | 41754 | 15.03 | 788 | | 45966 | 15.32 | 54034 | 2 |
| 59 | 42655 | 15.00 | 771 | .29 | 46885 | 15.29 | 53115 | 1 |
| 60 | 9.143555 | | 9.995753 | | 9.147803 | | 10.852197 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 82° |

| 8° | LOGARITHMIC | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
| 0 | 9.143555 | 14.96 | 9.995753 | .30 | 9.147803 | 15.26 | 10.852197 | 60 |
| 1 | 4453 | 14.93 | 735 | | 8718 | 15.23 | 1282 | 59 |
| 2 | 5349 | 14.90 | 717 | | 9.149632 | 15.20 | 10.850368 | 58 |
| 3 | 6243 | 14.87 | 699 | | 9.150544 | 15.17 | 10.849456 | 57 |
| 4 | 7136 | 14.84 | 681 | | 1454 | 15.14 | 8546 | 56 |
| 5 | 8026 | 14.81 | 664 | | 2363 | 15.11 | 7637 | 55 |
| 6 | 8915 | 14.78 | 646 | | 3269 | 15.08 | 6731 | 54 |
| 7 | 9.149802 | 14.75 | 628 | | 4174 | 15.05 | 5826 | 53 |
| 8 | 9.150686 | 14.72 | 610 | | 5077 | 15.02 | 4923 | 52 |
| 9 | 1569 | 14.69 | 591 | | 5978 | 14.99 | 4022 | 51 |
| 10 | 2451 | 14.66 | 573 | | 6877 | 14.96 | 3123 | 50 |
| 11 | 3330 | 14.63 | 9.995555 | | 7775 | 14.93 | 2225 | 49 |
| 12 | 4208 | 14.60 | 537 | | 8671 | 14.90 | 1329 | 48 |
| 13 | 5083 | 14.57 | 519 | .30 | 9.159565 | 14.87 | 10.840435 | 47 |
| 14 | 5957 | 14.54 | 501 | .31 | 9.160457 | 14.84 | 10.839543 | 46 |
| 15 | 6830 | 14.51 | 482 | | 1347 | 14.81 | 8653 | 45 |
| 16 | 7700 | 14.48 | 464 | | 2236 | 14.78 | 7764 | 44 |
| 17 | 8569 | 14.45 | 446 | | 3123 | 14.75 | 6877 | 43 |
| 18 | 9.159435 | 14.42 | 427 | | 4008 | 14.73 | 5992 | 42 |
| 19 | 9.160301 | 14.39 | 409 | | 4892 | 14.70 | 5108 | 41 |
| 20 | 1164 | 14.36 | 390 | | 5774 | 14.67 | 4226 | 40 |
| 21 | 2025 | 14.33 | 9.995372 | | 6654 | 14.64 | 3346 | 39 |
| 22 | 2885 | 14.30 | 353 | | 7532 | 14.61 | 2468 | 38 |
| 23 | 3743 | 14.27 | 334 | | 8409 | 14.58 | 1591 | 37 |
| 24 | 4600 | 14.24 | 316 | | 9.169284 | 14.55 | 10.830716 | 36 |
| 25 | 5454 | 14.22 | 297 | | 9.170157 | 14.53 | 10.829843 | 35 |
| 26 | 6307 | 14.19 | 278 | | 1029 | 14.50 | 8971 | 34 |
| 27 | 7159 | 14.16 | 260 | .31 | 1899 | 14.47 | 8101 | 33 |
| 28 | 8008 | 14.13 | 241 | .32 | 2767 | 14.44 | 7233 | 32 |
| 29 | 8856 | 14.10 | 222 | | 3634 | 14.42 | 6366 | 31 |
| 30 | 9.169702 | 14.07 | 203 | | 4499 | 14.39 | 5501 | 30 |
| 31 | 9.170547 | 14.05 | 9.995184 | | 5362 | 14.36 | 4638 | 29 |
| 32 | 1389 | 14.02 | 165 | | 6224 | 14.33 | 3776 | 28 |
| 33 | 2230 | 13.99 | 146 | | 7084 | 14.31 | 2916 | 27 |
| 34 | 3070 | 13.96 | 127 | | 7942 | 14.28 | 2058 | 26 |
| 35 | 3908 | 13.94 | 108 | | 8799 | 14.25 | 1201 | 25 |
| 36 | 4744 | 13.91 | 089 | | 9.179655 | 14.23 | 10.820345 | 24 |
| 37 | 5578 | 13.88 | 070 | | 9.180508 | 14.20 | 10.819492 | 23 |
| 38 | 6411 | 13.86 | 051 | | 1360 | 14.17 | 8640 | 22 |
| 39 | 7242 | 13.83 | 032 | | 2211 | 14.15 | 7789 | 21 |
| 40 | 8072 | 13.80 | 9.995013 | | 3059 | 14.12 | 6941 | 20 |
| 41 | 8900 | 13.77 | 9.994993 | | 3907 | 14.09 | 6093 | 19 |
| 42 | 9.179726 | 13.74 | 974 | | 4752 | 14.07 | 5248 | 18 |
| 43 | 9.180551 | 13.72 | 955 | | 5597 | 14.04 | 4403 | 17 |
| 44 | 1374 | 13.69 | 935 | .32 | 6439 | 14.02 | 3561 | 16 |
| 45 | 2196 | 13.67 | 916 | .33 | 7280 | 13.99 | 2720 | 15 |
| 46 | 3016 | 13.64 | 896 | | 8120 | 13.96 | 1880 | 14 |
| 47 | 3834 | 13.61 | 877 | | 8958 | 13.94 | 1042 | 13 |
| 48 | 4651 | 13.59 | 857 | | 9.189794 | 13.91 | 10.810206 | 12 |
| 49 | 5466 | 13.56 | 838 | | 9.190629 | 13.89 | 10.809371 | 11 |
| 50 | 6280 | 13.53 | 818 | | 1462 | 13.86 | 8538 | 10 |
| 51 | 7092 | 13.51 | 9.994798 | | 2294 | 13.84 | 7706 | 9 |
| 52 | 7903 | 13.48 | 779 | | 3124 | 13.81 | 6876 | 8 |
| 53 | 8712 | 13.46 | 759 | | 3953 | 13.79 | 6047 | 7 |
| 54 | 9.189519 | 13.43 | 739 | | 4780 | 13.76 | 5220 | 6 |
| 55 | 9.190325 | 13.41 | 719 | | 5606 | 13.74 | 4394 | 5 |
| 56 | 1130 | 13.38 | 700 | | 6430 | 13.71 | 3570 | 4 |
| 57 | 1933 | 13.36 | 680 | | 7253 | 13.69 | 2747 | 3 |
| 58 | 2734 | 13.33 | 660 | | 8074 | 13.66 | 1926 | 2 |
| 59 | 3534 | 13.30 | 640 | .33 | 8894 | 13.64 | 1106 | 1 |
| 60 | 9.194332 | | 9.994620 | | 9.199713 | | 10.800287 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 81° |

## 9° SINES AND TANGENTS.

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.194332 | 13.28 | 9.994620 | .33 | 9.199713 | 13.61 | 10.800287 | 60 |
| 1 | 5129 | 13.26 | 600 | .33 | 9.200529 | 13.59 | 10.799471 | 59 |
| 2 | 5925 | 13.23 | 580 | .33 | 1345 | 13.56 | 8655 | 58 |
| 3 | 6719 | 13.21 | 560 | .34 | 2159 | 13.54 | 7841 | 57 |
| 4 | 7511 | 13.18 | 540 | | 2971 | 13.52 | 7029 | 56 |
| 5 | 8302 | 13.16 | 519 | | 3782 | 13.49 | 6218 | 55 |
| 6 | 9091 | 13.13 | 499 | | 4592 | 13.47 | 5408 | 54 |
| 7 | 9.199879 | 13.11 | 479 | | 5400 | 13.45 | 4600 | 53 |
| 8 | 9.200666 | 13.08 | 459 | | 6207 | 13.42 | 3793 | 52 |
| 9 | 1451 | 13.06 | 438 | | 7013 | 13.40 | 2987 | 51 |
| 10 | 2234 | 13.04 | 418 | | 7817 | 13.38 | 2183 | 50 |
| 11 | 3017 | 13.01 | 9.994397 | | 8619 | 13.35 | 1381 | 49 |
| 12 | 3797 | 12.99 | 377 | | 9.209420 | 13.33 | 10.790580 | 48 |
| 13 | 4577 | 12.96 | 357 | | 9.210220 | 13.31 | 10.789780 | 47 |
| 14 | 5354 | 12.94 | 336 | | 1018 | 13.28 | 8982 | 46 |
| 15 | 6131 | 12.92 | 316 | | 1815 | 13.26 | 8185 | 45 |
| 16 | 6906 | 12.89 | 295 | .34 | 2611 | 13.24 | 7389 | 44 |
| 17 | 7679 | 12.87 | 274 | .35 | 3405 | 13.21 | 6595 | 43 |
| 18 | 8452 | 12.85 | 254 | | 4198 | 13.19 | 5802 | 42 |
| 19 | 9222 | 12.82 | 233 | | 4989 | 13.17 | 5011 | 41 |
| 20 | 9.209992 | 12.80 | 212 | | 5780 | 13.15 | 4220 | 40 |
| 21 | 9.210760 | 12.78 | 9.994191 | | 6568 | 13.12 | 3432 | 39 |
| 22 | 1526 | 12.75 | 171 | | 7356 | 13.10 | 2644 | 38 |
| 23 | 2291 | 12.73 | 150 | | 8142 | 13.08 | 1858 | 37 |
| 24 | 3055 | 12.71 | 129 | | 8926 | 13.05 | 1074 | 36 |
| 25 | 3818 | 12.68 | 108 | | 9.219710 | 13.03 | 10.780290 | 35 |
| 26 | 4579 | 12.66 | 087 | | 9.220492 | 13.01 | 10.779508 | 34 |
| 27 | 5338 | 12.64 | 066 | | 1272 | 12.99 | 8728 | 33 |
| 28 | 6097 | 12.61 | 045 | | 2052 | 12.97 | 7948 | 32 |
| 29 | 6854 | 12.59 | 024 | | 2830 | 12.94 | 7170 | 31 |
| 30 | 7609 | 12.57 | 9.994003 | | 3606 | 12.92 | 6394 | 30 |
| 31 | 8363 | 12.55 | 9.993981 | | 4382 | 12.90 | 5618 | 29 |
| 32 | 9116 | 12.53 | 960 | | 5156 | 12.88 | 4844 | 28 |
| 33 | 9.219868 | 12.50 | 939 | | 5929 | 12.86 | 4071 | 27 |
| 34 | 9.220618 | 12.48 | 918 | .35 | 6700 | 12.84 | 3300 | 26 |
| 35 | 1367 | 12.46 | 896 | .36 | 7471 | 12.81 | 2529 | 25 |
| 36 | 2115 | 12.44 | 875 | | 8239 | 12.79 | 1761 | 24 |
| 37 | 2861 | 12.42 | 854 | | 9007 | 12.77 | 0993 | 23 |
| 38 | 3606 | 12.39 | 832 | | 9.229773 | 12.75 | 10.770227 | 22 |
| 39 | 4349 | 12.37 | 811 | | 9.230539 | 12.73 | 10.769461 | 21 |
| 40 | 5092 | 12.35 | 789 | | 1302 | 12.71 | 8698 | 20 |
| 41 | 5833 | 12.33 | 9.993768 | | 2065 | 12.69 | 7935 | 19 |
| 42 | 6573 | 12.31 | 746 | | 2826 | 12.67 | 7174 | 18 |
| 43 | 7311 | 12.28 | 725 | | 3586 | 12.65 | 6414 | 17 |
| 44 | 8048 | 12.26 | 703 | | 4345 | 12.62 | 5655 | 16 |
| 45 | 8784 | 12.24 | 681 | | 5103 | 12.60 | 4897 | 15 |
| 46 | 9.229518 | 12.22 | 660 | | 5859 | 12.58 | 4141 | 14 |
| 47 | 9.230252 | 12.20 | 638 | | 6614 | 12.56 | 3386 | 13 |
| 48 | 0984 | 12.18 | 616 | .36 | 7368 | 12.54 | 2632 | 12 |
| 49 | 1714 | 12.16 | 594 | .37 | 8120 | 12.52 | 1880 | 11 |
| 50 | 2444 | 12.14 | 572 | | 8872 | 12.50 | 1128 | 10 |
| 51 | 3172 | 12.12 | 9.993550 | | 9.239622 | 12.48 | 10.760378 | 9 |
| 52 | 3899 | 12.09 | 528 | | 9.240371 | 12.46 | 10.759629 | 8 |
| 53 | 4625 | 12.07 | 506 | | 1118 | 12.44 | 8882 | 7 |
| 54 | 5349 | 12.05 | 484 | | 1865 | 12.42 | 8135 | 6 |
| 55 | 6073 | 12.03 | 462 | | 2610 | 12.40 | 7390 | 5 |
| 56 | 6795 | 12.01 | 440 | | 3354 | 12.38 | 6646 | 4 |
| 57 | 7515 | 11.99 | 418 | | 4097 | 12.36 | 5903 | 3 |
| 58 | 8235 | 11.97 | 396 | | 4839 | 12.34 | 5161 | 2 |
| 59 | 8953 | 11.95 | 374 | .37 | 5579 | 12.32 | 4421 | 1 |
| 60 | 9.239670 | | 9.993351 | | 9.246319 | | 10.753681 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 80° |

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.239670 | 11.93 | 9.993351 | .37 | 9.246319 | 12.30 | 10.753681 | 60 |
| 1 | 9.240386 | 11.91 | 329 | | 7057 | 12.28 | 2943 | 59 |
| 2 | 1101 | 11.89 | 307 | | 7794 | 12.26 | 2206 | 58 |
| 3 | 1814 | 11.87 | 285 | | 8530 | 12.24 | 1470 | 57 |
| 4 | 2526 | 11.85 | 262 | | 9264 | 12.22 | 0736 | 56 |
| 5 | 3237 | 11.83 | 240 | .37 | 9.249998 | 12.20 | 10.750002 | 55 |
| 6 | 3947 | 11.81 | 217 | .38 | 9.250730 | 12.18 | 10.749270 | 54 |
| 7 | 4656 | 11.79 | 195 | | 1461 | 12.17 | 8539 | 53 |
| 8 | 5363 | 11.77 | 172 | | 2191 | 12.15 | 7809 | 52 |
| 9 | 6069 | 11.75 | 149 | | 2920 | 12.13 | 7080 | 51 |
| 10 | 6775 | 11.73 | 127 | | 3648 | 12.11 | 6352 | 50 |
| 11 | 7478 | 11.71 | 9.993104 | | 4374 | 12.09 | 5626 | 49 |
| 12 | 8181 | 11.69 | 081 | | 5100 | 12.07 | 4900 | 48 |
| 13 | 8883 | 11.67 | 059 | | 5824 | 12.05 | 4176 | 47 |
| 14 | 9.249583 | 11.65 | 036 | | 6547 | 12.03 | 3453 | 46 |
| 15 | 9.250282 | 11.63 | 9.993013 | | 7269 | 12.01 | 2731 | 45 |
| 16 | 0980 | 11.61 | 9.992990 | | 7990 | 12.00 | 2010 | 44 |
| 17 | 1677 | 11.59 | 967 | | 8710 | 11.98 | 1290 | 43 |
| 18 | 2373 | 11.58 | 944 | | 9.259429 | 11.96 | 10.740571 | 42 |
| 19 | 3067 | 11.56 | 921 | | 9.260146 | 11.94 | 10.739854 | 41 |
| 20 | 3761 | 11.54 | 898 | | 0863 | 11.92 | 9137 | 40 |
| 21 | 4453 | 11.52 | 9.992875 | | 1578 | 11.90 | 8422 | 39 |
| 22 | 5144 | 11.50 | 852 | .38 | 2292 | 11.89 | 7708 | 38 |
| 23 | 5834 | 11.48 | 829 | .39 | 3005 | 11.87 | 6995 | 37 |
| 24 | 6523 | 11.46 | 806 | | 3717 | 11.85 | 6283 | 36 |
| 25 | 7211 | 11.44 | 783 | | 4428 | 11.83 | 5572 | 35 |
| 26 | 7898 | 11.42 | 759 | | 5138 | 11.81 | 4862 | 34 |
| 27 | 8583 | 11.41 | 736 | | 5847 | 11.79 | 4153 | 33 |
| 28 | 9268 | 11.39 | 713 | | 6555 | 11.78 | 3445 | 32 |
| 29 | 9.259951 | 11.37 | 690 | | 7261 | 11.76 | 2739 | 31 |
| 30 | 9.260633 | 11.35 | 666 | | 7967 | 11.74 | 2033 | 30 |
| 31 | 1314 | 11.33 | 9.992643 | | 8671 | 11.72 | 1329 | 29 |
| 32 | 1994 | 11.31 | 619 | | 9.269375 | 11.70 | 10.730625 | 28 |
| 33 | 2673 | 11.30 | 596 | | 9.270077 | 11.69 | 10.729923 | 27 |
| 34 | 3351 | 11.28 | 572 | | 0779 | 11.67 | 9221 | 26 |
| 35 | 4027 | 11.26 | 549 | | 1479 | 11.65 | 8521 | 25 |
| 36 | 4703 | 11.24 | 525 | | 2178 | 11.64 | 7822 | 24 |
| 37 | 5377 | 11.22 | 501 | .39 | 2876 | 11.62 | 7124 | 23 |
| 38 | 6051 | 11.20 | 478 | .40 | 3573 | 11.60 | 6427 | 22 |
| 39 | 6723 | 11.19 | 454 | | 4269 | 11.58 | 5731 | 21 |
| 40 | 7395 | 11.17 | 430 | | 4964 | 11.57 | 5036 | 20 |
| 41 | 8065 | 11.15 | 9.992406 | | 5658 | 11.55 | 4342 | 19 |
| 42 | 8734 | 11.13 | 382 | | 6351 | 11.53 | 3649 | 18 |
| 43 | 9.269402 | 11.12 | 359 | | 7043 | 11.51 | 2957 | 17 |
| 44 | 9.270069 | 11.10 | 335 | | 7734 | 11.50 | 2266 | 16 |
| 45 | 0735 | 11.08 | 311 | | 8424 | 11.48 | 1576 | 15 |
| 46 | 1400 | 11.06 | 287 | | 9113 | 11.46 | 0887 | 14 |
| 47 | 2064 | 11.05 | 263 | | 9.279801 | 11.45 | 10.720199 | 13 |
| 48 | 2726 | 11.03 | 239 | | 9.280488 | 11.43 | 10.719512 | 12 |
| 49 | 3388 | 11.01 | 214 | | 1174 | 11.41 | 8826 | 11 |
| 50 | 4049 | 10.99 | 190 | | 1858 | 11.40 | 8142 | 10 |
| 51 | 4708 | 10.98 | 9.992166 | | 2542 | 11.38 | 7458 | 9 |
| 52 | 5367 | 10.96 | 142 | .40 | 3225 | 11.36 | 6775 | 8 |
| 53 | 6024 | 10.94 | 117 | .41 | 3907 | 11.35 | 6093 | 7 |
| 54 | 6681 | 10.92 | 093 | | 4588 | 11.33 | 5412 | 6 |
| 55 | 7337 | 10.91 | 069 | | 5268 | 11.31 | 4732 | 5 |
| 56 | 7991 | 10.89 | 044 | | 5947 | 11.30 | 4053 | 4 |
| 57 | 8644 | 10.87 | 9.992020 | | 6624 | 11.28 | 3376 | 3 |
| 58 | 9297 | 10.86 | 9.991996 | | 7301 | 11.26 | 2699 | 2 |
| 59 | 9.279948 | 10.84 | 971 | .41 | 7977 | 11.25 | 2023 | 1 |
| 60 | 9.280599 | | 9.991947 | | 9.288652 | | 10.711348 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 79° |

## 11° SINES AND TANGENTS.

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.280599 | 10.82 | 9.991947 | .41 | 9.288652 | 11.23 | 10.711348 | 60 |
| 1 | 1248 | 10.81 | 922 | | 9326 | 11.22 | 0674 | 59 |
| 2 | 1897 | 10.79 | 897 | | 9.289999 | 11.20 | 10.710001 | 58 |
| 3 | 2544 | 10.77 | 873 | | 9.290671 | 11.18 | 10.709329 | 57 |
| 4 | 3190 | 10.76 | 848 | | 1342 | 11.17 | 8658 | 56 |
| 5 | 3836 | 10.74 | 823 | | 2013 | 11.15 | 7987 | 55 |
| 6 | 4480 | 10.72 | 799 | .41 | 2682 | 11.14 | 7318 | 54 |
| 7 | 5124 | 10.71 | 774 | .42 | 3350 | 11.12 | 6650 | 53 |
| 8 | 5766 | 10.69 | 749 | | 4017 | 11.11 | 5983 | 52 |
| 9 | 6408 | 10.67 | 724 | | 4684 | 11.09 | 5316 | 51 |
| 10 | 7048 | 10.66 | 699 | | 5349 | 11.07 | 4651 | 50 |
| 11 | 7687 | 10.64 | 9.991674 | | 9.296013 | 11.06 | 10.703987 | 49 |
| 12 | 8326 | 10.63 | 649 | | 6677 | 11.04 | 3323 | 48 |
| 13 | 8964 | 10.61 | 624 | | 7339 | 11.03 | 2661 | 47 |
| 14 | 9.289600 | 10.59 | 599 | | 8001 | 11.01 | 1999 | 46 |
| 15 | 9.290236 | 10.58 | 574 | | 8662 | 11.00 | 1338 | 45 |
| 16 | 0870 | 10.56 | 549 | | 9322 | 10.98 | 0678 | 44 |
| 17 | 1504 | 10.54 | 524 | | 9.299980 | 10.96 | 10.700020 | 43 |
| 18 | 2137 | 10.53 | 498 | | 9.300638 | 10.95 | 10.699362 | 42 |
| 19 | 2768 | 10.51 | 473 | | 1295 | 10.93 | 8705 | 41 |
| 20 | 3399 | 10.50 | 448 | | 1951 | 10.92 | 8049 | 40 |
| 21 | 9.294029 | 10.48 | 9.991422 | | 2607 | 10.90 | 7393 | 39 |
| 22 | 4658 | 10.46 | 397 | .42 | 3261 | 10.89 | 6739 | 38 |
| 23 | 5286 | 10.45 | 372 | .43 | 3914 | 10.87 | 6086 | 37 |
| 24 | 5913 | 10.43 | 346 | | 4567 | 10.86 | 5433 | 36 |
| 25 | 6539 | 10.42 | 321 | | 5218 | 10.84 | 4782 | 35 |
| 26 | 7164 | 10.40 | 295 | | 5869 | 10.83 | 4131 | 34 |
| 27 | 7788 | 10.39 | 270 | | 6519 | 10.81 | 3481 | 33 |
| 28 | 8412 | 10.37 | 244 | | 7168 | 10.80 | 2832 | 32 |
| 29 | 9034 | 10.36 | 218 | | 7816 | 10.78 | 2184 | 31 |
| 30 | 9.299655 | 10.34 | 193 | | 8463 | 10.77 | 1537 | 30 |
| 31 | 9.300276 | 10.32 | 9.991167 | | 9109 | 10.75 | 0891 | 29 |
| 32 | 0895 | 10.31 | 141 | | 9.309754 | 10.74 | 10.690246 | 28 |
| 33 | 1514 | 10.29 | 115 | | 9.310398 | 10.73 | 10.689602 | 27 |
| 34 | 2132 | 10.28 | 090 | | 1042 | 10.71 | 8958 | 26 |
| 35 | 2748 | 10.26 | 064 | | 1685 | 10.70 | 8315 | 25 |
| 36 | 3364 | 10.25 | 038 | | 2327 | 10.68 | 7673 | 24 |
| 37 | 3979 | 10.23 | 9.991012 | | 2967 | 10.67 | 7033 | 23 |
| 38 | 4593 | 10.22 | 9.990986 | | 3608 | 10.65 | 6392 | 22 |
| 39 | 5207 | 10.20 | 960 | .43 | 4247 | 10.64 | 5753 | 21 |
| 40 | 5819 | 10.19 | 934 | .44 | 4885 | 10.62 | 5115 | 20 |
| 41 | 6430 | 10.17 | 9.990908 | | 9.315523 | 10.61 | 10.684477 | 19 |
| 42 | 7041 | 10.16 | 882 | | 6159 | 10.60 | 3841 | 18 |
| 43 | 7650 | 10.14 | 855 | | 6795 | 10.58 | 3205 | 17 |
| 44 | 8259 | 10.13 | 829 | | 7430 | 10.57 | 2570 | 16 |
| 45 | 8867 | 10.11 | 803 | | 8064 | 10.55 | 1936 | 15 |
| 46 | 9.309474 | 10.10 | 777 | | 8697 | 10.54 | 1303 | 14 |
| 47 | 9.310080 | 10.08 | 750 | | 9330 | 10.53 | 0670 | 13 |
| 48 | 0685 | 10.07 | 724 | | 9.319961 | 10.51 | 10.680039 | 12 |
| 49 | 1289 | 10.06 | 697 | | 9.320592 | 10.50 | 10.679408 | 11 |
| 50 | 1893 | 10.04 | 671 | | 1222 | 10.48 | 8778 | 10 |
| 51 | 2495 | 10.03 | 9.990644 | | 1851 | 10.47 | 8149 | 9 |
| 52 | 3097 | 10.01 | 618 | | 2479 | 10.45 | 7521 | 8 |
| 53 | 3698 | 10.00 | 591 | | 3106 | 10.44 | 6894 | 7 |
| 54 | 4297 | 9.98 | 565 | | 3733 | 10.43 | 6267 | 6 |
| 55 | 4897 | 9.97 | 538 | .44 | 4358 | 10.41 | 5642 | 5 |
| 56 | 5495 | 9.96 | 511 | .45 | 4983 | 10.40 | 5017 | 4 |
| 57 | 6092 | 9.94 | 485 | | 5607 | 10.39 | 4393 | 3 |
| 58 | 6689 | 9.93 | 458 | | 6231 | 10.37 | 3769 | 2 |
| 59 | 7284 | 9.91 | 431 | .45 | 6853 | 10.36 | 3147 | 1 |
| 60 | 9.317879 | | 9.990404 | | 9.327475 | | 10.672525 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 78° |

## 12° LOGARITHMIC

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.317879 | 9.90 | 9.990404 | .45 | 9.327475 | 10.35 | 10.672525 | 60 |
| 1 | 8473 | 9.88 | 378 | | 8095 | 10.33 | 1905 | 59 |
| 2 | 9066 | 9.87 | 351 | | 8715 | 10.32 | 1285 | 58 |
| 3 | 9.319658 | 9.86 | 324 | | 9334 | 10.30 | 0666 | 57 |
| 4 | 9.320249 | 9.84 | 297 | | 9.329953 | 10.29 | 10.670047 | 56 |
| 5 | 0840 | 9.83 | 270 | | 9.330570 | 10.28 | 10.669430 | 55 |
| 6 | 1430 | 9.82 | 243 | | 1187 | 10.26 | 8813 | 54 |
| 7 | 2019 | 9.80 | 215 | | 1803 | 10.25 | 8197 | 53 |
| 8 | 2607 | 9.79 | 188 | | 2418 | 10.24 | 7582 | 52 |
| 9 | 3194 | 9.77 | 161 | | 3033 | 10.23 | 6967 | 51 |
| 10 | 3780 | 9.76 | 134 | .45 | 3646 | 10.21 | 6354 | 50 |
| 11 | 9.324366 | 9.75 | 9.990107 | .46 | 9.334259 | 10.20 | 10.665741 | 49 |
| 12 | 4950 | 9.73 | 079 | | 4871 | 10.19 | 5129 | 48 |
| 13 | 5534 | 9.72 | 052 | | 5482 | 10.17 | 4518 | 47 |
| 14 | 6117 | 9.70 | 9.990025 | | 6093 | 10.16 | 3907 | 46 |
| 15 | 6700 | 9.69 | 9.989997 | | 6702 | 10.15 | 3298 | 45 |
| 16 | 7281 | 9.68 | 970 | | 7311 | 10.13 | 2689 | 44 |
| 17 | 7862 | 9.66 | 942 | | 7919 | 10.12 | 2081 | 43 |
| 18 | 8442 | 9.65 | 915 | | 8527 | 10.11 | 1473 | 42 |
| 19 | 9021 | 9.64 | 887 | | 9133 | 10.10 | 0867 | 41 |
| 20 | 9.329599 | 9.62 | 860 | | 9.339739 | 10.08 | 10.660261 | 40 |
| 21 | 9.330176 | 9.61 | 9.989832 | | 9.340344 | 10.07 | 10.659656 | 39 |
| 22 | 0753 | 9.60 | 804 | | 0948 | 10.06 | 9052 | 38 |
| 23 | 1329 | 9.58 | 777 | .46 | 1552 | 10.04 | 8448 | 37 |
| 24 | 1903 | 9.57 | 749 | .47 | 2155 | 10.03 | 7845 | 36 |
| 25 | 2478 | 9.56 | 721 | | 2757 | 10.02 | 7243 | 35 |
| 26 | 3051 | 9.54 | 693 | | 3358 | 10.01 | 6642 | 34 |
| 27 | 3624 | 9.53 | 665 | | 3958 | 9.99 | 6042 | 33 |
| 28 | 4195 | 9.52 | 637 | | 4558 | 9.98 | 5442 | 32 |
| 29 | 4767 | 9.50 | 609 | | 5157 | 9.97 | 4843 | 31 |
| 30 | 5337 | 9.49 | 582 | | 5755 | 9.96 | 4245 | 30 |
| 31 | 9.335906 | 9.48 | 9.989553 | | 9.346353 | 9.94 | 10.653647 | 29 |
| 32 | 6475 | 9.46 | 525 | | 6949 | 9.93 | 3051 | 28 |
| 33 | 7043 | 9.45 | 497 | | 7545 | 9.92 | 2455 | 27 |
| 34 | 7610 | 9.44 | 469 | | 8141 | 9.91 | 1859 | 26 |
| 35 | 8176 | 9.43 | 441 | | 8735 | 9.90 | 1265 | 25 |
| 36 | 8742 | 9.41 | 413 | | 9329 | 9.88 | 0671 | 24 |
| 37 | 9307 | 9.40 | 384 | | 9.349922 | 9.87 | 10.650078 | 23 |
| 38 | 9.339871 | 9.39 | 356 | | 9.350514 | 9.86 | 10.649486 | 22 |
| 39 | 9.340434 | 9.37 | 328 | | 1106 | 9.85 | 8894 | 21 |
| 40 | 0996 | 9.36 | 300 | | 1697 | 9.83 | 8303 | 20 |
| 41 | 1558 | 9.35 | 9.989271 | | 2287 | 9.82 | 7713 | 19 |
| 42 | 2119 | 9.34 | 243 | | 2876 | 9.81 | 7124 | 18 |
| 43 | 2679 | 9.32 | 214 | | 3465 | 9.80 | 6535 | 17 |
| 44 | 3239 | 9.31 | 186 | | 4053 | 9.79 | 5947 | 16 |
| 45 | 3797 | 9.30 | 157 | .47 | 4640 | 9.77 | 5360 | 15 |
| 46 | 4355 | 9.29 | 128 | .48 | 5227 | 9.76 | 4773 | 14 |
| 47 | 4912 | 9.27 | 100 | | 5813 | 9.75 | 4187 | 13 |
| 48 | 5469 | 9.26 | 071 | | 6398 | 9.74 | 3602 | 12 |
| 49 | 6024 | 9.25 | 042 | | 6982 | 9.73 | 3018 | 11 |
| 50 | 6579 | 9.24 | 9.989014 | | 7566 | 9.71 | 2434 | 10 |
| 51 | 9.347134 | 9.22 | 9.988985 | | 9.358149 | 9.70 | 1851 | 9 |
| 52 | 7687 | 9.21 | 956 | | 8731 | 9.69 | 1269 | 8 |
| 53 | 8240 | 9.20 | 927 | | 9313 | 9.68 | 0687 | 7 |
| 54 | 8792 | 9.19 | 898 | | 9.359893 | 9.67 | 10.640107 | 6 |
| 55 | 9343 | 9.17 | 869 | | 9.360474 | 9.66 | 10.639526 | 5 |
| 56 | 9.349893 | 9.16 | 840 | .48 | 1053 | 9.65 | 8947 | 4 |
| 57 | 9.350443 | 9.15 | 811 | .49 | 1632 | 9.63 | 8368 | 3 |
| 58 | 0992 | 9.14 | 782 | .49 | 2210 | 9.62 | 7790 | 2 |
| 59 | 1540 | 9.13 | 753 | .49 | 2787 | 9.61 | 7213 | 1 |
| 60 | 9.352088 | | 9.988724 | | 9.363364 | | 10.636636 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 77° |

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.352088 | 9.11 | 9.988724 | .49 | 9.363364 | 9.60 | 10.636636 | 60 |
| 1 | 2635 | 9.10 | 8695 | | 3940 | 9.59 | 6060 | 59 |
| 2 | 3181 | 9.09 | 8666 | | 4515 | 9.58 | 5485 | 58 |
| 3 | 3726 | 9.08 | 8636 | | 5090 | 9.57 | 4910 | 57 |
| 4 | 4271 | 9.07 | 8607 | | 5664 | 9.55 | 4336 | 56 |
| 5 | 4815 | 9.05 | 8578 | | 6237 | 9.54 | 3763 | 55 |
| 6 | 5358 | 9.04 | 8548 | | 6810 | 9.53 | 3190 | 54 |
| 7 | 5901 | 9.03 | 8519 | | 7382 | 9.52 | 2618 | 53 |
| 8 | 6443 | 9.02 | 8489 | | 7953 | 9.51 | 2047 | 52 |
| 9 | 6984 | 9.01 | 8460 | | 8524 | 9.50 | 1476 | 51 |
| 10 | 7524 | 8.99 | 8430 | | 9094 | 9.49 | 0906 | 50 |
| 11 | 8064 | 8.98 | 9.988401 | | 9.369663 | 9.48 | 10.630337 | 49 |
| 12 | 8603 | 8.97 | 8371 | | 9.370232 | 9.46 | 10.629768 | 48 |
| 13 | 9141 | 8.96 | 8342 | .49 | 0799 | 9.45 | 9201 | 47 |
| 14 | 9.359678 | 8.95 | 8312 | .50 | 1367 | 9.44 | 8633 | 46 |
| 15 | 9.360215 | 8.93 | 8282 | | 1933 | 9.43 | 8067 | 45 |
| 16 | 0752 | 8.92 | 8252 | | 2499 | 9.42 | 7501 | 44 |
| 17 | 1287 | 8.91 | 8223 | | 3064 | 9.41 | 6936 | 43 |
| 18 | 1822 | 8.90 | 8193 | | 3629 | 9.40 | 6371 | 42 |
| 19 | 2356 | 8.89 | 8163 | | 4193 | 9.39 | 5807 | 41 |
| 20 | 2889 | 8.88 | 8133 | | 4756 | 9.38 | 5244 | 40 |
| 21 | 9.363422 | 8.87 | 9.988103 | | 9.375319 | 9.37 | 10.624681 | 39 |
| 22 | 3954 | 8.85 | 8073 | | 5881 | 9.35 | 4119 | 38 |
| 23 | 4485 | 8.84 | 8043 | | 6442 | 9.34 | 3558 | 37 |
| 24 | 5016 | 8.83 | 8013 | | 7003 | 9.33 | 2997 | 36 |
| 25 | 5546 | 8.82 | 7983 | | 7563 | 9.32 | 2437 | 35 |
| 26 | 6075 | 8.81 | 7953 | | 8122 | 9.31 | 1878 | 34 |
| 27 | 6604 | 8.80 | 7922 | | 8681 | 9.30 | 1319 | 33 |
| 28 | 7131 | 8.79 | 7892 | | 9239 | 9.29 | 0761 | 32 |
| 29 | 7659 | 8.78 | 7862 | .50 | 9797 | 9.28 | 0203 | 31 |
| 30 | 8185 | 8.76 | 7832 | .51 | 9.380354 | 9.27 | 10.619646 | 30 |
| 31 | 8711 | 8.75 | 9.987801 | | 9.380910 | 9.26 | 10.619090 | 29 |
| 32 | 9236 | 8.74 | 7771 | | 1466 | 9.25 | 8534 | 28 |
| 33 | 9.369761 | 8.73 | 7740 | | 2020 | 9.24 | 7980 | 27 |
| 34 | 9.370285 | 8.72 | 7710 | | 2575 | 9.23 | 7425 | 26 |
| 35 | 0808 | 8.71 | 7679 | | 3129 | 9.22 | 6871 | 25 |
| 36 | 1330 | 8.70 | 7649 | | 3682 | 9.21 | 6318 | 24 |
| 37 | 1852 | 8.69 | 7618 | | 4234 | 9.20 | 5766 | 23 |
| 38 | 2373 | 8.67 | 7588 | | 4786 | 9.19 | 5214 | 22 |
| 39 | 2894 | 8.66 | 7557 | | 5337 | 9.18 | 4663 | 21 |
| 40 | 3414 | 8.65 | 7526 | | 5888 | 9.17 | 4112 | 20 |
| 41 | 9.373933 | 8.64 | 9.987496 | | 9.386438 | 9.15 | 10.613562 | 19 |
| 42 | 4452 | 8.63 | 7465 | | 6987 | 9.14 | 3013 | 18 |
| 43 | 4970 | 8.62 | 7434 | .51 | 7536 | 9.13 | 2464 | 17 |
| 44 | 5487 | 8.61 | 7403 | .52 | 8084 | 9.12 | 1916 | 16 |
| 45 | 6003 | 8.60 | 7372 | | 8631 | 9.11 | 1369 | 15 |
| 46 | 6519 | 8.59 | 7341 | | 9178 | 9.10 | 0822 | 14 |
| 47 | 7035 | 8.58 | 7310 | | 9.389724 | 9.09 | 10.610276 | 13 |
| 48 | 7549 | 8.57 | 7279 | | 9.390270 | 9.08 | 10.609730 | 12 |
| 49 | 8063 | 8.56 | 7248 | | 0815 | 9.07 | 9185 | 11 |
| 50 | 8577 | 8.54 | 7217 | | 1360 | 9.06 | 8640 | 10 |
| 51 | 9089 | 8.53 | 9.987186 | | 1903 | 9.05 | 8097 | 9 |
| 52 | 9.379601 | 8.52 | 7155 | | 2447 | 9.04 | 7553 | 8 |
| 53 | 9.380113 | 8.51 | 7124 | | 2989 | 9.03 | 7011 | 7 |
| 54 | 0624 | 8.50 | 7092 | | 3531 | 9.02 | 6469 | 6 |
| 55 | 1134 | 8.49 | 7061 | | 4073 | 9.01 | 5927 | 5 |
| 56 | 1643 | 8.48 | 7030 | | 4614 | 9.00 | 5386 | 4 |
| 57 | 2152 | 8.47 | 6998 | | 5154 | 8.99 | 4846 | 3 |
| 58 | 2661 | 8.46 | 6967 | | 5694 | 8.98 | 4306 | 2 |
| 59 | 3168 | 8.45 | 6936 | .52 | 6233 | 8.97 | 3767 | 1 |
| 60 | 9.383675 | | 9.986904 | | 9.396771 | | 10.603229 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 76° |

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.383675 | 8.44 | 9.986904 | .52 | 9.396771 | 8.96 | 10.603229 | 60 |
| 1 | 4182 | 8.43 | 6873 | .53 | 7309 | 8.96 | 2691 | 59 |
| 2 | 4687 | 8.42 | 6841 | | 7846 | 8.95 | 2154 | 58 |
| 3 | 5192 | 8.41 | 6809 | | 8383 | 8.94 | 1617 | 57 |
| 4 | 5697 | 8.40 | 6778 | | 8919 | 8.93 | 1081 | 56 |
| 5 | 6201 | 8.39 | 6746 | | 9455 | 8.92 | 0545 | 55 |
| 6 | 6704 | 8.38 | 6714 | | 9.399990 | 8.91 | 10.600010 | 54 |
| 7 | 7207 | 8.37 | 6683 | | 9.400524 | 8.90 | 10.599476 | 53 |
| 8 | 7709 | 8.36 | 6651 | | 1058 | 8.89 | 8942 | 52 |
| 9 | 8210 | 8.35 | 6619 | | 1591 | 8.88 | 8409 | 51 |
| 10 | 8711 | 8.34 | 6587 | | 2124 | 8.87 | 7876 | 50 |
| 11 | 9211 | 8.33 | 9.986555 | | 2656 | 8.86 | 7344 | 49 |
| 12 | 9.389711 | 8.32 | 6523 | | 3187 | 8.85 | 6813 | 48 |
| 13 | 9.390210 | 8.31 | 6491 | | 3718 | 8.84 | 6282 | 47 |
| 14 | 0708 | 8.30 | 6459 | | 4249 | 8.83 | 5751 | 46 |
| 15 | 1206 | 8.28 | 6427 | | 4778 | 8.82 | 5222 | 45 |
| 16 | 1703 | 8.27 | 6395 | .53 | 5308 | 8.81 | 4692 | 44 |
| 17 | 2199 | 8.26 | 6363 | .54 | 5836 | 8.80 | 4164 | 43 |
| 18 | 2695 | 8.25 | 6331 | | 6364 | 8.79 | 3636 | 42 |
| 19 | 3191 | 8.24 | 6299 | | 6892 | 8.78 | 3108 | 41 |
| 20 | 3685 | 8.23 | 6266 | | 7419 | 8.77 | 2581 | 40 |
| 21 | 9.394179 | 8.22 | 9.986234 | | 9.407945 | 8.76 | 10.592055 | 39 |
| 22 | 4673 | 8.21 | 6202 | | 8471 | 8.75 | 1529 | 38 |
| 23 | 5166 | 8.20 | 6169 | | 8997 | 8.74 | 1003 | 37 |
| 24 | 5658 | 8.20 | 6137 | | 9.409521 | 8.74 | 10.590479 | 36 |
| 25 | 6150 | 8.18 | 6104 | | 9.410045 | 8.73 | 10.589955 | 35 |
| 26 | 6641 | 8.17 | 6072 | | 0569 | 8.72 | 9431 | 34 |
| 27 | 7132 | 8.17 | 6039 | | 1092 | 8.71 | 8908 | 33 |
| 28 | 7621 | 8.16 | 6007 | | 1615 | 8.70 | 8385 | 32 |
| 29 | 8111 | 8.15 | 5974 | | 2137 | 8.69 | 7863 | 31 |
| 30 | 8600 | 8.14 | 5942 | .54 | 2658 | 8.68 | 7342 | 30 |
| 31 | 9088 | 8.13 | 9.985909 | .55 | 9.413179 | 8.67 | 10.586821 | 29 |
| 32 | 9.399575 | 8.12 | 5876 | | 3699 | 8.66 | 6301 | 28 |
| 33 | 9.400062 | 8.11 | 5843 | | 4219 | 8.65 | 5781 | 27 |
| 34 | 0549 | 8.10 | 5811 | | 4738 | 8.64 | 5262 | 26 |
| 35 | 1035 | 8.09 | 5778 | | 5257 | 8.64 | 4743 | 25 |
| 36 | 1520 | 8.08 | 5745 | | 5775 | 8.63 | 4225 | 24 |
| 37 | 2005 | 8.07 | 5712 | | 6293 | 8.62 | 3707 | 23 |
| 38 | 2489 | 8.06 | 5679 | | 6810 | 8.61 | 3190 | 22 |
| 39 | 2972 | 8.05 | 5646 | | 7326 | 8.60 | 2674 | 21 |
| 40 | 3455 | 8.04 | 5613 | | 7842 | 8.59 | 2158 | 20 |
| 41 | 9.403938 | 8.03 | 9.985580 | | 8358 | 8.58 | 1642 | 19 |
| 42 | 4420 | 8.02 | 5547 | | 8873 | 8.57 | 1127 | 18 |
| 43 | 4901 | 8.01 | 5514 | | 9387 | 8.56 | 0613 | 17 |
| 44 | 5382 | 8.00 | 5480 | | 9.419901 | 8.55 | 10.580099 | 16 |
| 45 | 5862 | 7.99 | 5447 | .55 | 9.420415 | 8.55 | 10.579585 | 15 |
| 46 | 6341 | 7.98 | 5414 | .56 | 0927 | 8.54 | 9073 | 14 |
| 47 | 6820 | 7.97 | 5380 | | 1440 | 8.53 | 8560 | 13 |
| 48 | 7299 | 7.96 | 5347 | | 1952 | 8.52 | 8048 | 12 |
| 49 | 7777 | 7.95 | 5314 | | 2463 | 8.51 | 7537 | 11 |
| 50 | 8254 | 7.94 | 5280 | | 2974 | 8.50 | 7026 | 10 |
| 51 | 8731 | 7.94 | 9.985247 | | 9.423484 | 8.49 | 10.576516 | 9 |
| 52 | 9207 | 7.93 | 5213 | | 3993 | 8.48 | 6007 | 8 |
| 53 | 9.409682 | 7.92 | 5180 | | 4503 | 8.48 | 5497 | 7 |
| 54 | 9.410157 | 7.91 | 5146 | | 5011 | 8.47 | 4989 | 6 |
| 55 | 0632 | 7.90 | 5113 | | 5519 | 8.46 | 4481 | 5 |
| 56 | 1106 | 7.89 | 5079 | | 6027 | 8.45 | 3973 | 4 |
| 57 | 1579 | 7.88 | 5045 | | 6534 | 8.44 | 3466 | 3 |
| 58 | 2052 | 7.87 | 5011 | | 7041 | 8.43 | 2959 | 2 |
| 59 | 2524 | 7.86 | 4978 | .56 | 7547 | 8.43 | 2453 | 1 |
| 60 | 9.412996 | | 9.984944 | | 9.428052 | | 10.571948 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 75° |

## 15° SINES AND TANGENTS.

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.412996 | 7.85 | 9.984944 | .57 | 9.428052 | 8.42 | 10.571948 | 60 |
| 1 | 3467 | 7.84 | 4910 | | 8558 | 8.41 | 1442 | 59 |
| 2 | 3938 | 7.83 | 4876 | | 9062 | 8.40 | 0938 | 58 |
| 3 | 4408 | 7.83 | 4842 | | 9.429566 | 8.39 | 10.570434 | 57 |
| 4 | 4878 | 7.82 | 4808 | | 9.430070 | 8.38 | 10.569930 | 56 |
| 5 | 5347 | 7.81 | 4774 | | 0573 | 8.38 | 9427 | 55 |
| 6 | 5815 | 7.80 | 4740 | | 1075 | 8.37 | 8925 | 54 |
| 7 | 6283 | 7.79 | 4706 | | 1577 | 8.36 | 8423 | 53 |
| 8 | 6751 | 7.78 | 4672 | | 2079 | 8.35 | 7921 | 52 |
| 9 | 7217 | 7.77 | 4637 | | 2580 | 8.34 | 7420 | 51 |
| 10 | 7684 | 7.76 | 4603 | | 3080 | 8.33 | 6920 | 50 |
| 11 | 9.418150 | 7.75 | 9.984569 | | 9.433580 | 8.32 | 10.566420 | 49 |
| 12 | 8615 | 7.74 | 4535 | | 4080 | 8.32 | 5920 | 48 |
| 13 | 9079 | 7.73 | 4500 | | 4579 | 8.31 | 5421 | 47 |
| 14 | 9.419544 | 7.73 | 4466 | .57 | 5078 | 8.30 | 4922 | 46 |
| 15 | 9.420007 | 7.72 | 4432 | .58 | 5576 | 8.29 | 4424 | 45 |
| 16 | 0470 | 7.71 | 4397 | | 6073 | 8.28 | 3927 | 44 |
| 17 | 0933 | 7.70 | 4363 | | 6570 | 8.28 | 3430 | 43 |
| 18 | 1395 | 7.69 | 4328 | | 7067 | 8.27 | 2933 | 42 |
| 19 | 1857 | 7.68 | 4294 | | 7563 | 8.26 | 2437 | 41 |
| 20 | 2318 | 7.67 | 4259 | | 8059 | 8.25 | 1941 | 40 |
| 21 | 9.422778 | 7.67 | 9.984224 | | 8554 | 8.24 | 1446 | 39 |
| 22 | 3238 | 7.66 | 4190 | | 9048 | 8.23 | 0952 | 38 |
| 23 | 3697 | 7.65 | 4155 | | 9.439543 | 8.23 | 10.560457 | 37 |
| 24 | 4156 | 7.64 | 4120 | | 9.440036 | 8.22 | 10.559964 | 36 |
| 25 | 4615 | 7.63 | 4085 | | 0529 | 8.21 | 9471 | 35 |
| 26 | 5073 | 7.62 | 4050 | | 1022 | 8.20 | 8978 | 34 |
| 27 | 5530 | 7.61 | 4015 | | 1514 | 8.19 | 8486 | 33 |
| 28 | 5987 | 7.60 | 3981 | | 2006 | 8.19 | 7994 | 32 |
| 29 | 6443 | 7.60 | 3946 | | 2497 | 8.18 | 7503 | 31 |
| 30 | 6899 | 7.59 | 3911 | | 2988 | 8.17 | 7012 | 30 |
| 31 | 9.427354 | 7.58 | 9.983875 | .58 | 9.443479 | 8.16 | 10.556521 | 29 |
| 32 | 7809 | 7.57 | 3840 | .59 | 3968 | 8.16 | 6032 | 28 |
| 33 | 8263 | 7.56 | 3805 | | 4458 | 8.15 | 5542 | 27 |
| 34 | 8717 | 7.55 | 3770 | | 4947 | 8.14 | 5053 | 26 |
| 35 | 9170 | 7.54 | 3735 | | 5435 | 8.13 | 4565 | 25 |
| 36 | 9.429623 | 7.54 | 3700 | | 5923 | 8.12 | 4077 | 24 |
| 37 | 9.430075 | 7.53 | 3664 | | 6411 | 8.12 | 3589 | 23 |
| 38 | 0527 | 7.52 | 3629 | | 6898 | 8.11 | 3102 | 22 |
| 39 | 0978 | 7.51 | 3594 | | 7384 | 8.10 | 2616 | 21 |
| 40 | 1429 | 7.50 | 3558 | | 7870 | 8.09 | 2130 | 20 |
| 41 | 9.431879 | 7.49 | 9.983523 | | 8356 | 8.09 | 1644 | 19 |
| 42 | 2329 | 7.49 | 3487 | | 8841 | 8.08 | 1159 | 18 |
| 43 | 2778 | 7.48 | 3452 | | 9326 | 8.07 | 0674 | 17 |
| 44 | 3226 | 7.47 | 3416 | | 9.449810 | 8.06 | 10.550190 | 16 |
| 45 | 3675 | 7.46 | 3381 | | 9.450294 | 8.06 | 10.549706 | 15 |
| 46 | 4122 | 7.45 | 3345 | | 0777 | 8.05 | 9223 | 14 |
| 47 | 4569 | 7.44 | 3309 | .59 | 1260 | 8.04 | 8740 | 13 |
| 48 | 5016 | 7.44 | 3273 | .60 | 1743 | 8.03 | 8257 | 12 |
| 49 | 5462 | 7.43 | 3238 | | 2225 | 8.02 | 7775 | 11 |
| 50 | 5908 | 7.42 | 3202 | | 2706 | 8.02 | 7294 | 10 |
| 51 | 9.436353 | 7.41 | 9.983166 | | 9.453187 | 8.01 | 10.546813 | 9 |
| 52 | 6798 | 7.40 | 3130 | | 3668 | 8.00 | 6332 | 8 |
| 53 | 7242 | 7.40 | 3094 | | 4148 | 7.99 | 5852 | 7 |
| 54 | 7686 | 7.39 | 3058 | | 4628 | 7.99 | 5372 | 6 |
| 55 | 8129 | 7.38 | 3022 | | 5107 | 7.98 | 4893 | 5 |
| 56 | 8572 | 7.37 | 2986 | | 5586 | 7.97 | 4414 | 4 |
| 57 | 9014 | 7.36 | 2950 | | 6064 | 7.96 | 3936 | 3 |
| 58 | 9456 | 7.36 | 2914 | | 6542 | 7.96 | 3458 | 2 |
| 59 | 9.439897 | 7.35 | 2878 | .60 | 7019 | 7.95 | 2981 | 1 |
| 60 | 9.440338 | | 9.982842 | | 9.457496 | | 10.542504 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 74° |

| 16° | LOGARITHMIC | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
| 0 | 9.440338 | 7.34 | 9.982842 | .60 | 9.457496 | 7.94 | 10.542504 | 60 |
| 1 | 0778 | 7.33 | 2805 | .60 | 7973 | 7.93 | 2027 | 59 |
| 2 | 1218 | 7.32 | 2769 | .61 | 8449 | 7.93 | 1551 | 58 |
| 3 | 1658 | 7.31 | 2733 | | 8925 | 7.92 | 1075 | 57 |
| 4 | 2096 | 7.31 | 2696 | | 9400 | 7.91 | 0600 | 56 |
| 5 | 2535 | 7.30 | 2660 | | 9.459875 | 7.90 | 10.540125 | 55 |
| 6 | 2973 | 7.29 | 2624 | | 9.460349 | 7.90 | 10.539651 | 54 |
| 7 | 3410 | 7.28 | 2587 | | 0823 | 7.89 | 9177 | 53 |
| 8 | 3847 | 7.27 | 2551 | | 1297 | 7.88 | 8703 | 52 |
| 9 | 4284 | 7.27 | 2514 | | 1770 | 7.88 | 8230 | 51 |
| 10 | 4720 | 7.26 | 2477 | | 2242 | 7.87 | 7758 | 50 |
| 11 | 9.445155 | 7.25 | 9.982441 | | 9.462715 | 7.86 | 10.537285 | 49 |
| 12 | 5590 | 7.24 | 2404 | | 3186 | 7.85 | 6814 | 48 |
| 13 | 6025 | 7.23 | 2367 | | 3658 | 7.85 | 6342 | 47 |
| 14 | 6459 | 7.23 | 2331 | | 4128 | 7.84 | 5872 | 46 |
| 15 | 6893 | 7.22 | 2294 | | 4599 | 7.83 | 5401 | 45 |
| 16 | 7326 | 7.21 | 2257 | .61 | 5069 | 7.83 | 4931 | 44 |
| 17 | 7759 | 7.20 | 2220 | .62 | 5539 | 7.82 | 4461 | 43 |
| 18 | 8191 | 7.20 | 2183 | | 6008 | 7.81 | 3992 | 42 |
| 19 | 8623 | 7.19 | 2146 | | 6477 | 7.80 | 3523 | 41 |
| 20 | 9054 | 7.18 | 2109 | | 6945 | 7.80 | 3055 | 40 |
| 21 | 9485 | 7.17 | 9.982072 | | 9.467413 | 7.79 | 10.532587 | 39 |
| 22 | 9.449915 | 7.16 | 2035 | | 7880 | 7.78 | 2120 | 38 |
| 23 | 9.450345 | 7.16 | 1998 | | 8347 | 7.78 | 1653 | 37 |
| 24 | 0775 | 7.15 | 1961 | | 8814 | 7.77 | 1186 | 36 |
| 25 | 1204 | 7.14 | 1924 | | 9280 | 7.76 | 0720 | 35 |
| 26 | 1632 | 7.13 | 1886 | | 9.469746 | 7.75 | 10.530254 | 34 |
| 27 | 2060 | 7.13 | 1849 | | 9.470211 | 7.75 | 10.529789 | 33 |
| 28 | 2488 | 7.12 | 1812 | | 0676 | 7.74 | 9324 | 32 |
| 29 | 2915 | 7.11 | 1774 | | 1141 | 7.73 | 8859 | 31 |
| 30 | 3342 | 7.10 | 1737 | .62 | 1605 | 7.73 | 8395 | 30 |
| 31 | 9.453768 | 7.10 | 9.981699 | .63 | 9.472069 | 7.72 | 7931 | 29 |
| 32 | 4194 | 7.09 | 1662 | | 2532 | 7.71 | 7468 | 28 |
| 33 | 4619 | 7.08 | 1625 | | 2995 | 7.71 | 7005 | 27 |
| 34 | 5044 | 7.07 | 1587 | | 3457 | 7.70 | 6543 | 26 |
| 35 | 5469 | 7.07 | 1549 | | 3919 | 7.69 | 6081 | 25 |
| 36 | 5893 | 7.06 | 1512 | | 4381 | 7.69 | 5619 | 24 |
| 37 | 6316 | 7.05 | 1474 | | 4842 | 7.68 | 5158 | 23 |
| 38 | 6739 | 7.04 | 1436 | | 5303 | 7.67 | 4697 | 22 |
| 39 | 7162 | 7.04 | 1399 | | 5763 | 7.67 | 4237 | 21 |
| 40 | 7584 | 7.03 | 1361 | | 6223 | 7.66 | 3777 | 20 |
| 41 | 8006 | 7.02 | 9.981323 | | 9.476683 | 7.65 | 10.523317 | 19 |
| 42 | 8427 | 7.01 | 1285 | | 7142 | 7.65 | 2858 | 18 |
| 43 | 8848 | 7.01 | 1247 | | 7601 | 7.64 | 2399 | 17 |
| 44 | 9268 | 7.00 | 1209 | | 8059 | 7.63 | 1941 | 16 |
| 45 | 9.459688 | 6.99 | 1171 | .63 | 8517 | 7.63 | 1483 | 15 |
| 46 | 9.460108 | 6.98 | 1133 | .64 | 8975 | 7.62 | 1025 | 14 |
| 47 | 0527 | 6.98 | 1095 | | 9432 | 7.61 | 0568 | 13 |
| 48 | 0946 | 6.97 | 1057 | | 9.479889 | 7.61 | 10.520111 | 12 |
| 49 | 1364 | 6.96 | 1019 | | 9.480345 | 7.60 | 10.519655 | 11 |
| 50 | 1782 | 6.95 | 0981 | | 0801 | 7.59 | 9199 | 10 |
| 51 | 2199 | 6.95 | 9.980942 | | 1257 | 7.59 | 8743 | 9 |
| 52 | 2616 | 6.94 | 0904 | | 1712 | 7.58 | 8288 | 8 |
| 53 | 3032 | 6.93 | 0866 | | 2167 | 7.57 | 7833 | 7 |
| 54 | 3448 | 6.93 | 0827 | | 2621 | 7.57 | 7379 | 6 |
| 55 | 3864 | 6.92 | 0789 | | 3075 | 7.56 | 6925 | 5 |
| 56 | 4279 | 6.91 | 0750 | | 3529 | 7.55 | 6471 | 4 |
| 57 | 4694 | 6.90 | 0712 | | 3982 | 7.55 | 6018 | 3 |
| 58 | 5108 | 6.90 | 0673 | | 4435 | 7.54 | 5565 | 2 |
| 59 | 5522 | 6.89 | 0635 | .64 | 4887 | 7.53 | 5113 | 1 |
| 60 | 9.465935 | | 9.980596 | | 9.485339 | | 10.514661 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 73° |

## 17° SINES AND TANGENTS.

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.465935 | 6.88 | 9.980596 | .64 | 9.485339 | 7.53 | 10.514661 | 60 |
| 1 | 6348 | 6.88 | 0558 | .64 | 5791 | 7.52 | 4209 | 59 |
| 2 | 6761 | 6.87 | 0519 | .65 | 6242 | 7.51 | 3758 | 58 |
| 3 | 7173 | 6.86 | 0480 | | 6693 | 7.51 | 3307 | 57 |
| 4 | 7585 | 6.85 | 0442 | | 7143 | 7.50 | 2857 | 56 |
| 5 | 7996 | 6.85 | 0403 | | 7593 | 7.49 | 2407 | 55 |
| 6 | 8407 | 6.84 | 0364 | | 8043 | 7.49 | 1957 | 54 |
| 7 | 8817 | 6.83 | 0325 | | 8492 | 7.48 | 1508 | 53 |
| 8 | 9227 | 6.83 | 0286 | | 8941 | 7.47 | 1059 | 52 |
| 9 | 9.469637 | 6.82 | 0247 | | 9390 | 7.47 | 0610 | 51 |
| 10 | 9.470046 | 6.81 | 0208 | | 9.489838 | 7.46 | 10.510162 | 50 |
| 11 | 0455 | 6.80 | 9.980169 | | 9.490286 | 7.46 | 10.509714 | 49 |
| 12 | 0863 | 6.80 | 0130 | | 0733 | 7.45 | 9267 | 48 |
| 13 | 1271 | 6.79 | 0091 | | 1180 | 7.44 | 8820 | 47 |
| 14 | 1679 | 6.78 | 0052 | | 1627 | 7.44 | 8373 | 46 |
| 15 | 2086 | 6.78 | 9.980012 | | 2073 | 7.43 | 7927 | 45 |
| 16 | 2492 | 6.77 | 9.979973 | .65 | 2519 | 7.43 | 7481 | 44 |
| 17 | 2898 | 6.76 | 9934 | .66 | 2965 | 7.42 | 7035 | 43 |
| 18 | 3304 | 6.76 | 9895 | | 3410 | 7.41 | 6590 | 42 |
| 19 | 3710 | 6.75 | 9855 | | 3854 | 7.40 | 6146 | 41 |
| 20 | 4115 | 6.74 | 9816 | | 4299 | 7.40 | 5701 | 40 |
| 21 | 9.474519 | 6.74 | 9.979776 | | 9.494743 | 7.39 | 10.505257 | 39 |
| 22 | 4923 | 6.73 | 9737 | | 5186 | 7.39 | 4814 | 38 |
| 23 | 5327 | 6.72 | 9697 | | 5630 | 7.38 | 4370 | 37 |
| 24 | 5730 | 6.72 | 9658 | | 6073 | 7.37 | 3927 | 36 |
| 25 | 6133 | 6.71 | 9618 | | 6515 | 7.37 | 3485 | 35 |
| 26 | 6536 | 6.70 | 9579 | | 6957 | 7.36 | 3043 | 34 |
| 27 | 6938 | 6.69 | 9539 | | 7399 | 7.36 | 2601 | 33 |
| 28 | 7340 | 6.69 | 9499 | | 7841 | 7.35 | 2159 | 32 |
| 29 | 7741 | 6.68 | 9459 | | 8282 | 7.34 | 1718 | 31 |
| 30 | 8142 | 6.67 | 9420 | | 8722 | 7.34 | 1278 | 30 |
| 31 | 8542 | 6.67 | 9.979380 | | 9163 | 7.33 | 0837 | 29 |
| 32 | 8942 | 6.66 | 9340 | .66 | 9.499603 | 7.33 | 10.500397 | 28 |
| 33 | 9342 | 6.65 | 9300 | .67 | 9.500042 | 7.32 | 10.499958 | 27 |
| 34 | 9.479741 | 6.65 | 9260 | | 0481 | 7.31 | 9519 | 26 |
| 35 | 9.480140 | 6.64 | 9220 | | 0920 | 7.31 | 9080 | 25 |
| 36 | 0539 | 6.63 | 9180 | | 1359 | 7.30 | 8641 | 24 |
| 37 | 0937 | 6.63 | 9140 | | 1797 | 7.30 | 8203 | 23 |
| 38 | 1334 | 6.62 | 9100 | | 2235 | 7.29 | 7765 | 22 |
| 39 | 1731 | 6.61 | 9059 | | 2672 | 7.28 | 7328 | 21 |
| 40 | 2128 | 6.61 | 9019 | | 3109 | 7.28 | 6891 | 20 |
| 41 | 9.482525 | 6.60 | 9.978979 | | 9.503546 | 7.27 | 10.496454 | 19 |
| 42 | 2921 | 6.59 | 8939 | | 3982 | 7.27 | 6018 | 18 |
| 43 | 3316 | 6.59 | 8898 | | 4418 | 7.26 | 5582 | 17 |
| 44 | 3712 | 6.58 | 8858 | | 4854 | 7.25 | 5146 | 16 |
| 45 | 4107 | 6.57 | 8817 | | 5289 | 7.25 | 4711 | 15 |
| 46 | 4501 | 6.57 | 8777 | | 5724 | 7.24 | 4276 | 14 |
| 47 | 4895 | 6.56 | 8737 | .67 | 6159 | 7.24 | 3841 | 13 |
| 48 | 5289 | 6.55 | 8696 | .68 | 6593 | 7.23 | 3407 | 12 |
| 49 | 5682 | 6.55 | 8655 | | 7027 | 7.22 | 2973 | 11 |
| 50 | 6075 | 6.54 | 8615 | | 7460 | 7.22 | 2540 | 10 |
| 51 | 9.486467 | 6.53 | 9.978574 | | 7893 | 7.21 | 2107 | 9 |
| 52 | 6860 | 6.53 | 8533 | | 8326 | 7.21 | 1674 | 8 |
| 53 | 7251 | 6.52 | 8493 | | 8759 | 7.20 | 1241 | 7 |
| 54 | 7643 | 6.51 | 8452 | | 9191 | 7.19 | 0809 | 6 |
| 55 | 8034 | 6.51 | 8411 | | 9.509622 | 7.19 | 10.490378 | 5 |
| 56 | 8424 | 6.50 | 8370 | | 9.510054 | 7.18 | 10.489946 | 4 |
| 57 | 8814 | 6.50 | 8329 | | 0485 | 8.18 | 9515 | 3 |
| 58 | 9204 | 6.49 | 8288 | | 0916 | 7.17 | 9084 | 2 |
| 59 | 9593 | 6.48 | 8247 | .68 | 1346 | 7.17 | 8654 | 1 |
| 60 | 9.489982 | | 9.978206 | | 9.511776 | | 10.488224 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 72° |

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.489982 | 6.48 | 9.978206 | .68 | 9.511776 | 7.16 | 10.488224 | 60 |
| 1 | 9.490371 | 6.47 | 8165 | | 2206 | 7.16 | 7794 | 59 |
| 2 | 0759 | 6.46 | 8124 | .68 | 2635 | 7.15 | 7365 | 58 |
| 3 | 1147 | 6.46 | 8083 | .69 | 3064 | 7.14 | 6936 | 57 |
| 4 | 1535 | 6.45 | 8042 | | 3493 | 7.14 | 6507 | 56 |
| 5 | 1922 | 6.44 | 8001 | | 3921 | 7.13 | 6079 | 55 |
| 6 | 2308 | 6.44 | 7959 | | 4349 | 7.13 | 5651 | 54 |
| 7 | 2695 | 6.43 | 7918 | | 4777 | 7.12 | 5223 | 53 |
| 8 | 3081 | 6.42 | 7877 | | 5204 | 7.12 | 4796 | 52 |
| 9 | 3466 | 6.42 | 7835 | | 5631 | 7.11 | 4369 | 51 |
| 10 | 3851 | 6.41 | 7794 | | 6057 | 7.10 | 3943 | 50 |
| 11 | 9.494236 | 6.41 | 9.977752 | | 9.516484 | 7.10 | 10.483516 | 49 |
| 12 | 4621 | 6.40 | 7711 | | 6910 | 7.09 | 3090 | 48 |
| 13 | 5005 | 6.39 | 7669 | | 7335 | 7.09 | 2665 | 47 |
| 14 | 5388 | 6.39 | 7628 | | 7761 | 7.08 | 2239 | 46 |
| 15 | 5772 | 6.38 | 7586 | .69 | 8184 | 7.08 | 1816 | 45 |
| 16 | 6154 | 6.37 | 7544 | .70 | 8610 | 7.07 | 1390 | 44 |
| 17 | 6537 | 6.37 | 7503 | | 9034 | 7.06 | 0966 | 43 |
| 18 | 6919 | 6.36 | 7461 | | 9458 | 7.06 | 0542 | 42 |
| 19 | 7301 | 6.36 | 7419 | | 9.519882 | 7.05 | 10.480118 | 41 |
| 20 | 7682 | 6.35 | 7377 | | 9.520305 | 7.05 | 10.479695 | 40 |
| 21 | 9.498064 | 6.34 | 9.977335 | | 0728 | 7.04 | 9272 | 39 |
| 22 | 8444 | 6.34 | 7293 | | 1151 | 7.04 | 8849 | 38 |
| 23 | 8825 | 6.33 | 7251 | | 1573 | 7.03 | 8427 | 37 |
| 24 | 9204 | 6.32 | 7209 | | 1995 | 7.03 | 8005 | 36 |
| 25 | 9584 | 6.32 | 7167 | | 2417 | 7.02 | 7583 | 35 |
| 26 | 9.499963 | 6.31 | 7125 | | 2838 | 7.02 | 7162 | 34 |
| 27 | 9.500342 | 6.31 | 7083 | | 3259 | 7.01 | 6741 | 33 |
| 28 | 0721 | 6.30 | 7041 | | 3680 | 7.01 | 6320 | 32 |
| 29 | 1099 | 6.29 | 6999 | | 4100 | 7.00 | 5900 | 31 |
| 30 | 1476 | 6.29 | 6957 | | 4520 | 6.99 | 5480 | 30 |
| 31 | 9.501854 | 6.28 | 9.976914 | .70 | 9.524940 | 6.99 | 10.475060 | 29 |
| 32 | 2231 | 6.28 | 6872 | .71 | 5359 | 6.98 | 4641 | 28 |
| 33 | 2607 | 6.27 | 6830 | | 5778 | 6.98 | 4222 | 27 |
| 34 | 2984 | 6.26 | 6787 | | 6197 | 6.97 | 3803 | 26 |
| 35 | 3360 | 6.26 | 6745 | | 6615 | 6.97 | 3385 | 25 |
| 36 | 3735 | 6.25 | 6702 | | 7033 | 6.96 | 2967 | 24 |
| 37 | 4110 | 6.25 | 6660 | | 7451 | 6.96 | 2549 | 23 |
| 38 | 4485 | 6.24 | 6617 | | 7868 | 6.95 | 2132 | 22 |
| 39 | 4860 | 6.23 | 6574 | | 8285 | 6.95 | 1715 | 21 |
| 40 | 5234 | 6.23 | 6532 | | 8702 | 6.94 | 1298 | 20 |
| 41 | 9.505608 | 6.22 | 9.976489 | | 9.529119 | 6.93 | 10.470881 | 19 |
| 42 | 5981 | 6.22 | 6446 | | 9535 | 6.93 | 0465 | 18 |
| 43 | 6354 | 6.21 | 6404 | | 9.529950 | 6.93 | 10.470050 | 17 |
| 44 | 6727 | 6.20 | 6361 | | 9.530366 | 6.92 | 10.469634 | 16 |
| 45 | 7099 | 6.20 | 6318 | | 0781 | 6.91 | 9219 | 15 |
| 46 | 7471 | 6.19 | 6275 | .71 | 1196 | 6.91 | 8804 | 14 |
| 47 | 7843 | 6.19 | 6232 | .72 | 1611 | 6.90 | 8389 | 13 |
| 48 | 8214 | 6.18 | 6189 | | 2025 | 6.90 | 7975 | 12 |
| 49 | 8585 | 6.18 | 6146 | | 2439 | 6.89 | 7561 | 11 |
| 50 | 8956 | 6.17 | 6103 | | 2853 | 6.89 | 7147 | 10 |
| 51 | 9326 | 6.16 | 9.976060 | | 9.533266 | 6.88 | 10.466734 | 9 |
| 52 | 9.509696 | 6.16 | 6017 | | 3679 | 6.88 | 6321 | 8 |
| 53 | 9.510065 | 6.15 | 5974 | | 4092 | 6.87 | 5908 | 7 |
| 54 | 0434 | 6.15 | 5930 | | 4504 | 6.87 | 5496 | 6 |
| 55 | 0803 | 6.14 | 5887 | | 4916 | 6.86 | 5084 | 5 |
| 56 | 1172 | 6.13 | 5844 | | 5328 | 6.86 | 4672 | 4 |
| 57 | 1540 | 6.13 | 5800 | | 5739 | 6.85 | 4261 | 3 |
| 58 | 1907 | 6.12 | 5757 | | 6150 | 6.85 | 3850 | 2 |
| 59 | 2275 | 6.12 | 5714 | .72 | 6561 | 6.84 | 3439 | 1 |
| 60 | 9.512642 | | 9.975670 | | 9.536972 | | 10.463028 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 71° |

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.512642 | 6.11 | 9.975670 | .73 | 9.536972 | 6.84 | 10.463028 | 60 |
| 1 | 3009 | 6.11 | 5627 | | 7382 | 6.83 | 2618 | 59 |
| 2 | 3375 | 6.10 | 5583 | | 7792 | 6.83 | 2208 | 58 |
| 3 | 3741 | 6.09 | 5539 | | 8202 | 6.82 | 1798 | 57 |
| 4 | 4107 | 6.09 | 5496 | | 8611 | 6.82 | 1389 | 56 |
| 5 | 4472 | 6.08 | 5452 | | 9020 | 6.81 | 0980 | 55 |
| 6 | 4837 | 6.08 | 5408 | | 9429 | 6.81 | 0571 | 54 |
| 7 | 5202 | 6.07 | 5365 | | 9.539837 | 6.80 | 10.460163 | 53 |
| 8 | 5566 | 6.07 | 5321 | | 9.540245 | 6.80 | 10.459755 | 52 |
| 9 | 5930 | 6.06 | 5277 | | 0653 | 6.79 | 9347 | 51 |
| 10 | 6294 | 6.05 | 5233 | | 1061 | 6.79 | 8939 | 50 |
| 11 | 9.516657 | 6.05 | 9.975189 | | 1468 | 6.78 | 8532 | 49 |
| 12 | 7020 | 6.04 | 5145 | | 1875 | 6.78 | 8125 | 48 |
| 13 | 7382 | 6.04 | 5101 | | 2281 | 6.77 | 7719 | 47 |
| 14 | 7745 | 6.03 | 5057 | | 2688 | 6.77 | 7312 | 46 |
| 15 | 8107 | 6.03 | 5013 | .73 | 3094 | 6.76 | 6906 | 45 |
| 16 | 8468 | 6.02 | 4969 | .74 | 3499 | 6.76 | 6501 | 44 |
| 17 | 8829 | 6.01 | 4925 | | 3905 | 6.75 | 6095 | 43 |
| 18 | 9190 | 6.01 | 4880 | | 4310 | 6.75 | 5690 | 42 |
| 19 | 9551 | 6.00 | 4836 | | 4715 | 6.74 | 5285 | 41 |
| 20 | 9.519911 | 6.00 | 4792 | | 5119 | 6.74 | 4881 | 40 |
| 21 | 9.520271 | 5.99 | 9.974748 | | 9.545524 | 6.73 | 10.454476 | 39 |
| 22 | 0631 | 5.99 | 4703 | | 5928 | 6.73 | 4072 | 38 |
| 23 | 0990 | 5.98 | 4659 | | 6331 | 6.72 | 3669 | 37 |
| 24 | 1349 | 5.98 | 4614 | | 6735 | 6.72 | 3265 | 36 |
| 25 | 1707 | 5.97 | 4570 | | 7138 | 6.71 | 2862 | 35 |
| 26 | 2066 | 5.96 | 4525 | | 7540 | 6.71 | 2460 | 34 |
| 27 | 2424 | 5.96 | 4481 | | 7943 | 6.70 | 2057 | 33 |
| 28 | 2781 | 5.95 | 4436 | | 8345 | 6.70 | 1655 | 32 |
| 29 | 3138 | 5.95 | 4391 | .74 | 8747 | 6.69 | 1253 | 31 |
| 30 | 3495 | 5.94 | 4347 | .75 | 9149 | 6.69 | 0851 | 30 |
| 31 | 9.523852 | 5.94 | 9.974302 | | 9550 | 6.68 | 0450 | 29 |
| 32 | 4208 | 5.93 | 4257 | | 9.549951 | 6.68 | 10.450049 | 28 |
| 33 | 4564 | 5.93 | 4212 | | 9.550352 | 6.67 | 10.449648 | 27 |
| 34 | 4920 | 5.92 | 4167 | | 0752 | 6.67 | 9248 | 26 |
| 35 | 5275 | 5.91 | 4122 | | 1153 | 6.66 | 8847 | 25 |
| 36 | 5630 | 5.91 | 4077 | | 1552 | 6.66 | 8448 | 24 |
| 37 | 5984 | 5.90 | 4032 | | 1952 | 6.65 | 8048 | 23 |
| 38 | 6339 | 5.90 | 3987 | | 2351 | 6.65 | 7649 | 22 |
| 39 | 6693 | 5.89 | 3942 | | 2750 | 6.65 | 7250 | 21 |
| 40 | 7046 | 5.89 | 3897 | | 3149 | 6.64 | 6851 | 20 |
| 41 | 9.527400 | 5.88 | 9.973852 | | 9.553548 | 6.64 | 10.446452 | 19 |
| 42 | 7753 | 5.88 | 3807 | | 3946 | 6.63 | 6054 | 18 |
| 43 | 8105 | 5.87 | 3761 | .75 | 4344 | 6.63 | 5656 | 17 |
| 44 | 8458 | 5.87 | 3716 | .76 | 4741 | 6.62 | 5259 | 16 |
| 45 | 8810 | 5.86 | 3671 | | 5139 | 6.62 | 4861 | 15 |
| 46 | 9161 | 5.86 | 3625 | | 5536 | 6.61 | 4464 | 14 |
| 47 | 9513 | 5.85 | 3580 | | 5933 | 6.61 | 4067 | 13 |
| 48 | 9.529864 | 5.85 | 3535 | | 6329 | 6.60 | 3671 | 12 |
| 49 | 9.530215 | 5.84 | 3489 | | 6725 | 6.60 | 3275 | 11 |
| 50 | 0565 | 5.84 | 3444 | | 7121 | 6.59 | 2879 | 10 |
| 51 | 0915 | 5.83 | 9.973398 | | 9.557517 | 6.59 | 10.442483 | 9 |
| 52 | 1265 | 5.82 | 3352 | | 7913 | 6.59 | 2087 | 8 |
| 53 | 1614 | 5.82 | 3307 | | 8308 | 6.58 | 1692 | 7 |
| 54 | 1963 | 5.81 | 3261 | | 8703 | 6.58 | 1297 | 6 |
| 55 | 2312 | 5.81 | 3215 | | 9097 | 6.57 | 0903 | 5 |
| 56 | 2661 | 5.80 | 3169 | | 9491 | 6.57 | 0509 | 4 |
| 57 | 3009 | 5.80 | 3124 | | 9.559885 | 6.56 | 10.440115 | 3 |
| 58 | 3357 | 5.79 | 3078 | .76 | 9.560279 | 6.56 | 10.439721 | 2 |
| 59 | 3704 | 5.79 | 3032 | .77 | 0673 | 6.55 | 9327 | 1 |
| 60 | 9.534052 | | 9.972986 | | 9.561066 | | 10.438934 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 70° |

## 20° LOGARITHMIC

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.534052 | 5.78 | 9.972986 | .77 | 9.561066 | 6.55 | 10.438934 | 60 |
| 1 | 4399 | 5.78 | 2940 | | 1459 | 6.54 | 8541 | 59 |
| 2 | 4745 | 5.77 | 2894 | | 1851 | 6.54 | 8149 | 58 |
| 3 | 5092 | 5.77 | 2848 | | 2244 | 6.53 | 7756 | 57 |
| 4 | 5438 | 5.76 | 2802 | | 2636 | 6.53 | 7364 | 56 |
| 5 | 5783 | 5.76 | 2755 | | 3028 | 6.53 | 6972 | 55 |
| 6 | 6129 | 5.75 | 2709 | | 3419 | 6.52 | 6581 | 54 |
| 7 | 6474 | 5.74 | 2663 | | 3811 | 6.52 | 6189 | 53 |
| 8 | 6818 | 5.74 | 2617 | | 4202 | 6.51 | 5798 | 52 |
| 9 | 7163 | 5.73 | 2570 | | 4593 | 6.51 | 5407 | 51 |
| 10 | 7507 | 5.73 | 2524 | | 4983 | 6.50 | 5017 | 50 |
| 11 | 9.537851 | 5.72 | 9.972478 | .77 | 9.565373 | 6.50 | 10.434627 | 49 |
| 12 | 8194 | 5.72 | 2431 | .78 | 5763 | 6.49 | 4237 | 48 |
| 13 | 8538 | 5.71 | 2385 | | 6153 | 6.49 | 3847 | 47 |
| 14 | 8880 | 5.71 | 2338 | | 6542 | 6.49 | 3458 | 46 |
| 15 | 9223 | 5.70 | 2291 | | 6932 | 6.48 | 3068 | 45 |
| 16 | 9565 | 5.70 | 2245 | | 7320 | 6.48 | 2680 | 44 |
| 17 | 9.539907 | 5.69 | 2198 | | 7709 | 6.47 | 2291 | 43 |
| 18 | 9.540249 | 5.69 | 2151 | | 8098 | 6.47 | 1902 | 42 |
| 19 | 0590 | 5.68 | 2105 | | 8486 | 6.46 | 1514 | 41 |
| 20 | 0931 | 5.68 | 2058 | | 8873 | 6.46 | 1127 | 40 |
| 21 | 1272 | 5.67 | 9.972011 | | 9261 | 6.45 | 0739 | 39 |
| 22 | 1613 | 5.67 | 1964 | | 9.569648 | 6.45 | 10.430352 | 38 |
| 23 | 1953 | 5.66 | 1917 | | 9.570035 | 6.45 | 10.429965 | 37 |
| 24 | 2293 | 5.66 | 1870 | | 0422 | 6.44 | 9578 | 36 |
| 25 | 2632 | 5.65 | 1823 | | 0809 | 6.44 | 9191 | 35 |
| 26 | 2971 | 5.65 | 1776 | .78 | 1195 | 6.43 | 8805 | 34 |
| 27 | 3310 | 5.64 | 1729 | .79 | 1581 | 6.43 | 8419 | 33 |
| 28 | 3649 | 5.64 | 1682 | | 1967 | 6.42 | 8033 | 32 |
| 29 | 3987 | 5.63 | 1635 | | 2352 | 6.42 | 7648 | 31 |
| 30 | 4325 | 5.63 | 1588 | | 2738 | 6.42 | 7262 | 30 |
| 31 | 9.544663 | 5.62 | 9.971540 | | 9.573123 | 6.41 | 10.426877 | 29 |
| 32 | 5000 | 5.62 | 1493 | | 3507 | 6.41 | 6493 | 28 |
| 33 | 5338 | 5.61 | 1446 | | 3892 | 6.40 | 6108 | 27 |
| 34 | 5674 | 5.61 | 1398 | | 4276 | 6.40 | 5724 | 26 |
| 35 | 6011 | 5.60 | 1351 | | 4660 | 6.39 | 5340 | 25 |
| 36 | 6347 | 5.60 | 1303 | | 5044 | 6.39 | 4956 | 24 |
| 37 | 6683 | 5.59 | 1256 | | 5427 | 6.39 | 4573 | 23 |
| 38 | 7019 | 5.59 | 1208 | | 5810 | 6.38 | 4190 | 22 |
| 39 | 7354 | 5.58 | 1161 | | 6193 | 6.38 | 3807 | 21 |
| 40 | 7689 | 5.58 | 1113 | .79 | 6576 | 6.37 | 3424 | 20 |
| 41 | 8024 | 5.57 | 9.971066 | .80 | 9.576959 | 6.37 | 10.423041 | 19 |
| 42 | 8359 | 5.57 | 1018 | | 7341 | 6.36 | 2659 | 18 |
| 43 | 8693 | 5.56 | 0970 | | 7723 | 6.36 | 2277 | 17 |
| 44 | 9027 | 5.56 | 0922 | | 8104 | 6.36 | 1896 | 16 |
| 45 | 9360 | 5.55 | 0874 | | 8486 | 6.35 | 1514 | 15 |
| 46 | 9.549693 | 5.55 | 0827 | | 8867 | 6.35 | 1133 | 14 |
| 47 | 9.550026 | 5.54 | 0779 | | 9248 | 6.34 | 0752 | 13 |
| 48 | 0359 | 5.54 | 0731 | | 9.579629 | 6.34 | 10.420371 | 12 |
| 49 | 0692 | 5.53 | 0683 | | 9.580009 | 6.34 | 10.419991 | 11 |
| 50 | 1024 | 5.53 | 0635 | | 0389 | 6.33 | 9611 | 10 |
| 51 | 1356 | 5.52 | 9.970586 | | 0769 | 6.33 | 9231 | 9 |
| 52 | 1687 | 5.52 | 0538 | | 1149 | 6.32 | 8851 | 8 |
| 53 | 2018 | 5.52 | 0490 | | 1528 | 6.32 | 8472 | 7 |
| 54 | 2349 | 5.51 | 0442 | | 1907 | 6.32 | 8093 | 6 |
| 55 | 2680 | 5.51 | 0394 | .80 | 2286 | 6.31 | 7714 | 5 |
| 56 | 3010 | 5.50 | 0345 | .81 | 2665 | 6.31 | 7335 | 4 |
| 57 | 3341 | 5.50 | 0297 | | 3044 | 6.30 | 6956 | 3 |
| 58 | 3670 | 5.49 | 0249 | | 3422 | 6.30 | 6578 | 2 |
| 59 | 4000 | 5.49 | 0200 | .81 | 3800 | 6.29 | 6200 | 1 |
| 60 | 9.554329 | | 9.970152 | | 9.584177 | | 10.415823 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 69° |

## 21° SINES AND TANGENTS.

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.554329 | 5.48 | 9.970152 | .81 | 9.584177 | 6.29 | 10.415823 | 60 |
| 1 | 4658 | 5.48 | 0103 | | 4555 | 6.29 | 5445 | 59 |
| 2 | 4987 | 5.47 | 0055 | | 4932 | 6.28 | 5068 | 58 |
| 3 | 5315 | 5.47 | 9.970006 | | 5309 | 6.28 | 4691 | 57 |
| 4 | 5643 | 5.46 | 9.969957 | | 5686 | 6.27 | 4314 | 56 |
| 5 | 5971 | 5.46 | 9909 | | 6062 | 6.27 | 3938 | 55 |
| 6 | 6299 | 5.45 | 9860 | | 6439 | 6.27 | 3561 | 54 |
| 7 | 6626 | 5.45 | 9811 | | 6815 | 6.26 | 3185 | 53 |
| 8 | 6953 | 5.44 | 9762 | | 7190 | 6.26 | 2810 | 52 |
| 9 | 7280 | 5.44 | 9714 | | 7566 | 6.25 | 2434 | 51 |
| 10 | 7606 | 5.43 | 9665 | .81 | 7941 | 6.25 | 2059 | 50 |
| 11 | 9.557932 | 5.43 | 9.969616 | .82 | 8316 | 6.25 | 1684 | 49 |
| 12 | 8258 | 5.43 | 9567 | | 8691 | 6.24 | 1309 | 48 |
| 13 | 8583 | 5.42 | 9518 | | 9066 | 6.24 | 0934 | 47 |
| 14 | 8909 | 5.42 | 9469 | | 9440 | 6.23 | 0560 | 46 |
| 15 | 9234 | 5.41 | 9420 | | 9.589814 | 6.23 | 10.410186 | 45 |
| 16 | 9558 | 5.41 | 9370 | | 9.590188 | 6.23 | 10.409812 | 44 |
| 17 | 9.559883 | 5.40 | 9321 | | 0562 | 6.22 | 9438 | 43 |
| 18 | 9.560207 | 5.40 | 9272 | | 0935 | 6.22 | 9065 | 42 |
| 19 | 0531 | 5.39 | 9223 | | 1308 | 6.22 | 8692 | 41 |
| 20 | 0855 | 5.39 | 9173 | | 1681 | 6.21 | 8319 | 40 |
| 21 | 1178 | 5.38 | 9.969124 | | 2054 | 6.21 | 7946 | 39 |
| 22 | 1501 | 5.38 | 9075 | | 2426 | 6.20 | 7574 | 38 |
| 23 | 1824 | 5.37 | 9025 | | 2799 | 6.20 | 7201 | 37 |
| 24 | 2146 | 5.37 | 8976 | .82 | 3171 | 6.20 | 6829 | 36 |
| 25 | 2468 | 5.36 | 8926 | .83 | 3542 | 6.19 | 6458 | 35 |
| 26 | 2790 | 5.36 | 8877 | | 3914 | 6.19 | 6086 | 34 |
| 27 | 3112 | 5.36 | 8827 | | 4285 | 6.18 | 5715 | 33 |
| 28 | 3433 | 5.35 | 8777 | | 4656 | 6.18 | 5344 | 32 |
| 29 | 3755 | 5.35 | 8728 | | 5027 | 6.18 | 4973 | 31 |
| 30 | 4075 | 5.34 | 8678 | | 5398 | 6.17 | 4602 | 30 |
| 31 | 9.564396 | 5.34 | 9.968628 | | 9.595768 | 6.17 | 10.404232 | 29 |
| 32 | 4716 | 5.33 | 8578 | | 6138 | 6.16 | 3862 | 28 |
| 33 | 5036 | 5.33 | 8528 | | 6508 | 6.16 | 3492 | 27 |
| 34 | 5356 | 5.32 | 8479 | | 6878 | 6.16 | 3122 | 26 |
| 35 | 5676 | 5.32 | 8429 | | 7247 | 6.15 | 2753 | 25 |
| 36 | 5995 | 5.31 | 8379 | | 7616 | 6.15 | 2384 | 24 |
| 37 | 6314 | 5.31 | 8329 | | 7985 | 6.15 | 2015 | 23 |
| 38 | 6632 | 5.31 | 8278 | .83 | 8354 | 6.14 | 1646 | 22 |
| 39 | 6951 | 5.30 | 8228 | .84 | 8722 | 6.14 | 1278 | 21 |
| 40 | 7269 | 5.30 | 8178 | | 9091 | 6.13 | 0909 | 20 |
| 41 | 9.567587 | 5.29 | 9.968128 | | 9459 | 6.13 | 0541 | 19 |
| 42 | 7904 | 5.29 | 8078 | | 9.599827 | 6.13 | 10.400173 | 18 |
| 43 | 8222 | 5.28 | 8027 | | 9.600194 | 6.12 | 10.399806 | 17 |
| 44 | 8539 | 5.28 | 7977 | | 0562 | 6.12 | 9438 | 16 |
| 45 | 8856 | 5.28 | 7927 | | 0929 | 6.11 | 9071 | 15 |
| 46 | 9172 | 5.27 | 7876 | | 1296 | 6.11 | 8704 | 14 |
| 47 | 9488 | 5.27 | 7826 | | 1663 | 6.11 | 8337 | 13 |
| 48 | 9.569804 | 5.26 | 7775 | | 2029 | 6.10 | 7971 | 12 |
| 49 | 9.570120 | 5.26 | 7725 | | 2395 | 6.10 | 7605 | 11 |
| 50 | 0435 | 5.25 | 7674 | | 2761 | 6.10 | 7239 | 10 |
| 51 | 0751 | 5.25 | 9.967624 | | 9.603127 | 6.09 | 10.396873 | 9 |
| 52 | 1066 | 5.24 | 7573 | .84 | 3493 | 6.09 | 6507 | 8 |
| 53 | 1380 | 5.24 | 7522 | .85 | 3858 | 6.09 | 6142 | 7 |
| 54 | 1695 | 5.23 | 7471 | | 4223 | 6.08 | 5777 | 6 |
| 55 | 2009 | 5.23 | 7421 | | 4588 | 6.08 | 5412 | 5 |
| 56 | 2323 | 5.23 | 7370 | | 4953 | 6.07 | 5047 | 4 |
| 57 | 2636 | 5.22 | 7319 | | 5317 | 6.07 | 4683 | 3 |
| 58 | 2950 | 5.22 | 7268 | | 5682 | 6.07 | 4318 | 2 |
| 59 | 3263 | 5.21 | 7217 | .85 | 6046 | 6.06 | 3954 | 1 |
| 60 | 9.573575 | | 9.967166 | | 9.606410 | | 10.393590 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 68° |

## 22° LOGARITHMIC

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.573575 | 5.21 | 9.967166 | .85 | 9.606410 | 6.06 | 10.393590 | 60 |
| 1 | 3888 | 5.20 | 7115 | | 6773 | 6.06 | 3227 | 59 |
| 2 | 4200 | 5.20 | 7064 | | 7137 | 6.05 | 2863 | 58 |
| 3 | 4512 | 5.19 | 7013 | | 7500 | 6.05 | 2500 | 57 |
| 4 | 4824 | 5.19 | 6961 | | 7863 | 6.04 | 2137 | 56 |
| 5 | 5136 | 5.19 | 6910 | | 8225 | 6.04 | 1775 | 55 |
| 6 | 5447 | 5.18 | 6859 | | 8588 | 6.04 | 1412 | 54 |
| 7 | 5758 | 5.18 | 6808 | .85 | 8950 | 6.03 | 1050 | 53 |
| 8 | 6069 | 5.17 | 6756 | .86 | 9312 | 6.03 | 0688 | 52 |
| 9 | 6379 | 5.17 | 6705 | | 9.609674 | 6.03 | 10.390326 | 51 |
| 10 | 6689 | 5.16 | 6653 | | 9.610036 | 6.02 | 10.389964 | 50 |
| 11 | 9.576999 | 5.16 | 9.966602 | | 0397 | 6.02 | 9603 | 49 |
| 12 | 7309 | 5.16 | 6550 | | 0759 | 6.02 | 9241 | 48 |
| 13 | 7618 | 5.15 | 6499 | | 1120 | 6.01 | 8880 | 47 |
| 14 | 7927 | 5.15 | 6447 | | 1480 | 6.01 | 8520 | 46 |
| 15 | 8236 | 5.14 | 6395 | | 1841 | 6.01 | 8159 | 45 |
| 16 | 8545 | 5.14 | 6344 | | 2201 | 6.00 | 7799 | 44 |
| 17 | 8853 | 5.13 | 6292 | | 2561 | 6.00 | 7439 | 43 |
| 18 | 9162 | 5.13 | 6240 | | 2921 | 6.00 | 7079 | 42 |
| 19 | 9470 | 5.13 | 6188 | | 3281 | 5.99 | 6719 | 41 |
| 20 | 9.579777 | 5.12 | 6136 | .86 | 3641 | 5.99 | 6359 | 40 |
| 21 | 9.580085 | 5.12 | 9.966085 | .87 | 9.614000 | 5.98 | 10.386000 | 39 |
| 22 | 0392 | 5.11 | 6033 | | 4359 | 5.98 | 5641 | 38 |
| 23 | 0699 | 5.11 | 5981 | | 4718 | 5.98 | 5282 | 37 |
| 24 | 1005 | 5.11 | 5929 | | 5077 | 5.97 | 4923 | 36 |
| 25 | 1312 | 5.10 | 5876 | | 5435 | 5.97 | 4565 | 35 |
| 26 | 1618 | 5.10 | 5824 | | 5793 | 5.97 | 4207 | 34 |
| 27 | 1924 | 5.09 | 5772 | | 6151 | 5.96 | 3849 | 33 |
| 28 | 2229 | 5.09 | 5720 | | 6509 | 5.96 | 3491 | 32 |
| 29 | 2535 | 5.09 | 5668 | | 6867 | 5.96 | 3133 | 31 |
| 30 | 2840 | 5.08 | 5615 | | 7224 | 5.95 | 2776 | 30 |
| 31 | 9.583145 | 5.08 | 9.965563 | | 9.617582 | 5.95 | 10.382418 | 29 |
| 32 | 3449 | 5.07 | 5511 | | 7939 | 5.95 | 2061 | 28 |
| 33 | 3754 | 5.07 | 5458 | | 8295 | 5.94 | 1705 | 27 |
| 34 | 4058 | 5.06 | 5406 | .87 | 8652 | 5.94 | 1348 | 26 |
| 35 | 4361 | 5.06 | 5353 | .88 | 9008 | 5.94 | 0992 | 25 |
| 36 | 4665 | 5.06 | 5301 | | 9364 | 5.93 | 0636 | 24 |
| 37 | 4968 | 5.05 | 5248 | | 9.619720 | 5.93 | 10.380280 | 23 |
| 38 | 5272 | 5.05 | 5195 | | 9.620076 | 5.93 | 10.379924 | 22 |
| 39 | 5574 | 5.04 | 5143 | | 0432 | 5.92 | 9568 | 21 |
| 40 | 5877 | 5.04 | 5090 | | 0787 | 5.92 | 9213 | 20 |
| 41 | 9.586179 | 5.03 | 9.965037 | | 1142 | 5.92 | 8858 | 19 |
| 42 | 6482 | 5.03 | 4984 | | 1497 | 5.91 | 8503 | 18 |
| 43 | 6783 | 5.03 | 4931 | | 1852 | 5.91 | 8148 | 17 |
| 44 | 7085 | 5.02 | 4879 | | 2207 | 5.90 | 7793 | 16 |
| 45 | 7386 | 5.02 | 4826 | | 2561 | 5.90 | 7439 | 15 |
| 46 | 7688 | 5.01 | 4773 | | 2915 | 5.90 | 7085 | 14 |
| 47 | 7989 | 5.01 | 4720 | .88 | 3269 | 5.89 | 6731 | 13 |
| 48 | 8289 | 5.01 | 4666 | .89 | 3623 | 5.89 | 6377 | 12 |
| 49 | 8590 | 5.00 | 4613 | | 3976 | 5.89 | 6024 | 11 |
| 50 | 8890 | 5.00 | 4560 | | 4330 | 5.88 | 5670 | 10 |
| 51 | 9190 | 4.99 | 9.964507 | | 9.624683 | 5.88 | 10.375317 | 9 |
| 52 | 9489 | 4.99 | 4454 | | 5036 | 5.88 | 4964 | 8 |
| 53 | 9.589789 | 4.99 | 4400 | | 5388 | 5.87 | 4612 | 7 |
| 54 | 9.590088 | 4.98 | 4347 | | 5741 | 5.87 | 4259 | 6 |
| 55 | 0387 | 4.98 | 4294 | | 6093 | 5.87 | 3907 | 5 |
| 56 | 0686 | 4.97 | 4240 | | 6445 | 5.86 | 3555 | 4 |
| 57 | 0984 | 4.97 | 4187 | | 6797 | 5.86 | 3203 | 3 |
| 58 | 1282 | 4.97 | 4133 | | 7149 | 5.86 | 2851 | 2 |
| 59 | 1580 | 4.96 | 4080 | .89 | 7501 | 5.85 | 2499 | 1 |
| 60 | 9.591878 | | 9.964026 | | 9.627852 | | 10.372148 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 67° |

## 23° SINES AND TANGENTS.

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.591878 | 4.96 | 9.964026 | .89 | 9.627852 | 5.85 | 10.372148 | 60 |
| 1 | 2176 | 4.95 | 3972 | .89 | 8203 | 5.85 | 1797 | 59 |
| 2 | 2473 | 4.95 | 3919 | .89 | 8554 | 5.85 | 1446 | 58 |
| 3 | 2770 | 4.95 | 3865 | .90 | 8905 | 5.84 | 1095 | 57 |
| 4 | 3067 | 4.94 | 3811 | | 9255 | 5.84 | 0745 | 56 |
| 5 | 3363 | 4.94 | 3757 | | 9606 | 5.83 | 0394 | 55 |
| 6 | 3659 | 4.93 | 3704 | | 9.629956 | 5.83 | 10.370044 | 54 |
| 7 | 3955 | 4.93 | 3650 | | 9.630306 | 5.83 | 10.369694 | 53 |
| 8 | 4251 | 4.93 | 3596 | | 0656 | 5.83 | 9344 | 52 |
| 9 | 4547 | 4.92 | 3542 | | 1005 | 5.82 | 8995 | 51 |
| 10 | 4842 | 4.92 | 3488 | | 1355 | 5.82 | 8645 | 50 |
| 11 | 9.595137 | 4.91 | 9.963434 | | 1704 | 5.82 | 8296 | 49 |
| 12 | 5432 | 4.91 | 3379 | | 2053 | 5.81 | 7947 | 48 |
| 13 | 5727 | 4.91 | 3325 | | 2402 | 8.81 | 7598 | 47 |
| 14 | 6021 | 4.90 | 3271 | | 2750 | 5.81 | 7250 | 46 |
| 15 | 6315 | 4.90 | 3217 | | 3099 | 5.80 | 6901 | 45 |
| 16 | 6609 | 4.89 | 3163 | .90 | 3447 | 5.80 | 6553 | 44 |
| 17 | 6903 | 4.89 | 3108 | .91 | 3795 | 5.80 | 6205 | 43 |
| 18 | 7196 | 4.89 | 3054 | | 4143 | 5.79 | 5857 | 42 |
| 19 | 7490 | 4.88 | 2999 | | 4490 | 5.79 | 5510 | 41 |
| 20 | 7783 | 4.88 | 2945 | | 4838 | 5.79 | 5162 | 40 |
| 21 | 9.598075 | 4.87 | 9.962890 | | 9.635185 | 5.78 | 10.364815 | 39 |
| 22 | 8368 | 4.87 | 2836 | | 5532 | 5.78 | 4468 | 38 |
| 23 | 8660 | 4.87 | 2781 | | 5879 | 5.78 | 4121 | 37 |
| 24 | 8952 | 4.86 | 2727 | | 6226 | 5.77 | 3774 | 36 |
| 25 | 9244 | 4.86 | 2672 | | 6572 | 5.77 | 3428 | 35 |
| 26 | 9536 | 4.85 | 2617 | | 6919 | 5.77 | 3081 | 34 |
| 27 | 9.599827 | 4.85 | 2562 | | 7265 | 5.77 | 2735 | 33 |
| 28 | 9.600118 | 4.85 | 2508 | | 7611 | 5.76 | 2389 | 32 |
| 29 | 0409 | 4.84 | 2453 | .91 | 7956 | 5.76 | 2044 | 31 |
| 30 | 0700 | 4.84 | 2398 | .92 | 8302 | 5.76 | 1698 | 30 |
| 31 | 0990 | 4.84 | 9.962343 | | 8647 | 5.75 | 1353 | 29 |
| 32 | 1280 | 4.83 | 2288 | | 8992 | 5.75 | 1008 | 28 |
| 33 | 1570 | 4.83 | 2233 | | 9337 | 5.75 | 0663 | 27 |
| 34 | 1860 | 4.82 | 2178 | | 9.639682 | 5.74 | 10.360318 | 26 |
| 35 | 2150 | 4.82 | 2123 | | 9.640027 | 5.74 | 10.359973 | 25 |
| 36 | 2439 | 4.82 | 2067 | | 0371 | 5.74 | 9629 | 24 |
| 37 | 2728 | 4.81 | 2012 | | 0716 | 5.73 | 9284 | 23 |
| 38 | 3017 | 4.81 | 1957 | | 1060 | 5.73 | 8940 | 22 |
| 39 | 3305 | 4.81 | 1902 | | 1404 | 5.73 | 8596 | 21 |
| 40 | 3594 | 4.80 | 1846 | | 1747 | 5.72 | 8253 | 20 |
| 41 | 9.603882 | 4.80 | 9.961791 | | 2091 | 5.72 | 7909 | 19 |
| 42 | 4170 | 4.79 | 1735 | | 2434 | 5.72 | 7566 | 18 |
| 43 | 4457 | 4.79 | 1680 | .92 | 2777 | 5.72 | 7223 | 17 |
| 44 | 4745 | 4.79 | 1624 | .93 | 3120 | 5.71 | 6880 | 16 |
| 45 | 5032 | 4.78 | 1569 | | 3463 | 5.71 | 6537 | 15 |
| 46 | 5319 | 4.78 | 1513 | | 3806 | 5.71 | 6194 | 14 |
| 47 | 5606 | 4.78 | 1458 | | 4148 | 5.70 | 5852 | 13 |
| 48 | 5892 | 4.77 | 1402 | | 4490 | 5.70 | 5510 | 12 |
| 49 | 6179 | 4.77 | 1346 | | 4832 | 5.70 | 5168 | 11 |
| 50 | 6465 | 4.76 | 1290 | | 5174 | 5.69 | 4826 | 10 |
| 51 | 9.606751 | 4.76 | 9.961235 | | 9.645516 | 5.69 | 10.354484 | 9 |
| 52 | 7036 | 4.76 | 1179 | | 5857 | 5.69 | 4143 | 8 |
| 53 | 7322 | 4.75 | 1123 | | 6199 | 5.69 | 3801 | 7 |
| 54 | 7607 | 4.75 | 1067 | | 6540 | 5.68 | 3460 | 6 |
| 55 | 7892 | 4.74 | 1011 | | 6881 | 5.68 | 3119 | 5 |
| 56 | 8177 | 4.74 | 0955 | | 7222 | 5.68 | 2778 | 4 |
| 57 | 8461 | 4.74 | 0899 | .93 | 7562 | 5.67 | 2438 | 3 |
| 58 | 8745 | 4.73 | 0843 | .94 | 7903 | 5.67 | 2097 | 2 |
| 59 | 9029 | 4.73 | 0786 | .94 | 8243 | 5.67 | 1757 | 1 |
| 60 | 9.609313 | | 9.960730 | | 9.648583 | | 10.351417 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 66° |

| 24° | LOGARITHMIC | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
| 0 | 9.609313 | 4.73 | 9.960730 | .94 | 9.648583 | 5.66 | 10.351417 | 60 |
| 1 | 9597 | 4.72 | 0674 | | 8923 | 5.66 | 1077 | 59 |
| 2 | 9.609880 | 4.72 | 0618 | | 9263 | 5.66 | 0737 | 58 |
| 3 | 9.610164 | 4.72 | 0561 | | 9602 | 5.66 | 0398 | 57 |
| 4 | 0447 | 4.71 | 0505 | | 9.649942 | 5.65 | 10.350058 | 56 |
| 5 | 0729 | 4.71 | 0448 | | 9.650281 | 5.65 | 10.349719 | 55 |
| 6 | 1012 | 4.70 | 0392 | | 0620 | 5.65 | 9380 | 54 |
| 7 | 1294 | 4.70 | 0335 | | 0959 | 5.64 | 9041 | 53 |
| 8 | 1576 | 4.70 | 0279 | | 1297 | 5.64 | 8703 | 52 |
| 9 | 1858 | 4.69 | 0222 | | 1636 | 5.64 | 8364 | 51 |
| 10 | 2140 | 4.69 | 0165 | .94 | 1974 | 5.63 | 8026 | 50 |
| 11 | 9.612421 | 4.69 | 0109 | .95 | 9.652312 | 5.63 | 10.347688 | 49 |
| 12 | 2702 | 4.68 | 9.960052 | | 2650 | 5.63 | 7350 | 48 |
| 13 | 2983 | 4.68 | 9.959995 | | 2988 | 5.63 | 7012 | 47 |
| 14 | 3264 | 4.67 | 9938 | | 3326 | 5.62 | 6674 | 46 |
| 15 | 3545 | 4.67 | 9882 | | 3663 | 5.62 | 6337 | 45 |
| 16 | 3825 | 4.67 | 9825 | | 4000 | 5.62 | 6000 | 44 |
| 17 | 4105 | 4.66 | 9768 | | 4337 | 5.61 | 5663 | 43 |
| 18 | 4385 | 4.66 | 9711 | | 4674 | 5.61 | 5326 | 42 |
| 19 | 4665 | 4.66 | 9654 | | 5011 | 5.61 | 4989 | 41 |
| 20 | 4944 | 4.65 | 9596 | | 5348 | 5.61 | 4652 | 40 |
| 21 | 9.615223 | 4.65 | 9.959539 | | 9.655684 | 5.60 | 10.344316 | 39 |
| 22 | 5502 | 4.65 | 9482 | | 6020 | 5.60 | 3980 | 38 |
| 23 | 5781 | 4.64 | 9425 | | 6356 | 5.60 | 3644 | 37 |
| 24 | 6060 | 4.64 | 9368 | .95 | 6692 | 5.59 | 3308 | 36 |
| 25 | 6338 | 4.64 | 9310 | .96 | 7028 | 5.59 | 2972 | 35 |
| 26 | 6616 | 4.63 | 9253 | | 7364 | 5.59 | 2636 | 34 |
| 27 | 6894 | 4.63 | 9195 | | 7699 | 5.59 | 2301 | 33 |
| 28 | 7172 | 4.62 | 9138 | | 8034 | 5.58 | 1966 | 32 |
| 29 | 7450 | 4.62 | 9081 | | 8369 | 5.58 | 1631 | 31 |
| 30 | 7727 | 4.62 | 9023 | | 8704 | 5.58 | 1296 | 30 |
| 31 | 9.618004 | 4.61 | 9.958965 | | 9039 | 5.58 | 0961 | 29 |
| 32 | 8281 | 4.61 | 8908 | | 9373 | 5.57 | 0627 | 28 |
| 33 | 8558 | 4.61 | 8850 | | 9.659708 | 5.57 | 10.340292 | 27 |
| 34 | 8834 | 4.60 | 8792 | | 9.660042 | 5.57 | 10.339958 | 26 |
| 35 | 9110 | 4.60 | 8734 | | 0376 | 5.57 | 9624 | 25 |
| 36 | 9386 | 4.60 | 8677 | | 0710 | 5.56 | 9290 | 24 |
| 37 | 9662 | 4.59 | 8619 | | 1043 | 5.56 | 8957 | 23 |
| 38 | 9.619938 | 4.59 | 8561 | .96 | 1377 | 5.56 | 8623 | 22 |
| 39 | 9.620213 | 4.59 | 8503 | .97 | 1710 | 5.55 | 8290 | 21 |
| 40 | 0488 | 4.58 | 8445 | | 2043 | 5.55 | 7957 | 20 |
| 41 | 0763 | 4.58 | 9.958387 | | 9.662376 | 5.55 | 10.337624 | 19 |
| 42 | 1038 | 4.57 | 8329 | | 2709 | 5.54 | 7291 | 18 |
| 43 | 1313 | 4.57 | 8271 | | 3042 | 5.54 | 6958 | 17 |
| 44 | 1587 | 4.57 | 8213 | | 3375 | 5.54 | 6625 | 16 |
| 45 | 1861 | 4.56 | 8154 | | 3707 | 5.54 | 6293 | 15 |
| 46 | 2135 | 4.56 | 8096 | | 4039 | 5.53 | 5961 | 14 |
| 47 | 2409 | 4.56 | 8038 | | 4371 | 5.53 | 5629 | 13 |
| 48 | 2682 | 4.55 | 7979 | | 4703 | 5.53 | 5297 | 12 |
| 49 | 2956 | 4.55 | 7921 | | 5035 | 5.53 | 4965 | 11 |
| 50 | 3229 | 4.55 | 7863 | | 5366 | 5.52 | 4634 | 10 |
| 51 | 9.623502 | 4.54 | 9.957804 | .97 | 9.665698 | 5.52 | 10.334302 | 9 |
| 52 | 3774 | 4.54 | 7746 | .98 | 6029 | 5.52 | 3971 | 8 |
| 53 | 4047 | 4.54 | 7687 | | 6360 | 5.51 | 3640 | 7 |
| 54 | 4319 | 4.53 | 7628 | | 6691 | 5.51 | 3309 | 6 |
| 55 | 4591 | 4.53 | 7570 | | 7021 | 5.51 | 2979 | 5 |
| 56 | 4863 | 4.53 | 7511 | | 7352 | 5.51 | 2648 | 4 |
| 57 | 5135 | 4.52 | 7452 | | 7682 | 5.50 | 2318 | 3 |
| 58 | 5406 | 4.52 | 7393 | | 8013 | 5.50 | 1987 | 2 |
| 59 | 5677 | 4.52 | 7335 | .98 | 8343 | 5.50 | 1657 | 1 |
| 60 | 9.625948 | | 9.957276 | | 9.668673 | | 10.331327 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 65° |

## 25° SINES AND TANGENTS.

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.625948 | 4.51 | 9.957276 | .98 | 9.668673 | 5.50 | 10.331327 | 60 |
| 1 | 6219 | 4.51 | 7217 | | 9002 | 5.49 | 0998 | 59 |
| 2 | 6490 | 4.51 | 7158 | | 9332 | 5.49 | 0668 | 58 |
| 3 | 6760 | 4.50 | 7099 | | 9661 | 5.49 | 0339 | 57 |
| 4 | 7030 | 4.50 | 7040 | | 9.669991 | 5.48 | 10.330009 | 56 |
| 5 | 7300 | 4.50 | 6981 | .98 | 9.670320 | 5.48 | 10.329680 | 55 |
| 6 | 7570 | 4.49 | 6921 | .99 | 0649 | 5.48 | 9351 | 54 |
| 7 | 7840 | 4.49 | 6862 | | 0977 | 5.48 | 9023 | 53 |
| 8 | 8109 | 4.49 | 6803 | | 1306 | 5.47 | 8694 | 52 |
| 9 | 8378 | 4.48 | 6744 | | 1635 | 5.47 | 8365 | 51 |
| 10 | 8647 | 4.48 | 6684 | | 1963 | 5.47 | 8037 | 50 |
| 11 | 8916 | 4.47 | 9.956625 | | 9.672291 | 5.47 | 10.327709 | 49 |
| 12 | 9185 | 4.47 | 6566 | | 2619 | 5.46 | 7381 | 48 |
| 13 | 9453 | 4.47 | 6506 | | 2947 | 5.46 | 7053 | 47 |
| 14 | 9721 | 4.46 | 6447 | | 3274 | 5.46 | 6726 | 46 |
| 15 | 9.629989 | 4.46 | 6387 | | 3602 | 5.46 | 6398 | 45 |
| 16 | 9.630257 | 4.46 | 6327 | | 3929 | 5.45 | 6071 | 44 |
| 17 | 0524 | 4.46 | 6268 | .99 | 4257 | 5.45 | 5743 | 43 |
| 18 | 0792 | 4.45 | 6208 | 1.00 | 4584 | 5.45 | 5416 | 42 |
| 19 | 1059 | 4.45 | 6148 | | 4911 | 5.44 | 5089 | 41 |
| 20 | 1326 | 4.45 | 6089 | | 5237 | 5.44 | 4763 | 40 |
| 21 | 9.631593 | 4.44 | 9.956029 | | 9.675564 | 5.44 | 10.324436 | 39 |
| 22 | 1859 | 4.44 | 5969 | | 5890 | 5.44 | 4110 | 38 |
| 23 | 2125 | 4.44 | 5909 | | 6216 | 5.43 | 3784 | 37 |
| 24 | 2392 | 4.43 | 5849 | | 6543 | 5.43 | 3457 | 36 |
| 25 | 2658 | 4.43 | 5789 | | 6869 | 5.43 | 3131 | 35 |
| 26 | 2923 | 4.43 | 5729 | | 7194 | 5.43 | 2806 | 34 |
| 27 | 3189 | 4.42 | 5669 | | 7520 | 5.42 | 2480 | 33 |
| 28 | 3454 | 4.42 | 5609 | | 7846 | 5.42 | 2154 | 32 |
| 29 | 3719 | 4.42 | 5548 | | 8171 | 5.42 | 1829 | 31 |
| 30 | 3984 | 4.41 | 5488 | 1.00 | 8496 | 5.42 | 1504 | 30 |
| 31 | 9.634249 | 4.41 | 9.955428 | 1.01 | 8821 | 5.41 | 1179 | 29 |
| 32 | 4514 | 4.40 | 5368 | | 9146 | 5.41 | 0854 | 28 |
| 33 | 4778 | 4.40 | 5307 | | 9471 | 5.41 | 0529 | 27 |
| 34 | 5042 | 4.40 | 5247 | | 9.679795 | 5.41 | 10.320205 | 26 |
| 35 | 5306 | 4.39 | 5186 | | 9.680120 | 5.40 | 10.319880 | 25 |
| 36 | 5570 | 4.39 | 5126 | | 0444 | 5.40 | 9556 | 24 |
| 37 | 5834 | 4.39 | 5065 | | 0768 | 5.40 | 9232 | 23 |
| 38 | 6097 | 4.39 | 5005 | | 1092 | 5.40 | 8908 | 22 |
| 39 | 6360 | 4.38 | 4944 | | 1416 | 5.39 | 8584 | 21 |
| 40 | 6623 | 4.38 | 4883 | | 1740 | 5.39 | 8260 | 20 |
| 41 | 9.636886 | 4.37 | 9.954823 | | 9.682063 | 5.39 | 10.317937 | 19 |
| 42 | 7148 | 4.37 | 4762 | | 2387 | 5.39 | 7613 | 18 |
| 43 | 7411 | 4.37 | 4701 | | 2710 | 5.38 | 7290 | 17 |
| 44 | 7673 | 4.37 | 4640 | | 3033 | 5.38 | 6967 | 16 |
| 45 | 7935 | 4.36 | 4579 | 1.01 | 3356 | 5.38 | 6644 | 15 |
| 46 | 8197 | 4.36 | 4518 | 1.02 | 3679 | 5.38 | 6321 | 14 |
| 47 | 8458 | 4.36 | 4457 | | 4001 | 5.37 | 5999 | 13 |
| 48 | 8720 | 4.35 | 4396 | | 4324 | 5.37 | 5676 | 12 |
| 49 | 8981 | 4.35 | 4335 | | 4646 | 5.37 | 5354 | 11 |
| 50 | 9242 | 4.35 | 4274 | | 4968 | 5.37 | 5032 | 10 |
| 51 | 9503 | 4.34 | 9.954213 | | 9.685290 | 5.36 | 10.314710 | 9 |
| 52 | 9.639764 | 4.34 | 4152 | | 5612 | 5.36 | 4388 | 8 |
| 53 | 9.640024 | 4.34 | 4090 | | 5934 | 5.36 | 4066 | 7 |
| 54 | 0284 | 4.33 | 4029 | | 6255 | 5.36 | 3745 | 6 |
| 55 | 0544 | 4.33 | 3968 | | 6577 | 5.35 | 3423 | 5 |
| 56 | 0804 | 4.33 | 3906 | | 6898 | 5.35 | 3102 | 4 |
| 57 | 1064 | 4.32 | 3845 | | 7219 | 5.35 | 2781 | 3 |
| 58 | 1324 | 4.32 | 3783 | 1.02 | 7540 | 5.35 | 2460 | 2 |
| 59 | 1583 | 4.32 | 3722 | 1.03 | 7861 | 5.34 | 2139 | 1 |
| 60 | 9,641842 | | 9.953660 | | 9.688182 | | 10.311818 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 64° |

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.641842 | 4.31 | 9.953660 | 1.03 | 9.688182 | 5.34 | 10.311818 | 60 |
| 1 | 2101 | 4.31 | 3599 | | 8502 | 5.34 | 1498 | 59 |
| 2 | 2360 | 4.31 | 3537 | | 8823 | 5.34 | 1177 | 58 |
| 3 | 2618 | 4.30 | 3475 | | 9143 | 5.33 | 0857 | 57 |
| 4 | 2877 | 4.30 | 3413 | | 9463 | 5.33 | 0537 | 56 |
| 5 | 3135 | 4.30 | 3352 | | 9.689783 | 5.33 | 10.310217 | 55 |
| 6 | 3393 | 4.30 | 3290 | | 9.690103 | 5.33 | 10.309897 | 54 |
| 7 | 3650 | 4.29 | 3228 | | 0423 | 5.33 | 9577 | 53 |
| 8 | 3908 | 4.29 | 3166 | | 0742 | 5.32 | 9258 | 52 |
| 9 | 4165 | 4.29 | 3104 | | 1062 | 5.32 | 8938 | 51 |
| 10 | 4423 | 4.28 | 3042 | 1.03 | 1381 | 5.32 | 8619 | 50 |
| 11 | 9.644680 | 4.28 | 9.952980 | 1.04 | 9.691700 | 5.31 | 10.308300 | 49 |
| 12 | 4936 | 4.28 | 2918 | | 2019 | 5.31 | 7981 | 48 |
| 13 | 5193 | 4.27 | 2855 | | 2338 | 5.31 | 7662 | 47 |
| 14 | 5450 | 4.27 | 2793 | | 2656 | 5.31 | 7344 | 46 |
| 15 | 5706 | 4.27 | 2731 | | 2975 | 5.31 | 7025 | 45 |
| 16 | 5962 | 4.26 | 2669 | | 3293 | 5.30 | 6707 | 44 |
| 17 | 6218 | 4.26 | 2606 | | 3612 | 5.30 | 6388 | 43 |
| 18 | 6474 | 4.26 | 2544 | | 3930 | 5.30 | 6070 | 42 |
| 19 | 6729 | 4.26 | 2481 | | 4248 | 5.30 | 5752 | 41 |
| 20 | 6984 | 4.25 | 2419 | | 4566 | 5.29 | 5434 | 40 |
| 21 | 9.647240 | 4.25 | 9.952356 | | 9.694883 | 5.29 | 10.305117 | 39 |
| 22 | 7494 | 4.24 | 2294 | | 5201 | 5.29 | 4799 | 38 |
| 23 | 7749 | 4.24 | 2231 | 1.04 | 5518 | 5.29 | 4482 | 37 |
| 24 | 8004 | 4.24 | 2168 | 1.05 | 5836 | 5.29 | 4164 | 36 |
| 25 | 8258 | 4.24 | 2106 | | 6153 | 5.28 | 3847 | 35 |
| 26 | 8512 | 4.23 | 2043 | | 6470 | 5.28 | 3530 | 34 |
| 27 | 8766 | 4.23 | 1980 | | 6787 | 5.28 | 3213 | 33 |
| 28 | 9020 | 4.23 | 1917 | | 7103 | 5.28 | 2897 | 32 |
| 29 | 9274 | 4.22 | 1854 | | 7420 | 5.27 | 2580 | 31 |
| 30 | 9527 | 4.22 | 1791 | | 7736 | 5.27 | 2264 | 30 |
| 31 | 9.649781 | 4.22 | 9.951728 | | 9.698053 | 5.27 | 10.301947 | 29 |
| 32 | 9.650034 | 4.22 | 1665 | | 8369 | 5.27 | 1631 | 28 |
| 33 | 0287 | 4.21 | 1602 | | 8685 | 5.26 | 1315 | 27 |
| 34 | 0539 | 4.21 | 1539 | | 9001 | 5.26 | 0999 | 26 |
| 35 | 0792 | 4.21 | 1476 | | 9316 | 5.26 | 0684 | 25 |
| 36 | 1044 | 4.20 | 1412 | 1.05 | 9632 | 5.26 | 0368 | 24 |
| 37 | 1297 | 4.20 | 1349 | 1.06 | 9.699947 | 5.26 | 10.300053 | 23 |
| 38 | 1549 | 4.20 | 1286 | | 9.700263 | 5.25 | 10.299737 | 22 |
| 39 | 1800 | 4.19 | 1222 | | 0578 | 5.25 | 9422 | 21 |
| 40 | 2052 | 4.19 | 1159 | | 0893 | 5.25 | 9107 | 20 |
| 41 | 9.652304 | 4.19 | 9.951096 | | 1208 | 5.24 | 8792 | 19 |
| 42 | 2555 | 4.18 | 1032 | | 1523 | 5.24 | 8477 | 18 |
| 43 | 2806 | 4.18 | 0968 | | 1837 | 5.24 | 8163 | 17 |
| 44 | 3057 | 4.18 | 0905 | | 2152 | 5.24 | 7848 | 16 |
| 45 | 3308 | 4.18 | 0841 | | 2466 | 5.24 | 7534 | 15 |
| 46 | 3558 | 4.17 | 0778 | | 2781 | 5.23 | 7219 | 14 |
| 47 | 3808 | 4.17 | 0714 | | 3095 | 5.23 | 6905 | 13 |
| 48 | 4059 | 4.17 | 0650 | | 3409 | 5.23 | 6591 | 12 |
| 49 | 4309 | 4.16 | 0586 | 1.06 | 3722 | 5.23 | 6278 | 11 |
| 50 | 4558 | 4.16 | 0522 | 1.07 | 4036 | 5.22 | 5964 | 10 |
| 51 | 9.654808 | 4.16 | 9.950458 | | 9.704350 | 5.22 | 10.295650 | 9 |
| 52 | 5058 | 4.16 | 0394 | | 4663 | 5.22 | 5337 | 8 |
| 53 | 5307 | 4.15 | 0330 | | 4976 | 5.22 | 5024 | 7 |
| 54 | 5556 | 4.15 | 0266 | | 5290 | 5.22 | 4710 | 6 |
| 55 | 5805 | 4.15 | 0202 | | 5603 | 5.21 | 4397 | 5 |
| 56 | 6054 | 4.14 | 0138 | | 5916 | 5.21 | 4084 | 4 |
| 57 | 6302 | 4.14 | 0074 | | 6228 | 5.21 | 3772 | 3 |
| 58 | 6551 | 4.14 | 9.950010 | | 6541 | 5.21 | 3459 | 2 |
| 59 | 6799 | 4.13 | 9.949945 | 1.07 | 6854 | 5.21 | 3146 | 1 |
| 60 | 9.657047 | | 9.949881 | | 9.707166 | | 10.292834 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 63° |

## 27° SINES AND TANGENTS.

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.657047 | 4.13 | 9.949881 | 1.07 | 9.707166 | 5.20 | 10.292834 | 60 |
| 1 | 7295 | 4.13 | 9816 | 1.07 | 7478 | 5.20 | 2522 | 59 |
| 2 | 7542 | 4.12 | 9752 | 1.07 | 7790 | 5.20 | 2210 | 58 |
| 3 | 7790 | 4.12 | 9688 | 1.08 | 8102 | 5.20 | 1898 | 57 |
| 4 | 8037 | 4.12 | 9623 | | 8414 | 5.19 | 1586 | 56 |
| 5 | 8284 | 4.12 | 9558 | | 8726 | 5.19 | 1274 | 55 |
| 6 | 8531 | 4.11 | 9494 | | 9037 | 5.19 | 0963 | 54 |
| 7 | 8778 | 4.11 | 9429 | | 9349 | 5.19 | 0651 | 53 |
| 8 | 9025 | 4.11 | 9364 | | 9660 | 5.19 | 0340 | 52 |
| 9 | 9271 | 4.10 | 9300 | | 9.709971 | 5.18 | 10.290029 | 51 |
| 10 | 9517 | 4.10 | 9235 | | 9.710282 | 5.18 | 10.289718 | 50 |
| 11 | 9.659763 | 4.10 | 9.949170 | | 0593 | 5.18 | 9407 | 49 |
| 12 | 9.660009 | 4.09 | 9105 | | 0904 | 5.18 | 9096 | 48 |
| 13 | 0255 | 4.09 | 9040 | | 1215 | 5.18 | 8785 | 47 |
| 14 | 0501 | 4.09 | 8975 | | 1525 | 5.17 | 8475 | 46 |
| 15 | 0746 | 4.09 | 8910 | | 1836 | 5.17 | 8164 | 45 |
| 16 | 0991 | 4.08 | 8845 | 1.08 | 2146 | 5.17 | 7854 | 44 |
| 17 | 1236 | 4.08 | 8780 | 1.09 | 2456 | 5.17 | 7544 | 43 |
| 18 | 1481 | 4.08 | 8715 | | 2766 | 5.16 | 7234 | 42 |
| 19 | 1726 | 4.07 | 8650 | | 3076 | 5.16 | 6924 | 41 |
| 20 | 1970 | 4.07 | 8584 | | 3386 | 5.16 | 6614 | 40 |
| 21 | 9.662214 | 4.07 | 9.948519 | | 9.713696 | 5.16 | 10.286304 | 39 |
| 22 | 2459 | 4.07 | 8454 | | 4005 | 5.16 | 5995 | 38 |
| 23 | 2703 | 4.06 | 8388 | | 4314 | 5.15 | 5686 | 37 |
| 24 | 2946 | 4.06 | 8323 | | 4624 | 5.15 | 5376 | 36 |
| 25 | 3190 | 4.06 | 8257 | | 4933 | 5.15 | 5067 | 35 |
| 26 | 3433 | 4.05 | 8192 | | 5242 | 5.15 | 4758 | 34 |
| 27 | 3677 | 4.05 | 8126 | | 5551 | 5.14 | 4449 | 33 |
| 28 | 3920 | 4.05 | 8060 | 1.09 | 5860 | 5.14 | 4140 | 32 |
| 29 | 4163 | 4.05 | 7995 | 1.10 | 6168 | 5.14 | 3832 | 31 |
| 30 | 4406 | 4.04 | 7929 | | 6477 | 5.14 | 3523 | 30 |
| 31 | 9.664648 | 4.04 | 9.947863 | | 9.716785 | 5.14 | 10.283215 | 29 |
| 32 | 4891 | 4.04 | 7797 | | 7093 | 5.13 | 2907 | 28 |
| 33 | 5133 | 4.03 | 7731 | | 7401 | 5.13 | 2599 | 27 |
| 34 | 5375 | 4.03 | 7665 | | 7709 | 5.13 | 2291 | 26 |
| 35 | 5617 | 4.03 | 7600 | | 8017 | 5.13 | 1983 | 25 |
| 36 | 5859 | 4.02 | 7533 | | 8325 | 5.13 | 1675 | 24 |
| 37 | 6100 | 4.02 | 7467 | | 8633 | 5.12 | 1367 | 23 |
| 38 | 6342 | 4.02 | 7401 | | 8940 | 5.12 | 1060 | 22 |
| 39 | 6583 | 4.02 | 7335 | | 9248 | 5.12 | 0752 | 21 |
| 40 | 6824 | 4.01 | 7269 | | 9555 | 5.12 | 0445 | 20 |
| 41 | 9.667065 | 4.01 | 9.947203 | 1.10 | 9.719862 | 5.12 | 10.280138 | 19 |
| 42 | 7305 | 4.01 | 7136 | 1.11 | 9.720169 | 5.11 | 10.279831 | 18 |
| 43 | 7546 | 4.01 | 7070 | | 0476 | 5.11 | 9524 | 17 |
| 44 | 7786 | 4.00 | 7004 | | 0783 | 5.11 | 9217 | 16 |
| 45 | 8027 | 4.00 | 6937 | | 1089 | 5.11 | 8911 | 15 |
| 46 | 8267 | 4.00 | 6871 | | 1396 | 5.11 | 8604 | 14 |
| 47 | 8506 | 3.99 | 6804 | | 1702 | 5.10 | 8298 | 13 |
| 48 | 8746 | 3.99 | 6738 | | 2009 | 5.10 | 7991 | 12 |
| 49 | 8986 | 3.99 | 6671 | | 2315 | 5.10 | 7685 | 11 |
| 50 | 9225 | 3.99 | 6604 | | 2621 | 5.10 | 7379 | 10 |
| 51 | 9464 | 3.98 | 9.946538 | | 9.722927 | 5.10 | 10.277073 | 9 |
| 52 | 9703 | 3.98 | 6471 | | 3232 | 5.09 | 6768 | 8 |
| 53 | 9.669942 | 3.98 | 6404 | | 3538 | 5.09 | 6462 | 7 |
| 54 | 9.670181 | 3.97 | 6337 | 1.11 | 3844 | 5.09 | 6156 | 6 |
| 55 | 0419 | 3.97 | 6270 | 1.12 | 4149 | 5.09 | 5851 | 5 |
| 56 | 0658 | 3.97 | 6203 | | 4454 | 5.09 | 5546 | 4 |
| 57 | 0896 | 3.97 | 6136 | | 4760 | 5.08 | 5240 | 3 |
| 58 | 1134 | 3.96 | 6069 | | 5065 | 5.08 | 4935 | 2 |
| 59 | 1372 | 3.96 | 6002 | 1.12 | 5370 | 5.08 | 4630 | 1 |
| 60 | 9.671609 | | 9.945935 | | 9.725674 | | 10.274326 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 62° |

| 28° | LOGARITHMIC | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
| 0 | 9.671609 | 3.96 | 9.945935 | 1.12 | 9.725674 | 5.08 | 10.274326 | 60 |
| 1 | 1847 | 3.95 | 5868 | | 5979 | 5.08 | 4021 | 59 |
| 2 | 2084 | 3.95 | 5800 | | 6284 | 5.07 | 3716 | 58 |
| 3 | 2321 | 3.95 | 5733 | | 6588 | 5.07 | 3412 | 57 |
| 4 | 2558 | 3.95 | 5666 | | 6892 | 5.07 | 3108 | 56 |
| 5 | 2795 | 3.94 | 5598 | | 7197 | 5.07 | 2803 | 55 |
| 6 | 3032 | 3.94 | 5531 | 1.12 | 7501 | 5.07 | 2499 | 54 |
| 7 | 3268 | 3.94 | 5464 | 1.13 | 7805 | 5.06 | 2195 | 53 |
| 8 | 3505 | 3.94 | 5396 | | 8109 | 5.06 | 1891 | 52 |
| 9 | 3741 | 3.93 | 5328 | | 8412 | 5.06 | 1588 | 51 |
| 10 | 3977 | 3.93 | 5261 | | 8716 | 5.06 | 1284 | 50 |
| 11 | 9.674213 | 3.93 | 9.945193 | | 9020 | 5.06 | 0980 | 49 |
| 12 | 4448 | 3.92 | 5125 | | 9323 | 5.05 | 0677 | 48 |
| 13 | 4684 | 3.92 | 5058 | | 9626 | 5.05 | 0374 | 47 |
| 14 | 4919 | 3.92 | 4990 | | 9.729929 | 5.05 | 10.270071 | 46 |
| 15 | 5155 | 3.92 | 4922 | | 9.730233 | 5.05 | 10.269767 | 45 |
| 16 | 5390 | 3.91 | 4854 | | 0535 | 5.05 | 9465 | 44 |
| 17 | 5624 | 3.91 | 4786 | | 0838 | 5.04 | 9162 | 43 |
| 18 | 5859 | 3.91 | 4718 | | 1141 | 5.04 | 8859 | 42 |
| 19 | 6094 | 3.91 | 4650 | 1.13 | 1444 | 5.04 | 8556 | 41 |
| 20 | 6328 | 3.90 | 4582 | 1.14 | 1746 | 5.04 | 8254 | 40 |
| 21 | 9.676562 | 3.90 | 9.944514 | | 9.732048 | 5.04 | 10.267952 | 39 |
| 22 | 6796 | 3.90 | 4446 | | 2351 | 5.03 | 7649 | 38 |
| 23 | 7030 | 3.90 | 4377 | | 2653 | 5.03 | 7347 | 37 |
| 24 | 7264 | 3.89 | 4309 | | 2955 | 5.03 | 7045 | 36 |
| 25 | 7498 | 3.89 | 4241 | | 3257 | 5.03 | 6743 | 35 |
| 26 | 7731 | 3.89 | 4172 | | 3558 | 5.03 | 6442 | 34 |
| 27 | 7964 | 3.88 | 4104 | | 3860 | 5.02 | 6140 | 33 |
| 28 | 8197 | 3.88 | 4036 | | 4162 | 5.02 | 5838 | 32 |
| 29 | 8430 | 3.88 | 3967 | | 4463 | 5.02 | 5537 | 31 |
| 30 | 8663 | 3.88 | 3899 | | 4764 | 5.02 | 5236 | 30 |
| 31 | 9.678895 | 3.87 | 9.943830 | | 9.735066 | 5.02 | 10.264934 | 29 |
| 32 | 9128 | 3.87 | 3761 | 1.14 | 5367 | 5.02 | 4633 | 28 |
| 33 | 9360 | 3.87 | 3693 | 1.15 | 5668 | 5.01 | 4332 | 27 |
| 34 | 9592 | 3.87 | 3624 | | 5969 | 5.01 | 4031 | 26 |
| 35 | 9.679824 | 3.86 | 3555 | | 6269 | 5.01 | 3731 | 25 |
| 36 | 9.680056 | 3.86 | 3486 | | 6570 | 5.01 | 3430 | 24 |
| 37 | 0288 | 3.86 | 3417 | | 6871 | 5.01 | 3129 | 23 |
| 38 | 0519 | 3.85 | 3348 | | 7171 | 5.00 | 2829 | 22 |
| 39 | 0750 | 3.85 | 3279 | | 7471 | 5.00 | 2529 | 21 |
| 40 | 0982 | 3.85 | 3210 | | 7771 | 5.00 | 2229 | 20 |
| 41 | 9.681213 | 3.85 | 9.943141 | | 9.738071 | 5.00 | 10.261929 | 19 |
| 42 | 1443 | 3.84 | 3072 | | 8371 | 5.00 | 1629 | 18 |
| 43 | 1674 | 3.84 | 3003 | | 8671 | 4.99 | 1329 | 17 |
| 44 | 1905 | 3.84 | 2934 | | 8971 | 4.99 | 1029 | 16 |
| 45 | 2135 | 3.84 | 2864 | 1.15 | 9271 | 4.99 | 0729 | 15 |
| 46 | 2365 | 3.83 | 2795 | 1.16 | 9570 | 4.99 | 0430 | 14 |
| 47 | 2595 | 3.83 | 2726 | | 9.739870 | 4.99 | 10.260130 | 13 |
| 48 | 2825 | 3.83 | 2656 | | 9.740169 | 4.99 | 10.259831 | 12 |
| 49 | 3055 | 3.83 | 2587 | | 0468 | 4.98 | 9532 | 11 |
| 50 | 3284 | 3.82 | 2517 | | 0767 | 4.98 | 9233 | 10 |
| 51 | 9.683514 | 3.82 | 9.942448 | | 1066 | 4.98 | 8934 | 9 |
| 52 | 3743 | 3.82 | 2378 | | 1365 | 4.98 | 8635 | 8 |
| 53 | 3972 | 3.82 | 2308 | | 1664 | 4.98 | 8336 | 7 |
| 54 | 4201 | 3.81 | 2239 | | 1962 | 4.97 | 8038 | 6 |
| 55 | 4430 | 3.81 | 2169 | | 2261 | 4.97 | 7739 | 5 |
| 56 | 4658 | 3.81 | 2099 | | 2559 | 4.97 | 7441 | 4 |
| 57 | 4887 | 3.80 | 2029 | | 2858 | 4.97 | 7142 | 3 |
| 58 | 5115 | 3.80 | 1959 | 1.16 | 3156 | 4.97 | 6844 | 2 |
| 59 | 5343 | 3.80 | 1889 | 1.17 | 3454 | 4.97 | 6546 | 1 |
| 60 | 9.685571 | | 9.941819 | | 9.743752 | | 10.256248 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 61° |

## 29° SINES AND TANGENTS.

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.685571 | 3.80 | 9.941819 | 1.17 | 9.743752 | 4.96 | 10.256248 | 60 |
| 1 | 5799 | 3.79 | 1749 | | 4050 | 4.96 | 5950 | 59 |
| 2 | 6027 | 3.79 | 1679 | | 4348 | 4.96 | 5652 | 58 |
| 3 | 6254 | 3.79 | 1609 | | 4645 | 4.96 | 5355 | 57 |
| 4 | 6482 | 3.79 | 1539 | | 4943 | 4.96 | 5057 | 56 |
| 5 | 6709 | 3.78 | 1469 | | 5240 | 4.95 | 4760 | 55 |
| 6 | 6936 | 3.78 | 1398 | | 5538 | 4.95 | 4462 | 54 |
| 7 | 7163 | 3.78 | 1328 | | 5835 | 4.95 | 4165 | 53 |
| 8 | 7389 | 3.78 | 1258 | | 6132 | 4.95 | 3868 | 52 |
| 9 | 7616 | 3.77 | 1187 | | 6429 | 4.95 | 3571 | 51 |
| 10 | 7843 | 3.77 | 1117 | 1.17 | 6726 | 4.95 | 3274 | 50 |
| 11 | 9.688069 | 3.77 | 9.941046 | 1.18 | 9.747023 | 4.94 | 10.252977 | 49 |
| 12 | 8295 | 3.77 | 0975 | | 7319 | 4.94 | 2681 | 48 |
| 13 | 8521 | 3.76 | 0905 | | 7616 | 4.94 | 2384 | 47 |
| 14 | 8747 | 3.76 | 0834 | | 7913 | 4.94 | 2087 | 46 |
| 15 | 8972 | 3.76 | 0763 | | 8209 | 4.94 | 1791 | 45 |
| 16 | 9198 | 3.76 | 0693 | | 8505 | 4.93 | 1495 | 44 |
| 17 | 9423 | 3.75 | 0622 | | 8801 | 4.93 | 1199 | 43 |
| 18 | 9648 | 3.75 | 0551 | | 9097 | 4.93 | 0903 | 42 |
| 19 | 9.689873 | 3.75 | 0480 | | 9393 | 4.93 | 0607 | 41 |
| 20 | 9.690098 | 3.75 | 0409 | | 9689 | 4.93 | 0311 | 40 |
| 21 | 0323 | 3.74 | 9.940338 | | 9.749985 | 4.93 | 10.250015 | 39 |
| 22 | 0548 | 3.74 | 0267 | | 9.750281 | 4.93 | 10.249719 | 38 |
| 23 | 0772 | 3.74 | 0196 | 1.18 | 0576 | 4.92 | 9424 | 37 |
| 24 | 0996 | 3.74 | 0125 | 1.19 | 0872 | 4.92 | 9128 | 36 |
| 25 | 1220 | 3.73 | 9.940054 | | 1167 | 4.92 | 8833 | 35 |
| 26 | 1444 | 3.73 | 9.939982 | | 1462 | 4.92 | 8538 | 34 |
| 27 | 1668 | 3.73 | 9911 | | 1757 | 4.92 | 8243 | 33 |
| 28 | 1892 | 3.73 | 9840 | | 2052 | 4.91 | 7948 | 32 |
| 29 | 2115 | 3.72 | 9768 | | 2347 | 4.91 | 7653 | 31 |
| 30 | 2339 | 3.72 | 9697 | | 2642 | 4.91 | 7358 | 30 |
| 31 | 9.692562 | 3.72 | 9.939625 | | 9.752937 | 4.91 | 10.247063 | 29 |
| 32 | 2785 | 3.71 | 9554 | | 3231 | 4.91 | 6769 | 28 |
| 33 | 3008 | 3.71 | 9482 | | 3526 | 4.91 | 6474 | 27 |
| 34 | 3231 | 3.71 | 9410 | | 3820 | 4.90 | 6180 | 26 |
| 35 | 3453 | 3.71 | 9339 | 1.19 | 4115 | 4.90 | 5885 | 25 |
| 36 | 3676 | 3.70 | 9267 | 1.20 | 4409 | 4.90 | 5591 | 24 |
| 37 | 3898 | 3.70 | 9195 | | 4703 | 4.90 | 5297 | 23 |
| 38 | 4120 | 3.70 | 9123 | | 4997 | 4.90 | 5003 | 22 |
| 39 | 4342 | 3.70 | 9052 | | 5291 | 4.90 | 4709 | 21 |
| 40 | 4564 | 3.69 | 8980 | | 5585 | 4.89 | 4415 | 20 |
| 41 | 9.694786 | 3.69 | 9.938908 | | 9.755878 | 4.89 | 10.244122 | 19 |
| 42 | 5007 | 3.69 | 8836 | | 6172 | 4.89 | 3828 | 18 |
| 43 | 5229 | 3.69 | 8763 | | 6465 | 4.89 | 3535 | 17 |
| 44 | 5450 | 3.68 | 8691 | | 6759 | 4.89 | 3241 | 16 |
| 45 | 5671 | 3.68 | 8619 | | 7052 | 4.89 | 2948 | 15 |
| 46 | 5892 | 3.68 | 8547 | | 7345 | 4.88 | 2655 | 14 |
| 47 | 6113 | 3.68 | 8475 | 1.20 | 7638 | 4.88 | 2362 | 13 |
| 48 | 6334 | 3.67 | 8402 | 1.21 | 7931 | 4.88 | 2069 | 12 |
| 49 | 6554 | 3.67 | 8330 | | 8224 | 4.88 | 1776 | 11 |
| 50 | 6775 | 3.67 | 8258 | | 8517 | 4.88 | 1483 | 10 |
| 51 | 9.696995 | 3.67 | 9.938185 | | 9.758810 | 4.88 | 10.241190 | 9 |
| 52 | 7215 | 3.66 | 8113 | | 9102 | 4.87 | 0898 | 8 |
| 53 | 7435 | 3.66 | 8040 | | 9395 | 4.87 | 0605 | 7 |
| 54 | 7654 | 3.66 | 7967 | | 9687 | 4.87 | 0313 | 6 |
| 55 | 7874 | 3.66 | 7895 | | 9.759979 | 4.87 | 10.240021 | 5 |
| 56 | 8094 | 3.65 | 7822 | | 9.760272 | 4.87 | 10.239728 | 4 |
| 57 | 8313 | 3.65 | 7749 | | 0564 | 4.87 | 9436 | 3 |
| 58 | 8532 | 3.65 | 7676 | | 0856 | 4.86 | 9144 | 2 |
| 59 | 8751 | 3.65 | 7604 | 1.21 | 1148 | 4.86 | 8852 | 1 |
| 60 | 9.698970 | | 9.937531 | | 9.761439 | | 10.238561 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 60° |

| 30° | LOGARITHMIC | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
| 0 | 9.698970 | 3.64 | 9.937531 | 1.21 | 9.761439 | 4.86 | 10.238561 | 60 |
| 1 | 9189 | 3.64 | 7458 | 1.22 | 1731 | 4.86 | 8269 | 59 |
| 2 | 9407 | 3.64 | 7385 | | 2023 | 4.86 | 7977 | 58 |
| 3 | 9626 | 3.64 | 7312 | | 2314 | 4.86 | 7686 | 57 |
| 4 | 9.699844 | 3.63 | 7238 | | 2606 | 4.85 | 7394 | 56 |
| 5 | 9.700062 | 3.63 | 7165 | | 2897 | 4.85 | 7103 | 55 |
| 6 | 0280 | 3.63 | 7092 | | 3188 | 4.85 | 6812 | 54 |
| 7 | 0498 | 3.63 | 7019 | | 3479 | 4.85 | 6521 | 53 |
| 8 | 0716 | 3.63 | 6946 | | 3770 | 4.85 | 6230 | 52 |
| 9 | 0933 | 3.62 | 6872 | | 4061 | 4.85 | 5939 | 51 |
| 10 | 1151 | 3.62 | 6799 | | 4352 | 4.84 | 5648 | 50 |
| 11 | 9.701368 | 3.62 | 9.936725 | 1.22 | 9.764643 | 4.84 | 10.235357 | 49 |
| 12 | 1585 | 3.62 | 6652 | 1.23 | 4933 | 4.84 | 5067 | 48 |
| 13 | 1802 | 3.61 | 6578 | | 5224 | 4.84 | 4776 | 47 |
| 14 | 2019 | 3.61 | 6505 | | 5514 | 4.84 | 4486 | 46 |
| 15 | 2236 | 3.61 | 6431 | | 5805 | 4.84 | 4195 | 45 |
| 16 | 2452 | 3.61 | 6357 | | 6095 | 4.84 | 3905 | 44 |
| 17 | 2669 | 3.60 | 6284 | | 6385 | 4.83 | 3615 | 43 |
| 18 | 2885 | 3.60 | 6210 | | 6675 | 4.83 | 3325 | 42 |
| 19 | 3101 | 3.60 | 6136 | | 6965 | 4.83 | 3035 | 41 |
| 20 | 3317 | 3.60 | 6062 | | 7255 | 4.83 | 2745 | 40 |
| 21 | 9.703533 | 3.59 | 9.935988 | | 9.767545 | 4.83 | 10.232455 | 39 |
| 22 | 3749 | 3.59 | 5914 | | 7834 | 4.83 | 2166 | 38 |
| 23 | 3964 | 3.59 | 5840 | 1.23 | 8124 | 4.82 | 1876 | 37 |
| 24 | 4179 | 3.59 | 5766 | 1.24 | 8414 | 4.82 | 1586 | 36 |
| 25 | 4395 | 3.59 | 5692 | | 8703 | 4.82 | 1297 | 35 |
| 26 | 4610 | 3.58 | 5618 | | 8992 | 4.82 | 1008 | 34 |
| 27 | 4825 | 3.58 | 5543 | | 9281 | 4.82 | 0719 | 33 |
| 28 | 5040 | 3.58 | 5469 | | 9571 | 4.82 | 0429 | 32 |
| 29 | 5254 | 3.58 | 5395 | | 9.769860 | 4.81 | 10.230140 | 31 |
| 30 | 5469 | 3.57 | 5320 | | 9.770148 | 4.81 | 10.229852 | 30 |
| 31 | 9.705683 | 3.57 | 9.935246 | | 0437 | 4.81 | 9563 | 29 |
| 32 | 5898 | 3.57 | 5171 | | 0726 | 4.81 | 9274 | 28 |
| 33 | 6112 | 3.57 | 5097 | | 1015 | 4.81 | 8985 | 27 |
| 34 | 6326 | 3.56 | 5022 | | 1303 | 4.81 | 8697 | 26 |
| 35 | 6539 | 3.56 | 4948 | | 1592 | 4.81 | 8408 | 25 |
| 36 | 6753 | 3.56 | 4873 | 1.24 | 1880 | 4.80 | 8120 | 24 |
| 37 | 6967 | 3.56 | 4798 | 1.25 | 2168 | 4.80 | 7832 | 23 |
| 38 | 7180 | 3.55 | 4723 | | 2457 | 4.80 | 7543 | 22 |
| 39 | 7393 | 3.55 | 4649 | | 2745 | 4.80 | 7255 | 21 |
| 40 | 7606 | 3.55 | 4574 | | 3033 | 4.80 | 6967 | 20 |
| 41 | 9.707819 | 3.55 | 9.934499 | | 9.773321 | 4.80 | 10.226679 | 19 |
| 42 | 8032 | 3.54 | 4424 | | 3608 | 4.79 | 6392 | 18 |
| 43 | 8245 | 3.54 | 4349 | | 3896 | 4.79 | 6104 | 17 |
| 44 | 8458 | 3.54 | 4274 | | 4184 | 4.79 | 5816 | 16 |
| 45 | 8670 | 3.54 | 4199 | | 4471 | 4.79 | 5529 | 15 |
| 46 | 8882 | 3.53 | 4123 | | 4759 | 4.79 | 5241 | 14 |
| 47 | 9094 | 3.53 | 4048 | | 5046 | 4.79 | 4954 | 13 |
| 48 | 9306 | 3.53 | 3973 | 1.25 | 5333 | 4.79 | 4667 | 12 |
| 49 | 9518 | 3.53 | 3898 | 1.26 | 5621 | 4.78 | 4379 | 11 |
| 50 | 9730 | 3.53 | 3822 | | 5908 | 4.78 | 4092 | 10 |
| 51 | 9.709941 | 3.52 | 9.933747 | | 9.776195 | 4.78 | 10.223805 | 9 |
| 52 | 9.710153 | 3.52 | 3671 | | 6482 | 4.78 | 3518 | 8 |
| 53 | 0364 | 3.52 | 3596 | | 6768 | 4.78 | 3232 | 7 |
| 54 | 0575 | 3.52 | 3520 | | 7055 | 4.78 | 2945 | 6 |
| 55 | 0786 | 3.51 | 3445 | | 7342 | 4.78 | 2658 | 5 |
| 56 | 0997 | 3.51 | 3369 | | 7628 | 4.77 | 2372 | 4 |
| 57 | 1208 | 3.51 | 3293 | | 7915 | 4.77 | 2085 | 3 |
| 58 | 1419 | 3.51 | 3217 | | 8201 | 4.77 | 1799 | 2 |
| 59 | 1629 | 3.50 | 3141 | 1.26 | 8488 | 4.77 | 1512 | 1 |
| 60 | 9.711839 | | 9.933066 | | 9.778774 | | 10.221226 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 59° |

## 31° SINES AND TANGENTS.

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.711839 | 3.50 | 9.933066 | 1.26 | 9.778774 | 4.77 | 10.221226 | 60 |
| 1 | 2050 | 3.50 | 2990 | 1.27 | 9060 | 4.77 | 0940 | 59 |
| 2 | 2260 | 3.50 | 2914 | | 9346 | 4.76 | 0654 | 58 |
| 3 | 2469 | 3.49 | 2838 | | 9632 | 4.76 | 0368 | 57 |
| 4 | 2679 | 3.49 | 2762 | | 9.779918 | 4.76 | 10.220082 | 56 |
| 5 | 2889 | 3.49 | 2685 | | 9.780203 | 4.76 | 10.219797 | 55 |
| 6 | 3098 | 3.49 | 2609 | | 0489 | 4.76 | 9511 | 54 |
| 7 | 3308 | 3.49 | 2533 | | 0775 | 4.76 | 9225 | 53 |
| 8 | 3517 | 3.48 | 2457 | | 1060 | 4.76 | 8940 | 52 |
| 9 | 3726 | 3.48 | 2380 | | 1346 | 4.75 | 8654 | 51 |
| 10 | 3935 | 3.48 | 2304 | | 1631 | 4.75 | 8369 | 50 |
| 11 | 9.714144 | 3.48 | 9.932228 | | 9.781916 | 4.75 | 10.218084 | 49 |
| 12 | 4352 | 3.47 | 2151 | 1.27 | 2201 | 4.75 | 7799 | 48 |
| 13 | 4561 | 3.47 | 2075 | 1.28 | 2486 | 4.75 | 7514 | 47 |
| 14 | 4769 | 3.47 | 1998 | | 2771 | 4.75 | 7229 | 46 |
| 15 | 4978 | 3.47 | 1921 | | 3056 | 4.75 | 6944 | 45 |
| 16 | 5186 | 3.47 | 1845 | | 3341 | 4.75 | 6659 | 44 |
| 17 | 5394 | 3.46 | 1768 | | 3626 | 4.74 | 6374 | 43 |
| 18 | 5602 | 3.46 | 1691 | | 3910 | 4.74 | 6090 | 42 |
| 19 | 5809 | 3.46 | 1614 | | 4195 | 4.74 | 5805 | 41 |
| 20 | 6017 | 3.46 | 1537 | | 4479 | 4.74 | 5521 | 40 |
| 21 | 9.716224 | 3.45 | 9.931460 | | 9.784764 | 4.74 | 10.215236 | 39 |
| 22 | 6432 | 3.45 | 1383 | | 5048 | 4.74 | 4952 | 38 |
| 23 | 6639 | 3.45 | 1306 | 1.28 | 5332 | 4.73 | 4668 | 37 |
| 24 | 6846 | 3.45 | 1229 | 1.29 | 5616 | 4.73 | 4384 | 36 |
| 25 | 7053 | 3.45 | 1152 | | 5900 | 4.73 | 4100 | 35 |
| 26 | 7259 | 3.44 | 1075 | | 6184 | 4.73 | 3816 | 34 |
| 27 | 7466 | 3.44 | 0998 | | 6468 | 4.73 | 3532 | 33 |
| 28 | 7673 | 3.44 | 0921 | | 6752 | 4.73 | 3248 | 32 |
| 29 | 7879 | 3.44 | 0843 | | 7036 | 4.73 | 2964 | 31 |
| 30 | 8085 | 3.43 | 0766 | | 7319 | 4.72 | 2681 | 30 |
| 31 | 9.718291 | 3.43 | 9.930688 | | 9.787603 | 4.72 | 10.212397 | 29 |
| 32 | 8497 | 3.43 | 0611 | | 7886 | 4.72 | 2114 | 28 |
| 33 | 8703 | 3.43 | 0533 | | 8170 | 4.72 | 1830 | 27 |
| 34 | 8909 | 3.43 | 0456 | | 8453 | 4.72 | 1547 | 26 |
| 35 | 9114 | 3.42 | 0378 | 1.29 | 8736 | 4.72 | 1264 | 25 |
| 36 | 9320 | 3.42 | 0300 | 1.30 | 9019 | 4.72 | 0981 | 24 |
| 37 | 9525 | 3.42 | 0223 | | 9302 | 4.71 | 0698 | 23 |
| 38 | 9730 | 3.42 | 0145 | | 9585 | 4.71 | 0415 | 22 |
| 39 | 9.719935 | 3.41 | 9.930067 | | 9.789868 | 4.71 | 10.210132 | 21 |
| 40 | 9.720140 | 3.41 | 9.929989 | | 9.790151 | 4.71 | 10.209849 | 20 |
| 41 | 0345 | 3.41 | 9911 | | 0434 | 4.71 | 9566 | 19 |
| 42 | 0549 | 3.41 | 9833 | | 0716 | 4.71 | 9284 | 18 |
| 43 | 0754 | 3.40 | 9755 | | 0999 | 4.71 | 9001 | 17 |
| 44 | 0958 | 3.40 | 9677 | | 1281 | 4.71 | 8719 | 16 |
| 45 | 1162 | 3.40 | 9599 | | 1563 | 4.70 | 8437 | 15 |
| 46 | 1366 | 3.40 | 9521 | | 1846 | 4.70 | 8154 | 14 |
| 47 | 1570 | 3.40 | 9442 | 1.30 | 2128 | 4.70 | 7872 | 13 |
| 48 | 1774 | 3.39 | 9364 | 1.31 | 2410 | 4.70 | 7590 | 12 |
| 49 | 1978 | 3.39 | 9286 | | 2692 | 4.70 | 7308 | 11 |
| 50 | 2181 | 3.39 | 9207 | | 2974 | 4.70 | 7026 | 10 |
| 51 | 9.722385 | 3.39 | 9.929129 | | 9.793256 | 4.70 | 10.206744 | 9 |
| 52 | 2588 | 3.39 | 9050 | | 3538 | 4.69 | 6462 | 8 |
| 53 | 2791 | 3.38 | 8972 | | 3819 | 4.69 | 6181 | 7 |
| 54 | 2994 | 3.38 | 8893 | | 4101 | 4.69 | 5899 | 6 |
| 55 | 3197 | 3.38 | 8815 | | 4383 | 4.69 | 5617 | 5 |
| 56 | 3400 | 3.38 | 8736 | | 4664 | 4.69 | 5336 | 4 |
| 57 | 3603 | 3.37 | 8657 | | 4946 | 4.69 | 5054 | 3 |
| 58 | 3805 | 3.37 | 8578 | | 5227 | 4.69 | 4773 | 2 |
| 59 | 4007 | 3.37 | 8499 | 1.31 | 5508 | 4.68 | 4492 | 1 |
| 60 | 9.724210 | | 9.928420 | | 9.795789 | | 10.204211 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 58° |

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.724210 | 3.37 | 9.928420 | 1.32 | 9.795789 | 4.68 | 10.204211 | 60 |
| 1 | 4412 | 3.37 | 8342 | | 6070 | 4.68 | 3930 | 59 |
| 2 | 4614 | 3.36 | 8263 | | 6351 | 4.68 | 3649 | 58 |
| 3 | 4816 | 3.36 | 8183 | | 6632 | 4.68 | 3368 | 57 |
| 4 | 5017 | 3.36 | 8104 | | 6913 | 4.68 | 3087 | 56 |
| 5 | 5219 | 3.36 | 8025 | | 7194 | 4.68 | 2806 | 55 |
| 6 | 5420 | 3.35 | 7946 | | 7476 | 4.68 | 2524 | 54 |
| 7 | 5622 | 3.35 | 7867 | | 7755 | 4.68 | 2245 | 53 |
| 8 | 5823 | 3.35 | 7787 | | 8036 | 4.67 | 1964 | 52 |
| 9 | 6024 | 3.35 | 7708 | | 8316 | 4.67 | 1684 | 51 |
| 10 | 6225 | 3.35 | 7629 | | 8596 | 4.67 | 1404 | 50 |
| 11 | 9.726426 | 3.34 | 9.927549 | 1.32 | 8877 | 4.67 | 1123 | 49 |
| 12 | 6626 | 3.34 | 7470 | 1.33 | 9157 | 4.67 | 0843 | 48 |
| 13 | 6827 | 3.34 | 7390 | | 9437 | 4.67 | 0563 | 47 |
| 14 | 7027 | 3.34 | 7310 | | 9717 | 4.67 | 0283 | 46 |
| 15 | 7228 | 3.34 | 7231 | | 9.799997 | 4.66 | 10.200003 | 45 |
| 16 | 7428 | 3.33 | 7151 | | 9.800277 | 4.66 | 10.199723 | 44 |
| 17 | 7628 | 3.33 | 7071 | | 0557 | 4.66 | 9443 | 43 |
| 18 | 7828 | 3.33 | 6991 | | 0836 | 4.66 | 9164 | 42 |
| 19 | 8027 | 3.33 | 6911 | | 1116 | 4.66 | 8884 | 41 |
| 20 | 8227 | 3.33 | 6831 | | 1396 | 4.66 | 8604 | 40 |
| 21 | 9.728427 | 3.32 | 9.926751 | | 1675 | 4.66 | 8325 | 39 |
| 22 | 8626 | 3.32 | 6671 | | 1955 | 4.66 | 8045 | 38 |
| 23 | 8825 | 3.32 | 6591 | 1.33 | 2234 | 4.65 | 7766 | 37 |
| 24 | 9024 | 3.32 | 6511 | 1.34 | 2513 | 4.65 | 7487 | 36 |
| 25 | 9223 | 3.31 | 6431 | | 2792 | 4.65 | 7208 | 35 |
| 26 | 9422 | 3.31 | 6351 | | 3072 | 4.65 | 6928 | 34 |
| 27 | 9621 | 3.31 | 6270 | | 3351 | 4.65 | 6649 | 33 |
| 28 | 9.729820 | 3.31 | 6190 | | 3630 | 4.65 | 6370 | 32 |
| 29 | 9.730018 | 3.30 | 6110 | | 3909 | 4.65 | 6091 | 31 |
| 30 | 0216 | 3.30 | 6029 | | 4187 | 4.65 | 5813 | 30 |
| 31 | 0415 | 3.30 | 9.925949 | | 9.804466 | 4.64 | 10.195534 | 29 |
| 32 | 0613 | 3.30 | 5868 | | 4745 | 4.64 | 5255 | 28 |
| 33 | 0811 | 3.30 | 5788 | | 5023 | 4.64 | 4977 | 27 |
| 34 | 1009 | 3.29 | 5707 | | 5302 | 4.64 | 4698 | 26 |
| 35 | 1206 | 3.29 | 5626 | 1.34 | 5580 | 4.64 | 4420 | 25 |
| 36 | 1404 | 3.29 | 5545 | 1.35 | 5859 | 4.64 | 4141 | 24 |
| 37 | 1602 | 3.29 | 5465 | | 6137 | 4.64 | 3863 | 23 |
| 38 | 1799 | 3.29 | 5384 | | 6415 | 4.63 | 3585 | 22 |
| 39 | 1996 | 3.28 | 5303 | | 6693 | 4.63 | 3307 | 21 |
| 40 | 2193 | 3.28 | 5222 | | 6971 | 4.63 | 3029 | 20 |
| 41 | 9.732390 | 3.28 | 9.925141 | | 9.807249 | 4.63 | 10.192751 | 19 |
| 42 | 2587 | 3.28 | 5060 | | 7527 | 4.63 | 2473 | 18 |
| 43 | 2784 | 3.28 | 4979 | | 7805 | 4.63 | 2195 | 17 |
| 44 | 2980 | 3.27 | 4897 | | 8083 | 4.63 | 1917 | 16 |
| 45 | 3177 | 3.27 | 4816 | 1.35 | 8361 | 4.63 | 1639 | 15 |
| 46 | 3373 | 3.27 | 4735 | 1.36 | 8638 | 4.62 | 1362 | 14 |
| 47 | 3569 | 3.27 | 4654 | | 8916 | 4.62 | 1084 | 13 |
| 48 | 3765 | 3.27 | 4572 | | 9193 | 4.62 | 0807 | 12 |
| 49 | 3961 | 3.26 | 4491 | | 9471 | 4.62 | 0529 | 11 |
| 50 | 4157 | 3.26 | 4409 | | 9.809748 | 4.62 | 10.190252 | 10 |
| 51 | 9.734353 | 3.26 | 9.924328 | | 9.810025 | 4.62 | 10.189975 | 9 |
| 52 | 4549 | 3.26 | 4246 | | 0302 | 4.62 | 9698 | 8 |
| 53 | 4744 | 3.25 | 4164 | | 0580 | 4.62 | 9420 | 7 |
| 54 | 4939 | 3.25 | 4083 | | 0857 | 4.62 | 9143 | 6 |
| 55 | 5135 | 3.25 | 4001 | | 1134 | 4.61 | 8866 | 5 |
| 56 | 5330 | 3.25 | 3919 | | 1410 | 4.61 | 8590 | 4 |
| 57 | 5525 | 3.25 | 3837 | 1.36 | 1687 | 4.61 | 8313 | 3 |
| 58 | 5719 | 3.24 | 3755 | 1.37 | 1964 | 4.61 | 8036 | 2 |
| 59 | 5914 | 3.24 | 3673 | 1.37 | 2241 | 4.61 | 7759 | 1 |
| 60 | 9.736109 | | 9.923591 | | 9.812517 | | 10.187483 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 57° |

## 33° SINES AND TANGENTS.

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.736109 | 3.24 | 9.923591 | 1.37 | 9.812517 | 4.61 | 10.187483 | 60 |
| 1 | 6303 | 3.24 | 3509 | | 2794 | 4.61 | 7206 | 59 |
| 2 | 6498 | 3.24 | 3427 | | 3070 | 4.61 | 6930 | 58 |
| 3 | 6692 | 3.23 | 3345 | | 3347 | 4.60 | 6653 | 57 |
| 4 | 6886 | 3.23 | 3263 | | 3623 | 4.60 | 6377 | 56 |
| 5 | 7080 | 3.23 | 3181 | | 3899 | 4.60 | 6101 | 55 |
| 6 | 7274 | 3.23 | 3098 | | 4176 | 4.60 | 5824 | 54 |
| 7 | 7467 | 3.23 | 3016 | | 4452 | 4.60 | 5548 | 53 |
| 8 | 7661 | 3.22 | 2933 | | 4728 | 4.60 | 5272 | 52 |
| 9 | 7855 | 3.22 | 2851 | 1.37 | 5004 | 4.60 | 4996 | 51 |
| 10 | 8048 | 3.22 | 2768 | 1.38 | 5280 | 4.60 | 4720 | 50 |
| 11 | 9.738241 | 3.22 | 9.922686 | | 9.815555 | 4.59 | 10.184445 | 49 |
| 12 | 8434 | 3.22 | 2603 | | 5831 | 4.59 | 4169 | 48 |
| 13 | 8627 | 3.21 | 2520 | | 6107 | 4.59 | 3893 | 47 |
| 14 | 8820 | 3.21 | 2438 | | 6382 | 4.59 | 3618 | 46 |
| 15 | 9013 | 3.21 | 2355 | | 6658 | 4.59 | 3342 | 45 |
| 16 | 9206 | 3.21 | 2272 | | 6933 | 4.59 | 3067 | 44 |
| 17 | 9398 | 3.21 | 2189 | | 7209 | 4.59 | 2791 | 43 |
| 18 | 9590 | 3.20 | 2106 | | 7484 | 4.59 | 2516 | 42 |
| 19 | 9783 | 3.20 | 2023 | | 7759 | 4.59 | 2241 | 41 |
| 20 | 9.739975 | 3.20 | 1940 | 1.38 | 8035 | 4.58 | 1965 | 40 |
| 21 | 9.740167 | 3.20 | 9.921857 | 1.39 | 9.818310 | 4.58 | 10.181690 | 39 |
| 22 | 0359 | 3.20 | 1774 | | 8585 | 4.58 | 1415 | 38 |
| 23 | 0550 | 3.19 | 1691 | | 8860 | 4.58 | 1140 | 37 |
| 24 | 0742 | 3.19 | 1607 | | 9135 | 4.58 | 0865 | 36 |
| 25 | 0934 | 3.19 | 1524 | | 9410 | 4.58 | 0590 | 35 |
| 26 | 1125 | 3.19 | 1441 | | 9684 | 4.58 | 0316 | 34 |
| 27 | 1316 | 3.19 | 1357 | | 9.819959 | 4.58 | 10.180041 | 33 |
| 28 | 1508 | 3.18 | 1274 | | 9.820234 | 4.58 | 10.179766 | 32 |
| 29 | 1699 | 3.18 | 1190 | | 0508 | 4.57 | 9492 | 31 |
| 30 | 1889 | 3.18 | 1107 | | 0783 | 4.57 | 9217 | 30 |
| 31 | 9.742080 | 3.18 | 9.921023 | 1.39 | 1057 | 4.57 | 8943 | 29 |
| 32 | 2271 | 3.18 | 0939 | 1.40 | 1332 | 4.57 | 8668 | 28 |
| 33 | 2462 | 3.17 | 0856 | | 1606 | 4.57 | 8394 | 27 |
| 34 | 2652 | 3.17 | 0772 | | 1880 | 4.57 | 8120 | 26 |
| 35 | 2842 | 3.17 | 0688 | | 2154 | 4.57 | 7846 | 25 |
| 36 | 3033 | 3.17 | 0604 | | 2429 | 4.57 | 7571 | 24 |
| 37 | 3223 | 3.17 | 0520 | | 2703 | 4.57 | 7297 | 23 |
| 38 | 3413 | 3.16 | 0436 | | 2977 | 4.56 | 7023 | 22 |
| 39 | 3602 | 3.16 | 0352 | | 3251 | 4.56 | 6749 | 21 |
| 40 | 3792 | 3.16 | 0268 | | 3524 | 4.56 | 6476 | 20 |
| 41 | 9.743982 | 3.16 | 0184 | | 9.823798 | 4.56 | 10.176202 | 19 |
| 42 | 4171 | 3.16 | 0099 | | 4072 | 4.56 | 5928 | 18 |
| 43 | 4361 | 3.15 | 9.920015 | 1.40 | 4345 | 4.56 | 5655 | 17 |
| 44 | 4550 | 3.15 | 9.919931 | 1.41 | 4619 | 4.56 | 5381 | 16 |
| 45 | 4739 | 3.15 | 9846 | | 4893 | 4.56 | 5107 | 15 |
| 46 | 4928 | 3.15 | 9762 | | 5166 | 4.56 | 4834 | 14 |
| 47 | 5117 | 3.15 | 9677 | | 5439 | 4.55 | 4561 | 13 |
| 48 | 5306 | 3.14 | 9593 | | 5713 | 4.55 | 4287 | 12 |
| 49 | 5494 | 3.14 | 9508 | | 5986 | 4.55 | 4014 | 11 |
| 50 | 5683 | 3.14 | 9424 | | 6259 | 4.55 | 3741 | 10 |
| 51 | 9.745871 | 3.14 | 9.919339 | | 9.826532 | 4.55 | 10.173468 | 9 |
| 52 | 6060 | 3.14 | 9254 | | 6805 | 4.55 | 3195 | 8 |
| 53 | 6248 | 3.13 | 9169 | | 7078 | 4.55 | 2922 | 7 |
| 54 | 6436 | 3.13 | 9085 | 1.41 | 7351 | 4.55 | 2649 | 6 |
| 55 | 6624 | 3.13 | 9000 | 1.42 | 7624 | 4.55 | 2376 | 5 |
| 56 | 6812 | 3.13 | 8915 | | 7897 | 4.54 | 2103 | 4 |
| 57 | 6999 | 3.13 | 8830 | | 8170 | 4.54 | 1830 | 3 |
| 58 | 7187 | 3.12 | 8745 | | 8442 | 4.54 | 1558 | 2 |
| 59 | 7374 | 3.12 | 8659 | 1.42 | 8715 | 4.54 | 1285 | 1 |
| 60 | 9.747562 | | 9.918574 | | 9.828987 | | 10.171013 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 56° |

## 34° LOGARITHMIC

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.747562 | 3.12 | 9.918574 | 1.42 | 9.828987 | 4.54 | 10.171013 | 60 |
| 1 | 7749 | 3.12 | 8489 | | 9260 | 4.54 | 0740 | 59 |
| 2 | 7936 | 3.12 | 8404 | | 9532 | 4.54 | 0468 | 58 |
| 3 | 8123 | 3.11 | 8318 | | 9.829805 | 4.54 | 10.170195 | 57 |
| 4 | 8310 | 3.11 | 8232 | | 9.830077 | 4.54 | 10.169923 | 56 |
| 5 | 8497 | 3.11 | 8147 | 1.42 | 0349 | 4.53 | 9651 | 55 |
| 6 | 8683 | 3.11 | 8062 | 1.43 | 0621 | 4.53 | 9379 | 54 |
| 7 | 8870 | 3.11 | 7976 | | 0893 | 4.53 | 9107 | 53 |
| 8 | 9056 | 3.10 | 7891 | | 1165 | 4.53 | 8835 | 52 |
| 9 | 9243 | 3.10 | 7805 | | 1437 | 4.53 | 8563 | 51 |
| 10 | 9429 | 3.10 | 7719 | | 1709 | 4.53 | 8291 | 50 |
| 11 | 9615 | 3.10 | 9.917634 | | 9.831981 | 4.53 | 10.168019 | 49 |
| 12 | 9801 | 3.10 | 7548 | | 2253 | 4.53 | 7747 | 48 |
| 13 | 9.740987 | 3.09 | 7462 | | 2525 | 4.53 | 7475 | 47 |
| 14 | 9.750172 | 3.09 | 7376 | | 2796 | 4.53 | 7204 | 46 |
| 15 | 0358 | 3.09 | 7290 | | 3068 | 4.52 | 6932 | 45 |
| 16 | 0543 | 3.09 | 7204 | 1.43 | 3339 | 4.52 | 6661 | 44 |
| 17 | 0729 | 3.09 | 7118 | 1.44 | 3611 | 4.52 | 6389 | 43 |
| 18 | 0914 | 3.08 | 7032 | | 3882 | 4.52 | 6118 | 42 |
| 19 | 1099 | 3.08 | 6946 | | 4154 | 4.52 | 5846 | 41 |
| 20 | 1284 | 3.08 | 6859 | | 4425 | 4.52 | 5575 | 40 |
| 21 | 9.751469 | 3.08 | 9.916773 | | 9.834696 | 4.52 | 10.165304 | 39 |
| 22 | 1654 | 3.08 | 6687 | | 4967 | 4.52 | 5033 | 38 |
| 23 | 1839 | 3.08 | 6600 | | 5238 | 4.52 | 4762 | 37 |
| 24 | 2023 | 3.07 | 6514 | | 5509 | 4.52 | 4491 | 36 |
| 25 | 2208 | 3.07 | 6427 | | 5780 | 4.51 | 4220 | 35 |
| 26 | 2392 | 3.07 | 6341 | | 6051 | 4.51 | 3949 | 34 |
| 27 | 2576 | 3.07 | 6254 | 1.44 | 6322 | 4.51 | 3678 | 33 |
| 28 | 2760 | 3.07 | 6167 | 1.45 | 6593 | 4.51 | 3407 | 32 |
| 29 | 2944 | 3.06 | 6081 | | 6864 | 4.51 | 3136 | 31 |
| 30 | 3128 | 3.06 | 5994 | | 7134 | 4.51 | 2866 | 30 |
| 31 | 9.753312 | 3.06 | 9.915907 | | 9.837405 | 4.51 | 10.162595 | 29 |
| 32 | 3495 | 3.06 | 5820 | | 7675 | 4.51 | 2325 | 28 |
| 33 | 3679 | 3.06 | 5733 | | 7946 | 4.51 | 2054 | 27 |
| 34 | 3862 | 3.05 | 5646 | | 8216 | 4.50 | 1784 | 26 |
| 35 | 4046 | 3.05 | 5559 | | 8487 | 4.50 | 1513 | 25 |
| 36 | 4229 | 3.05 | 5472 | | 8757 | 4.50 | 1243 | 24 |
| 37 | 4412 | 3.05 | 5385 | | 9027 | 4.50 | 0973 | 23 |
| 38 | 4595 | 3.05 | 5297 | | 9297 | 4.50 | 0703 | 22 |
| 39 | 4778 | 3.04 | 5210 | 1.45 | 9568 | 4.50 | 0432 | 21 |
| 40 | 4960 | 3.04 | 5123 | 1.46 | 9.839838 | 4.50 | 10.160162 | 20 |
| 41 | 9.755143 | 3.04 | 9.915035 | | 9.840108 | 4.50 | 10.159892 | 19 |
| 42 | 5326 | 3.04 | 4948 | | 0378 | 4.50 | 9622 | 18 |
| 43 | 5508 | 3.04 | 4860 | | 0648 | 4.50 | 9352 | 17 |
| 44 | 5690 | 3.04 | 4773 | | 0917 | 4.49 | 9083 | 16 |
| 45 | 5872 | 3.03 | 4685 | | 1187 | 4.49 | 8813 | 15 |
| 46 | 6054 | 3.03 | 4598 | | 1457 | 4.49 | 8543 | 14 |
| 47 | 6236 | 3.03 | 4510 | | 1727 | 4.49 | 8273 | 13 |
| 48 | 6418 | 3.03 | 4422 | | 1996 | 4.49 | 8004 | 12 |
| 49 | 6600 | 3.03 | 4334 | 1.46 | 2266 | 4.49 | 7734 | 11 |
| 50 | 6782 | 3.02 | 4246 | 1.47 | 2535 | 4.49 | 7465 | 10 |
| 51 | 9.756963 | 3.02 | 9.914158 | | 9.842805 | 4.49 | 10.157195 | 9 |
| 52 | 7144 | 3.02 | 4070 | | 3074 | 4.49 | 6926 | 8 |
| 53 | 7326 | 3.02 | 3982 | | 3343 | 4.49 | 6657 | 7 |
| 54 | 7507 | 3.02 | 3894 | | 3612 | 4.49 | 6388 | 6 |
| 55 | 7688 | 3.01 | 3806 | | 3882 | 4.48 | 6118 | 5 |
| 56 | 7869 | 3.01 | 3718 | | 4151 | 4.48 | 5849 | 4 |
| 57 | 8050 | 3.01 | 3630 | | 4420 | 4.48 | 5580 | 3 |
| 58 | 8230 | 3.01 | 3541 | | 4689 | 4.48 | 5311 | 2 |
| 59 | 8411 | 3.01 | 3453 | 1.47 | 4958 | 4.48 | 5042 | 1 |
| 60 | 9.758591 | | 9.913365 | | 9.845227 | | 10.154773 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 55° |

## 35° SINES AND TANGENTS.

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.758591 | 3.01 | 9.913365 | 1.47 | 9.845227 | 4.48 | 10.154773 | 60 |
| 1 | 8772 | 3.00 | 3276 | 1.47 | 5496 | 4.48 | 4504 | 59 |
| 2 | 8952 | 3.00 | 3187 | 1.48 | 5764 | 4.48 | 4236 | 58 |
| 3 | 9132 | 3.00 | 3099 | | 6033 | 4.48 | 3967 | 57 |
| 4 | 9312 | 3.00 | 3010 | | 6302 | 4.48 | 3698 | 56 |
| 5 | 9492 | 3.00 | 2922 | | 6570 | 4.47 | 3430 | 55 |
| 6 | 9672 | 2.99 | 2833 | | 6839 | 4.47 | 3161 | 54 |
| 7 | 9.759852 | 2.99 | 2744 | | 7107 | 4.47 | 2893 | 53 |
| 8 | 9.760031 | 2.99 | 2655 | | 7376 | 4.47 | 2624 | 52 |
| 9 | 0211 | 2.99 | 2566 | | 7644 | 4.47 | 2356 | 51 |
| 10 | 0390 | 2.99 | 2477 | | 7913 | 4.47 | 2087 | 50 |
| 11 | 0569 | 2.98 | 9.912388 | 1.48 | 9.848181 | 4.47 | 10.151819 | 49 |
| 12 | 0748 | 2.98 | 2299 | 1.49 | 8449 | 4.47 | 1551 | 48 |
| 13 | 0927 | 2.98 | 2210 | | 8717 | 4.47 | 1283 | 47 |
| 14 | 1106 | 2.98 | 2121 | | 8986 | 4.47 | 1014 | 46 |
| 15 | 1285 | 2.98 | 2031 | | 9254 | 4.47 | 0746 | 45 |
| 16 | 1464 | 2.98 | 1942 | | 9522 | 4.47 | 0478 | 44 |
| 17 | 1642 | 2.97 | 1853 | | 9.849790 | 4.46 | 10.150210 | 43 |
| 18 | 1821 | 2.97 | 1763 | | 9.850057 | 4.46 | 10.149943 | 42 |
| 19 | 1999 | 2.97 | 1674 | | 0325 | 4.46 | 9675 | 41 |
| 20 | 2177 | 2.97 | 1584 | | 0593 | 4.46 | 9407 | 40 |
| 21 | 9.762356 | 2.97 | 9.911495 | | 0861 | 4.46 | 9139 | 39 |
| 22 | 2534 | 2.96 | 1405 | 1.49 | 1129 | 4.46 | 8871 | 38 |
| 23 | 2712 | 2.96 | 1315 | 1.50 | 1396 | 4.46 | 8604 | 37 |
| 24 | 2889 | 2.96 | 1226 | | 1664 | 4.46 | 8336 | 36 |
| 25 | 3067 | 2.96 | 1136 | | 1931 | 4.46 | 8069 | 35 |
| 26 | 3245 | 2.96 | 1046 | | 2199 | 4.46 | 7801 | 34 |
| 27 | 3422 | 2.96 | 0956 | | 2466 | 4.46 | 7534 | 33 |
| 28 | 3600 | 2.95 | 0866 | | 2733 | 4.45 | 7267 | 32 |
| 29 | 3777 | 2.95 | 0776 | | 3001 | 4.45 | 6999 | 31 |
| 30 | 3954 | 2.95 | 0686 | | 3268 | 4.45 | 6732 | 30 |
| 31 | 9.764131 | 2.95 | 9.910596 | | 9.853535 | 4.45 | 10.146465 | 29 |
| 32 | 4308 | 2.95 | 0506 | 1.50 | 3802 | 4.45 | 6198 | 28 |
| 33 | 4485 | 2.94 | 0415 | 1.51 | 4069 | 4.45 | 5931 | 27 |
| 34 | 4662 | 2.94 | 0325 | | 4336 | 4.45 | 5664 | 26 |
| 35 | 4838 | 2.94 | 0235 | | 4603 | 4.45 | 5397 | 25 |
| 36 | 5015 | 2.94 | 0144 | | 4870 | 4.45 | 5130 | 24 |
| 37 | 5191 | 2.94 | 9.910054 | | 5137 | 4.45 | 4863 | 23 |
| 38 | 5367 | 2.94 | 9.909963 | | 5404 | 4.45 | 4596 | 22 |
| 39 | 5544 | 2.93 | 9873 | | 5671 | 4.44 | 4329 | 21 |
| 40 | 5720 | 2.93 | 9782 | | 5938 | 4.44 | 4062 | 20 |
| 41 | 9.765896 | 2.93 | 9691 | | 9.856204 | 4.44 | 10.143796 | 19 |
| 42 | 6072 | 2.93 | 9601 | | 6471 | 4.44 | 3529 | 18 |
| 43 | 6247 | 2.93 | 9510 | | 6737 | 4.44 | 3263 | 17 |
| 44 | 6423 | 2.93 | 9419 | 1.51 | 7004 | 4.44 | 2996 | 16 |
| 45 | 6598 | 2.92 | 9328 | 1.52 | 7270 | 4.44 | 2730 | 15 |
| 46 | 6774 | 2.92 | 9237 | | 7537 | 4.44 | 2463 | 14 |
| 47 | 6949 | 2.92 | 9146 | | 7803 | 4.44 | 2197 | 13 |
| 48 | 7124 | 2.92 | 9055 | | 8069 | 4.44 | 1931 | 12 |
| 49 | 7300 | 2.92 | 8964 | | 8336 | 4.44 | 1664 | 11 |
| 50 | 7475 | 2.91 | 8873 | | 8602 | 4.43 | 1398 | 10 |
| 51 | 9.767649 | 2.91 | 9.908781 | | 8868 | 4.43 | 1132 | 9 |
| 52 | 7824 | 2.91 | 8690 | | 9134 | 4.43 | 0866 | 8 |
| 53 | 7999 | 2.91 | 8599 | | 9400 | 4.43 | 0600 | 7 |
| 54 | 8173 | 2.91 | 8507 | 1.52 | 9666 | 4.43 | 0334 | 6 |
| 55 | 8348 | 2.90 | 8416 | 1.53 | 9.859932 | 4.43 | 10.140068 | 5 |
| 56 | 8522 | 2.90 | 8324 | | 9.860198 | 4.43 | 10.139802 | 4 |
| 57 | 8697 | 2.90 | 8233 | | 0464 | 4.43 | 9536 | 3 |
| 58 | 8871 | 2.90 | 8141 | | 0730 | 4.43 | 9270 | 2 |
| 59 | 9045 | 2.90 | 8049 | 1.53 | 0995 | 4.43 | 9005 | 1 |
| 60 | 9.769219 | | 9.907958 | | 9.861261 | | 10.138739 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 54° |

## 36° LOGARITHMIC

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.769219 | 2.90 | 9.907958 | 1.53 | 9.861261 | 4.43 | 10.138739 | 60 |
| 1 | 9393 | 2.89 | 7866 | | 1527 | 4.43 | 8473 | 59 |
| 2 | 9566 | 2.89 | 7774 | | 1792 | 4.42 | 8208 | 58 |
| 3 | 9740 | 2.89 | 7682 | | 2058 | 4.42 | 7942 | 57 |
| 4 | 9.769913 | 2.89 | 7590 | | 2323 | 4.42 | 7677 | 56 |
| 5 | 9.770087 | 2.89 | 7498 | | 2589 | 4.42 | 7411 | 55 |
| 6 | 0260 | 2.88 | 7406 | 1.53 | 2854 | 4.42 | 7146 | 54 |
| 7 | 0433 | 2.88 | 7314 | 1.54 | 3119 | 4.42 | 6881 | 53 |
| 8 | 0606 | 2.88 | 7222 | | 3385 | 4.42 | 6615 | 52 |
| 9 | 0779 | 2.88 | 7129 | | 3650 | 4.42 | 6350 | 51 |
| 10 | 0952 | 2.88 | 7037 | | 3915 | 4.42 | 6085 | 50 |
| 11 | 9.771125 | 2.88 | 9.906945 | | 9.864180 | 4.42 | 10.135820 | 49 |
| 12 | 1298 | 2.88 | 6852 | | 4445 | 4.42 | 5555 | 48 |
| 13 | 1470 | 2.87 | 6760 | | 4710 | 4.42 | 5290 | 47 |
| 14 | 1643 | 2.87 | 6667 | | 4975 | 4.41 | 5025 | 46 |
| 15 | 1815 | 2.87 | 6575 | | 5240 | 4.41 | 4760 | 45 |
| 16 | 1987 | 2.87 | 6482 | 1.54 | 5505 | 4.41 | 4495 | 44 |
| 17 | 2159 | 2.87 | 6389 | 1.55 | 5770 | 4.41 | 4230 | 43 |
| 18 | 2331 | 2.86 | 6296 | | 6035 | 4.41 | 3965 | 42 |
| 19 | 2503 | 2.86 | 6204 | | 6300 | 4.41 | 3700 | 41 |
| 20 | 2675 | 2.86 | 6111 | | 6564 | 4.41 | 3436 | 40 |
| 21 | 9.772847 | 2.86 | 9.906018 | | 9.866829 | 4.41 | 10.133171 | 39 |
| 22 | 3018 | 2.86 | 5925 | | 7094 | 4.41 | 2906 | 38 |
| 23 | 3190 | 2.86 | 5832 | | 7358 | 4.41 | 2642 | 37 |
| 24 | 3361 | 2.85 | 5739 | | 7623 | 4.41 | 2377 | 36 |
| 25 | 3533 | 2.85 | 5645 | | 7887 | 4.41 | 2113 | 35 |
| 26 | 3704 | 2.85 | 5552 | | 8152 | 4.40 | 1848 | 34 |
| 27 | 3875 | 2.85 | 5459 | 1.55 | 8416 | 4.40 | 1584 | 33 |
| 28 | 4046 | 2.85 | 5366 | 1.56 | 8680 | 4.40 | 1320 | 32 |
| 29 | 4217 | 2.85 | 5272 | | 8945 | 4.40 | 1055 | 31 |
| 30 | 4388 | 2.84 | 5179 | | 9209 | 4.40 | 0791 | 30 |
| 31 | 9.774558 | 2.84 | 9.905085 | | 9473 | 4.40 | 0527 | 29 |
| 32 | 4729 | 2.84 | 4992 | | 9.869737 | 4.40 | 10.130263 | 28 |
| 33 | 4899 | 2.84 | 4898 | | 9.870001 | 4.40 | 10.129999 | 27 |
| 34 | 5070 | 2.84 | 4804 | | 0265 | 4.40 | 9735 | 26 |
| 35 | 5240 | 2.84 | 4711 | | 0529 | 4.40 | 9471 | 25 |
| 36 | 5410 | 2.83 | 4617 | | 0793 | 4.40 | 9207 | 24 |
| 37 | 5580 | 2.83 | 4523 | 1.56 | 1057 | 4.40 | 8943 | 23 |
| 38 | 5750 | 2.83 | 4429 | 1.57 | 1321 | 4.40 | 8679 | 22 |
| 39 | 5920 | 2.83 | 4335 | | 1585 | 4.40 | 8415 | 21 |
| 40 | 6090 | 2.83 | 4241 | | 1849 | 4.39 | 8151 | 20 |
| 41 | 9.776259 | 2.83 | 9.904147 | | 9.872112 | 4.39 | 10.127888 | 19 |
| 42 | 6429 | 2.82 | 4053 | | 2376 | 4.39 | 7624 | 18 |
| 43 | 6598 | 2.82 | 3959 | | 2640 | 4.39 | 7360 | 17 |
| 44 | 6768 | 2.82 | 3864 | | 2903 | 4.39 | 7097 | 16 |
| 45 | 6937 | 2.82 | 3770 | | 3167 | 4.39 | 6833 | 15 |
| 46 | 7106 | 2.82 | 3676 | | 3430 | 4.39 | 6570 | 14 |
| 47 | 7275 | 2.81 | 3581 | | 3694 | 4.39 | 6306 | 13 |
| 48 | 7444 | 2.81 | 3487 | 1.57 | 3957 | 4.39 | 6043 | 12 |
| 49 | 7613 | 2.81 | 3392 | 1.58 | 4220 | 4.39 | 5780 | 11 |
| 50 | 7781 | 2.81 | 3298 | | 4484 | 4.39 | 5516 | 10 |
| 51 | 9.777950 | 2.81 | 9.903203 | | 9.874747 | 4.39 | 10.125253 | 9 |
| 52 | 8119 | 2.81 | 3108 | | 5010 | 4.39 | 4990 | 8 |
| 53 | 8287 | 2.80 | 3014 | | 5273 | 4.38 | 4727 | 7 |
| 54 | 8455 | 2.80 | 2919 | | 5537 | 4.38 | 4463 | 6 |
| 55 | 8624 | 2.80 | 2824 | | 5800 | 4.38 | 4200 | 5 |
| 56 | 8792 | 2.80 | 2729 | | 6063 | 4.38 | 3937 | 4 |
| 57 | 8960 | 2.80 | 2634 | | 6326 | 4.38 | 3674 | 3 |
| 58 | 9128 | 2.80 | 2539 | 1.58 | 6589 | 4.38 | 3411 | 2 |
| 59 | 9295 | 2.79 | 2444 | 1.59 | 6852 | 4.38 | 3148 | 1 |
| 60 | 9.779463 | | 9.902349 | | 9.877114 | | 10.122886 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 53° |

## 37° SINES AND TANGENTS.

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.779463 | 2.79 | 9.902349 | 1.59 | 9.877114 | 4.38 | 10.122886 | 60 |
| 1 | 9631 | 2.79 | 2253 | | 7377 | 4.38 | 2623 | 59 |
| 2 | 9798 | 2.79 | 2158 | | 7640 | 4.38 | 2360 | 58 |
| 3 | 9966 | 2.79 | 2063 | | 7903 | 4.38 | 2097 | 57 |
| 4 | 9.780133 | 2.79 | 1967 | | 8165 | 4.38 | 1835 | 56 |
| 5 | 0300 | 2.78 | 1872 | | 8428 | 4.38 | 1572 | 55 |
| 6 | 0467 | 2.78 | 1776 | | 8691 | 4.38 | 1309 | 54 |
| 7 | 0634 | 2.78 | 1681 | | 8953 | 4.37 | 1047 | 53 |
| 8 | 0801 | 2.78 | 1585 | | 9216 | 4.37 | 0784 | 52 |
| 9 | 0968 | 2.78 | 1490 | 1.59 | 9478 | 4.37 | 0522 | 51 |
| 10 | 1134 | 2.78 | 1394 | 1.60 | 9.879741 | 4.37 | 10.120259 | 50 |
| 11 | 9.781301 | 2.77 | 9.901298 | | 9.880003 | 4.37 | 10.119997 | 49 |
| 12 | 1468 | 2.77 | 1202 | | 0265 | 4.37 | 9735 | 48 |
| 13 | 1634 | 2.77 | 1106 | | 0528 | 4.37 | 9472 | 47 |
| 14 | 1800 | 2.77 | 1010 | | 0790 | 4.37 | 9210 | 46 |
| 15 | 1966 | 2.77 | 0914 | | 1052 | 4.37 | 8948 | 45 |
| 16 | 2132 | 2.77 | 0818 | | 1314 | 4.37 | 8686 | 44 |
| 17 | 2298 | 2.76 | 0722 | | 1576 | 4.37 | 8424 | 43 |
| 18 | 2464 | 2.76 | 0626 | | 1839 | 4.37 | 8161 | 42 |
| 19 | 2630 | 2.76 | 0529 | 1.60 | 2101 | 4.37 | 7899 | 41 |
| 20 | 2796 | 2.76 | 0433 | 1.61 | 2363 | 4.36 | 7637 | 40 |
| 21 | 9.782961 | 2.76 | 0337 | | 9.882625 | 4.36 | 10.117375 | 39 |
| 22 | 3127 | 2.76 | 0240 | | 2887 | 4.36 | 7113 | 38 |
| 23 | 3292 | 2.75 | 0144 | | 3148 | 4.36 | 6852 | 37 |
| 24 | 3458 | 2.75 | 9.900047 | | 3410 | 4.36 | 6590 | 36 |
| 25 | 3623 | 2.75 | 9.899951 | | 3672 | 4.36 | 6328 | 35 |
| 26 | 3788 | 2.75 | 9854 | | 3934 | 4.36 | 6066 | 34 |
| 27 | 3953 | 2.75 | 9757 | | 4196 | 4.36 | 5804 | 33 |
| 28 | 4118 | 2.74 | 9660 | | 4457 | 4.36 | 5543 | 32 |
| 29 | 4282 | 2.74 | 9564 | 1.61 | 4719 | 4.36 | 5281 | 31 |
| 30 | 4447 | 2.74 | 9467 | 1.62 | 4980 | 4.36 | 5020 | 30 |
| 31 | 9.784612 | 2.74 | 9.899370 | | 9.885242 | 4.36 | 10.114758 | 29 |
| 32 | 4776 | 2.74 | 9273 | | 5504 | 4.36 | 4496 | 28 |
| 33 | 4941 | 2.74 | 9176 | | 5765 | 4.36 | 4235 | 27 |
| 34 | 5105 | 2.74 | 9078 | | 6026 | 4.36 | 3974 | 26 |
| 35 | 5269 | 2.73 | 8981 | | 6288 | 4.36 | 3712 | 25 |
| 36 | 5433 | 2.73 | 8884 | | 6549 | 4.35 | 3451 | 24 |
| 37 | 5597 | 2.73 | 8787 | | 6811 | 4.35 | 3189 | 23 |
| 38 | 5761 | 2.73 | 8689 | | 7072 | 4.35 | 2928 | 22 |
| 39 | 5925 | 2.73 | 8592 | 1.62 | 7333 | 4.35 | 2667 | 21 |
| 40 | 6089 | 2.73 | 8494 | 1.63 | 7594 | 4.35 | 2406 | 20 |
| 41 | 9.786252 | 2.72 | 9.898397 | | 9.887855 | 4.35 | 10.112145 | 19 |
| 42 | 6416 | 2.72 | 8299 | | 8116 | 4.35 | 1884 | 18 |
| 43 | 6579 | 2.72 | 8202 | | 8378 | 4.35 | 1622 | 17 |
| 44 | 6742 | 2.72 | 8104 | | 8639 | 4.35 | 1361 | 16 |
| 45 | 6906 | 2.72 | 8006 | | 8900 | 4.35 | 1100 | 15 |
| 46 | 7069 | 2.72 | 7908 | | 9161 | 4.35 | 0839 | 14 |
| 47 | 7232 | 2.71 | 7810 | | 9421 | 4.35 | 0579 | 13 |
| 48 | 7395 | 2.71 | 7712 | | 9682 | 4.35 | 0318 | 12 |
| 49 | 7557 | 2.71 | 7614 | | 9.889943 | 4.35 | 10.110057 | 11 |
| 50 | 7720 | 2.71 | 7516 | 1.63 | 9.890204 | 4.34 | 10.109796 | 10 |
| 51 | 9.787883 | 2.71 | 9.897418 | 1.64 | 0465 | 4.34 | 9535 | 9 |
| 52 | 8045 | 2.71 | 7320 | | 0725 | 4.34 | 9275 | 8 |
| 53 | 8208 | 2.71 | 7222 | | 0986 | 4.34 | 9014 | 7 |
| 54 | 8370 | 2.70 | 7123 | | 1247 | 4.34 | 8753 | 6 |
| 55 | 8532 | 2.70 | 7025 | | 1507 | 4.34 | 8493 | 5 |
| 56 | 8694 | 2.70 | 6926 | | 1768 | 4.34 | 8232 | 4 |
| 57 | 8856 | 2.70 | 6828 | | 2028 | 4.34 | 7972 | 3 |
| 58 | 9018 | 2.70 | 6729 | | 2289 | 4.34 | 7711 | 2 |
| 59 | 9180 | 2.70 | 6631 | 1.64 | 2549 | 4.34 | 7451 | 1 |
| 60 | 9.789342 | | 9.896532 | | 9.892810 | | 10.107190 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 52° |

## 38° LOGARITHMIC

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.789342 | 2.69 | 9.896532 | 1.64 | 9.892810 | 4.34 | 10.107190 | 60 |
| 1 | 9504 | 2.69 | 6433 | 1.65 | 3070 | 4.34 | 6930 | 59 |
| 2 | 9665 | 2.69 | 6335 | | 3331 | 4.34 | 6669 | 58 |
| 3 | 9827 | 2.69 | 6236 | | 3591 | 4.34 | 6409 | 57 |
| 4 | 9.789988 | 2.69 | 6137 | | 3851 | 4.34 | 6149 | 56 |
| 5 | 9.790149 | 2.69 | 6038 | | 4111 | 4.34 | 5889 | 55 |
| 6 | 0310 | 2.68 | 5939 | | 4371 | 4.34 | 5629 | 54 |
| 7 | 0471 | 2.68 | 5840 | | 4632 | 4.33 | 5368 | 53 |
| 8 | 0632 | 2.68 | 5741 | | 4892 | 4.33 | 5108 | 52 |
| 9 | 0793 | 2.68 | 5641 | | 5152 | 4.33 | 4848 | 51 |
| 10 | 0954 | 2.68 | 5542 | 1.65 | 5412 | 4.33 | 4588 | 50 |
| 11 | 9.791115 | 2.68 | 9.895443 | 1.66 | 9.895672 | 4.33 | 10.104328 | 49 |
| 12 | 1275 | 2.67 | 5343 | | 5932 | 4.33 | 4068 | 48 |
| 13 | 1436 | 2.67 | 5244 | | 6192 | 4.33 | 3808 | 47 |
| 14 | 1596 | 2.67 | 5145 | | 6452 | 4.33 | 3548 | 46 |
| 15 | 1757 | 2.67 | 5045 | | 6712 | 4.33 | 3288 | 45 |
| 16 | 1917 | 2.67 | 4945 | | 6971 | 4.33 | 3029 | 44 |
| 17 | 2077 | 2.67 | 4846 | | 7231 | 4.33 | 2769 | 43 |
| 18 | 2237 | 2.66 | 4746 | | 7491 | 4.33 | 2509 | 42 |
| 19 | 2397 | 2.66 | 4646 | | 7751 | 4.33 | 2249 | 41 |
| 20 | 2557 | 2.66 | 4546 | 1.66 | 8010 | 4.33 | 1990 | 40 |
| 21 | 9.792716 | 2.66 | 9.894446 | 1.67 | 9.898270 | 4.33 | 10.101730 | 39 |
| 22 | 2876 | 2.66 | 4346 | | 8530 | 4.33 | 1470 | 38 |
| 23 | 3035 | 2.66 | 4246 | | 8789 | 4.32 | 1211 | 37 |
| 24 | 3195 | 2.66 | 4146 | | 9049 | 4.32 | 0951 | 36 |
| 25 | 3354 | 2.65 | 4046 | | 9308 | 4.32 | 0692 | 35 |
| 26 | 3514 | 2.65 | 3946 | | 9568 | 4.32 | 0432 | 34 |
| 27 | 3673 | 2.65 | 3846 | | 9.899827 | 4.32 | 10.100173 | 33 |
| 28 | 3832 | 2.65 | 3745 | | 9.900087 | 4.32 | 10.099913 | 32 |
| 29 | 3991 | 2.65 | 3645 | | 0346 | 4.32 | 9654 | 31 |
| 30 | 4150 | 2.64 | 3544 | 1.67 | 0605 | 4.32 | 9395 | 30 |
| 31 | 9.794308 | 2.64 | 9.893444 | 1.68 | 0864 | 4.32 | 9136 | 29 |
| 32 | 4467 | 2.64 | 3343 | | 1124 | 4.32 | 8876 | 28 |
| 33 | 4626 | 2.64 | 3243 | | 1383 | 4.32 | 8617 | 27 |
| 34 | 4784 | 2.64 | 3142 | | 1642 | 4.32 | 8358 | 26 |
| 35 | 4942 | 2.64 | 3041 | | 1901 | 4.32 | 8099 | 25 |
| 36 | 5101 | 2.64 | 2940 | | 2160 | 4.32 | 7840 | 24 |
| 37 | 5259 | 2.63 | 2839 | | 2420 | 4.32 | 7580 | 23 |
| 38 | 5417 | 2.63 | 2739 | | 2679 | 4.32 | 7321 | 22 |
| 39 | 5575 | 2.63 | 2638 | | 2938 | 4.32 | 7062 | 21 |
| 40 | 5733 | 2.63 | 2536 | 1.68 | 3197 | 4.31 | 6803 | 20 |
| 41 | 9.795891 | 2.63 | 9.892435 | 1.69 | 9.903456 | 4.31 | 10.096544 | 19 |
| 42 | 6049 | 2.63 | 2334 | | 3714 | 4.31 | 6286 | 18 |
| 43 | 6206 | 2.63 | 2233 | | 3973 | 4.31 | 6027 | 17 |
| 44 | 6364 | 2.62 | 2132 | | 4232 | 4.31 | 5768 | 16 |
| 45 | 6521 | 2.62 | 2030 | | 4491 | 4.31 | 5509 | 15 |
| 46 | 6679 | 2.62 | 1929 | | 4750 | 4.31 | 5250 | 14 |
| 47 | 6836 | 2.62 | 1827 | | 5008 | 4.31 | 4992 | 13 |
| 48 | 6993 | 2.62 | 1726 | | 5267 | 4.31 | 4733 | 12 |
| 49 | 7150 | 2.62 | 1624 | 1.69 | 5526 | 4.31 | 4474 | 11 |
| 50 | 7307 | 2.61 | 1523 | 1.70 | 5785 | 4.31 | 4215 | 10 |
| 51 | 9.797464 | 2.61 | 9.891421 | | 9.906043 | 4.31 | 10.093957 | 9 |
| 52 | 7621 | 2.61 | 1319 | | 6302 | 4.31 | 3698 | 8 |
| 53 | 7777 | 2.61 | 1217 | | 6560 | 4.31 | 3440 | 7 |
| 54 | 7934 | 2.61 | 1115 | | 6819 | 4.31 | 3181 | 6 |
| 55 | 8091 | 2.61 | 1013 | | 7077 | 4.31 | 2923 | 5 |
| 56 | 8247 | 2.61 | 0911 | | 7336 | 4.31 | 2664 | 4 |
| 57 | 8403 | 2.60 | 0809 | | 7594 | 4.31 | 2406 | 3 |
| 58 | 8560 | 2.60 | 0707 | | 7853 | 4.31 | 2147 | 2 |
| 59 | 8716 | 2.60 | 0605 | 1.70 | 8111 | 4.30 | 1889 | 1 |
| 60 | 9.798872 | | 9.890503 | | 9.908369 | | 10.091631 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 51° |

## 39° SINES AND TANGENTS.

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.798872 | 2.60 | 9.890503 | 1.70 | 9.908369 | 4.30 | 10.091631 | 60 |
| 1 | 9028 | 2.60 | 0400 | 1.71 | 8628 | 4.30 | 1372 | 59 |
| 2 | 9184 | 2.60 | 0298 | | 8886 | 4.30 | 1114 | 58 |
| 3 | 9339 | 2.59 | 0195 | | 9144 | 4.30 | 0856 | 57 |
| 4 | 9495 | 2.59 | 9.890093 | | 9402 | 4.30 | 0598 | 56 |
| 5 | 9651 | 2.59 | 9.889990 | | 9660 | 4.30 | 0340 | 55 |
| 6 | 9806 | 2.59 | 9888 | | 9.909918 | 4.30 | 10.090082 | 54 |
| 7 | 9.799962 | 2.59 | 9785 | | 9.910177 | 4.30 | 10.089823 | 53 |
| 8 | 9.800117 | 2.59 | 9682 | | 0435 | 4.30 | 9565 | 52 |
| 9 | 0272 | 2.58 | 9579 | | 0693 | 4.30 | 9307 | 51 |
| 10 | 0427 | 2.58 | 9477 | 1.71 | 0951 | 4.30 | 9049 | 50 |
| 11 | 0582 | 2.58 | 9.889374 | 1.72 | 1209 | 4.30 | 8791 | 49 |
| 12 | 0737 | 2.58 | 9271 | | 1467 | 4.30 | 8533 | 48 |
| 13 | 0892 | 2.58 | 9168 | | 1725 | 4.30 | 8275 | 47 |
| 14 | 1047 | 2.58 | 9064 | | 1982 | 4.30 | 8018 | 46 |
| 15 | 1201 | 2.58 | 8961 | | 2240 | 4.30 | 7760 | 45 |
| 16 | 1356 | 2.57 | 8858 | | 2498 | 4.30 | 7502 | 44 |
| 17 | 1511 | 2.57 | 8755 | | 2756 | 4.30 | 7244 | 43 |
| 18 | 1665 | 2.57 | 8651 | | 3014 | 4.29 | 6986 | 42 |
| 19 | 1819 | 2.57 | 8548 | 1.72 | 3271 | 4.29 | 6729 | 41 |
| 20 | 1973 | 2.57 | 8444 | 1.73 | 3529 | 4.29 | 6471 | 40 |
| 21 | 9.302128 | 2.57 | 9.888341 | | 9.913787 | 4.29 | 10.086213 | 39 |
| 22 | 2282 | 2.56 | 8237 | | 4044 | 4.29 | 5956 | 38 |
| 23 | 2436 | 2.56 | 8134 | | 4302 | 4.29 | 5698 | 37 |
| 24 | 2589 | 2.56 | 8030 | | 4560 | 4.29 | 5440 | 36 |
| 25 | 2743 | 2.56 | 7926 | | 4817 | 4.29 | 5183 | 35 |
| 26 | 2897 | 2.56 | 7822 | | 5075 | 4.29 | 4925 | 34 |
| 27 | 3050 | 2.56 | 7718 | | 5332 | 4.29 | 4668 | 33 |
| 28 | 3204 | 2.56 | 7614 | | 5590 | 4.29 | 4410 | 32 |
| 29 | 3357 | 2.55 | 7510 | 1.73 | 5847 | 4.29 | 4153 | 31 |
| 30 | 3511 | 2.55 | 7406 | 1.74 | 6104 | 4.29 | 3896 | 30 |
| 31 | 9.803664 | 2.55 | 9.887302 | | 9.916362 | 4.29 | 10.083638 | 29 |
| 32 | 3817 | 2.55 | 7198 | | 6619 | 4.29 | 3381 | 28 |
| 33 | 3970 | 2.55 | 7093 | | 6877 | 4.29 | 3123 | 27 |
| 34 | 4123 | 2.55 | 6989 | | 7134 | 4.29 | 2866 | 26 |
| 35 | 4276 | 2.54 | 6885 | | 7391 | 4.29 | 2609 | 25 |
| 36 | 4428 | 2.54 | 6780 | | 7648 | 4.29 | 2352 | 24 |
| 37 | 4581 | 2.54 | 6676 | | 7906 | 4.29 | 2094 | 23 |
| 38 | 4734 | 2.54 | 6571 | | 8163 | 4.28 | 1837 | 22 |
| 39 | 4886 | 2.54 | 6466 | 1.74 | 8420 | 4.28 | 1580 | 21 |
| 40 | 5039 | 2.54 | 6362 | 1.75 | 8677 | 4.28 | 1323 | 20 |
| 41 | 9.805191 | 2.54 | 9.886257 | | 8934 | 4.28 | 1066 | 19 |
| 42 | 5343 | 2.53 | 6152 | | 9191 | 4.28 | 0809 | 18 |
| 43 | 5495 | 2.53 | 6047 | | 9448 | 4.28 | 0552 | 17 |
| 44 | 5647 | 2.53 | 5942 | | 9705 | 4.28 | 0295 | 16 |
| 45 | 5799 | 2.53 | 5837 | | 9.919962 | 4.28 | 10.080038 | 15 |
| 46 | 5951 | 2.53 | 5732 | | 9.920219 | 4.28 | 10.079781 | 14 |
| 47 | 6103 | 2.53 | 5627 | | 0476 | 4.28 | 9524 | 13 |
| 48 | 6254 | 2.53 | 5522 | | 0733 | 4.28 | 9267 | 12 |
| 49 | 6406 | 2.52 | 5416 | 1.75 | 0990 | 4.28 | 9010 | 11 |
| 50 | 6557 | 2.52 | 5311 | 1.76 | 1247 | 4.28 | 8753 | 10 |
| 51 | 9.806709 | 2.52 | 9.885205 | | 1503 | 4.28 | 8497 | 9 |
| 52 | 6860 | 2.52 | 5100 | | 1760 | 4.28 | 8240 | 8 |
| 53 | 7011 | 2.52 | 4994 | | 2017 | 4.28 | 7983 | 7 |
| 54 | 7163 | 2.52 | 4889 | | 2274 | 4.28 | 7726 | 6 |
| 55 | 7314 | 2.52 | 4783 | | 2530 | 4.28 | 7470 | 5 |
| 56 | 7465 | 2.51 | 4677 | | 2787 | 4.28 | 7213 | 4 |
| 57 | 7615 | 2.51 | 4572 | 1.76 | 3044 | 4.28 | 6956 | 3 |
| 58 | 7766 | 2.51 | 4466 | 1.77 | 3300 | 4.28 | 6700 | 2 |
| 59 | 7917 | 2.51 | 4360 | 1.77 | 3557 | 4.27 | 6443 | 1 |
| 60 | 9.808067 | | 9.884254 | | 9.923814 | | 10.076186 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 50° |

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.808067 | 2.51 | 9.884254 | 1.77 | 9.923814 | 4.28 | 10.076186 | 60 |
| 1 | 8218 | 2.51 | 4148 | | 4070 | 4.27 | 5930 | 59 |
| 2 | 8368 | 2.51 | 4042 | | 4327 | 4.27 | 5673 | 58 |
| 3 | 8519 | 2.50 | 3936 | | 4583 | 4.27 | 5417 | 57 |
| 4 | 8669 | 2.50 | 3829 | | 4840 | 4.27 | 5160 | 56 |
| 5 | 8819 | 2.50 | 3723 | | 5096 | 4.27 | 4904 | 55 |
| 6 | 8969 | 2.50 | 3617 | | 5352 | 4.27 | 4648 | 54 |
| 7 | 9119 | 2.50 | 3510 | | 5609 | 4.27 | 4391 | 53 |
| 8 | 9269 | 2.50 | 3404 | 1.77 | 5865 | 4.27 | 4135 | 52 |
| 9 | 9419 | 2.49 | 3297 | 1.78 | 6122 | 4.27 | 3878 | 51 |
| 10 | 9569 | 2.49 | 3191 | | 6378 | 4.27 | 3622 | 50 |
| 11 | 9718 | 2.49 | 9.883084 | | 9.926634 | 4.27 | 10.073366 | 49 |
| 12 | 9.809868 | 2.49 | 2977 | | 6890 | 4.27 | 3110 | 48 |
| 13 | 9.810017 | 2.49 | 2871 | | 7147 | 4.27 | 2853 | 47 |
| 14 | 0167 | 2.49 | 2764 | | 7403 | 4.27 | 2597 | 46 |
| 15 | 0316 | 2.48 | 2657 | | 7659 | 4.27 | 2341 | 45 |
| 16 | 0465 | 2.48 | 2550 | | 7915 | 4.27 | 2085 | 44 |
| 17 | 0614 | 2.48 | 2443 | 1.78 | 8171 | 4.27 | 1829 | 43 |
| 18 | 0763 | 2.48 | 2336 | 1.79 | 8427 | 4.27 | 1573 | 42 |
| 19 | 0912 | 2.48 | 2229 | | 8684 | 4.27 | 1316 | 41 |
| 20 | 1061 | 2.48 | 2121 | | 8940 | 4.27 | 1060 | 40 |
| 21 | 9.811210 | 2.48 | 9.882014 | | 9196 | 4.27 | 0804 | 39 |
| 22 | 1358 | 2.48 | 1907 | | 9452 | 4.27 | 0548 | 38 |
| 23 | 1507 | 2.47 | 1799 | | 9708 | 4.27 | 0292 | 37 |
| 24 | 1655 | 2.47 | 1692 | | 9.929964 | 4.27 | 10.070036 | 36 |
| 25 | 1804 | 2.47 | 1584 | | 9.930220 | 4.26 | 10.069780 | 35 |
| 26 | 1952 | 2.47 | 1477 | | 0475 | 4.26 | 9525 | 34 |
| 27 | 2100 | 2.47 | 1369 | 1.79 | 0731 | 4.26 | 9269 | 33 |
| 28 | 2248 | 2.47 | 1261 | 1.80 | 0987 | 4.26 | 9013 | 32 |
| 29 | 2396 | 2.46 | 1153 | | 1243 | 4.26 | 8757 | 31 |
| 30 | 2544 | 2.46 | 1046 | | 1499 | 4.26 | 8501 | 30 |
| 31 | 9.812692 | 2.46 | 9.880938 | | 9.931755 | 4.26 | 10.068245 | 29 |
| 32 | 2840 | 2.46 | 0830 | | 2010 | 4.26 | 7990 | 28 |
| 33 | 2988 | 2.46 | 0722 | | 2266 | 4.26 | 7734 | 27 |
| 34 | 3135 | 2.46 | 0613 | | 2522 | 4.26 | 7478 | 26 |
| 35 | 3283 | 2.46 | 0505 | | 2778 | 4.26 | 7222 | 25 |
| 36 | 3430 | 2.46 | 0397 | 1.80 | 3033 | 4.26 | 6967 | 24 |
| 37 | 3578 | 2.45 | 0289 | 1.81 | 3289 | 4.26 | 6711 | 23 |
| 38 | 3725 | 2.45 | 0180 | | 3545 | 4.26 | 6455 | 22 |
| 39 | 3872 | 2.45 | 9.880072 | | 3800 | 4.26 | 6200 | 21 |
| 40 | 4019 | 2.45 | 9.879963 | | 4056 | 4.26 | 5944 | 20 |
| 41 | 9.814166 | 2.45 | 9855 | | 9.934311 | 4.26 | 10.065689 | 19 |
| 42 | 4313 | 2.45 | 9746 | | 4567 | 4.26 | 5433 | 18 |
| 43 | 4460 | 2.44 | 9637 | | 4822 | 4.26 | 5178 | 17 |
| 44 | 4607 | 2.44 | 9529 | | 5078 | 4.26 | 4922 | 16 |
| 45 | 4753 | 2.44 | 9420 | | 5333 | 4.26 | 4667 | 15 |
| 46 | 4900 | 2.44 | 9311 | 1.81 | 5589 | 4.26 | 4411 | 14 |
| 47 | 5046 | 2.44 | 9202 | 1.82 | 5844 | 4.26 | 4156 | 13 |
| 48 | 5193 | 2.44 | 9093 | | 6100 | 4.26 | 3900 | 12 |
| 49 | 5339 | 2.44 | 8984 | | 6355 | 4.26 | 3645 | 11 |
| 50 | 5485 | 2.43 | 8875 | | 6611 | 4.26 | 3389 | 10 |
| 51 | 9.815632 | 2.43 | 9.878766 | | 9.936866 | 4.25 | 10.063134 | 9 |
| 52 | 5778 | 2.43 | 8656 | | 7121 | 4.25 | 2879 | 8 |
| 53 | 5924 | 2.43 | 8547 | | 7377 | 4.25 | 2623 | 7 |
| 54 | 6069 | 2.43 | 8438 | | 7632 | 4.25 | 2368 | 6 |
| 55 | 6215 | 2.43 | 8328 | 1.82 | 7887 | 4.25 | 2113 | 5 |
| 56 | 6361 | 2.43 | 8219 | 1.83 | 8142 | 4.25 | 1858 | 4 |
| 57 | 6507 | 2.42 | 8109 | | 8398 | 4.25 | 1602 | 3 |
| 58 | 6652 | 2.42 | 7999 | | 8653 | 4.25 | 1347 | 2 |
| 59 | 6798 | 2.42 | 7890 | 1.83 | 8908 | 4.25 | 1092 | 1 |
| 60 | 9.816943 | | 9.877780 | | 9.939163 | | 10.060837 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 49° |

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.816943 | 2.42 | 9.877780 | 1.83 | 9.939163 | 4.25 | 10.060837 | 60 |
| 1 | 7088 | 2.42 | 7670 | | 9418 | 4.25 | 0582 | 59 |
| 2 | 7233 | 2.42 | 7560 | | 9673 | 4.25 | 0327 | 58 |
| 3 | 7379 | 2.42 | 7450 | | 9.939928 | 4.25 | 10.060072 | 57 |
| 4 | 7524 | 2.42 | 7340 | 1.83 | 9.940183 | 4.25 | 10.059817 | 56 |
| 5 | 7668 | 2.41 | 7230 | 1.84 | 0439 | 4.25 | 9561 | 55 |
| 6 | 7813 | 2.41 | 7120 | | 0694 | 4.25 | 9306 | 54 |
| 7 | 7958 | 2.41 | 7010 | | 0949 | 4.25 | 9051 | 53 |
| 8 | 8103 | 2.41 | 6899 | | 1204 | 4.25 | 8796 | 52 |
| 9 | 8247 | 2.41 | 6789 | | 1459 | 4.25 | 8541 | 51 |
| 10 | 8392 | 2.41 | 6678 | | 1713 | 4.25 | 8287 | 50 |
| 11 | 9.818536 | 2.40 | 9.876568 | | 9.941968 | 4.25 | 10.058032 | 49 |
| 12 | 8681 | 2.40 | 6457 | | 2223 | 4.25 | 7777 | 48 |
| 13 | 8825 | 2.40 | 6347 | 1.84 | 2478 | 4.25 | 7522 | 47 |
| 14 | 8969 | 2.40 | 6236 | 1.85 | 2733 | 4.25 | 7267 | 46 |
| 15 | 9113 | 2.40 | 6125 | | 2988 | 4.25 | 7012 | 45 |
| 16 | 9257 | 2.40 | 6014 | | 3243 | 4.25 | 6757 | 44 |
| 17 | 9401 | 2.40 | 5904 | | 3498 | 4.25 | 6502 | 43 |
| 18 | 9545 | 2.40 | 5793 | | 3752 | 4.25 | 6248 | 42 |
| 19 | 9689 | 2.39 | 5682 | | 4007 | 4.25 | 5993 | 41 |
| 20 | 9832 | 2.39 | 5571 | | 4262 | 4.25 | 5738 | 40 |
| 21 | 9.819976 | 2.39 | 9.875459 | | 9.944517 | 4.25 | 10.055483 | 39 |
| 22 | 9.820120 | 2.39 | 5348 | | 4771 | 4.24 | 5229 | 38 |
| 23 | 0263 | 2.39 | 5237 | 1.85 | 5026 | 4.24 | 4974 | 37 |
| 24 | 0406 | 2.39 | 5126 | 1.86 | 5281 | 4.24 | 4719 | 36 |
| 25 | 0550 | 2.38 | 5014 | | 5535 | 4.34 | 4465 | 35 |
| 26 | 0693 | 2.38 | 4903 | | 5790 | 4.24 | 4210 | 34 |
| 27 | 0836 | 2.38 | 4791 | | 6045 | 4.24 | 3955 | 33 |
| 28 | 0979 | 2.38 | 4680 | | 6299 | 4.24 | 3701 | 32 |
| 29 | 1122 | 2.38 | 4568 | | 6554 | 4.24 | 3446 | 31 |
| 30 | 1265 | 2.38 | 4456 | | 6808 | 4.24 | 3192 | 30 |
| 31 | 9.821407 | 2.38 | 9.874344 | 1.86 | 9.947063 | 4.24 | 10.052937 | 29 |
| 32 | 1550 | 2.38 | 4232 | 1.87 | 7318 | 4.24 | 2682 | 28 |
| 33 | 1693 | 2.37 | 4121 | | 7572 | 4.24 | 2428 | 27 |
| 34 | 1835 | 2.37 | 4009 | | 7827 | 4.24 | 2173 | 26 |
| 35 | 1977 | 2.37 | 3896 | | 8081 | 4.24 | 1919 | 25 |
| 36 | 2120 | 2.37 | 3784 | | 8335 | 4.24 | 1665 | 24 |
| 37 | 2262 | 2.37 | 3672 | | 8590 | 4.24 | 1410 | 23 |
| 38 | 2404 | 2.37 | 3560 | | 8844 | 4.24 | 1156 | 22 |
| 39 | 2546 | 2.37 | 3448 | | 9099 | 4.24 | 0901 | 21 |
| 40 | 2688 | 2.36 | 3335 | | 9353 | 4.24 | 0647 | 20 |
| 41 | 9.822830 | 2.36 | 9.873223 | 1.87 | 9608 | 4.24 | 0392 | 19 |
| 42 | 2972 | 2.36 | 3110 | 1.88 | 9.949862 | 4.24 | 10.050138 | 18 |
| 43 | 3114 | 2.36 | 2998 | | 9.950116 | 4.24 | 10.049884 | 17 |
| 44 | 3255 | 2.36 | 2885 | | 0371 | 4.24 | 9629 | 16 |
| 45 | 3397 | 2.36 | 2772 | | 0625 | 4.24 | 9375 | 15 |
| 46 | 3539 | 2.36 | 2659 | | 0879 | 4.24 | 9121 | 14 |
| 47 | 3680 | 2.35 | 2547 | | 1133 | 4.24 | 8867 | 13 |
| 48 | 3821 | 2.35 | 2434 | | 1388 | 4.24 | 8612 | 12 |
| 49 | 3963 | 2.35 | 2321 | | 1642 | 4.24 | 8358 | 11 |
| 50 | 4104 | 2.35 | 2208 | 1.88 | 1896 | 4.24 | 8104 | 10 |
| 51 | 9.824245 | 2.35 | 9.872095 | 1.89 | 9.952150 | 4.24 | 10.047850 | 9 |
| 52 | 4386 | 2.35 | 1981 | | 2405 | 4.24 | 7595 | 8 |
| 53 | 4527 | 2.35 | 1868 | | 2659 | 4.24 | 7341 | 7 |
| 54 | 4668 | 2.34 | 1755 | | 2913 | 4.24 | 7087 | 6 |
| 55 | 4808 | 2.34 | 1641 | | 3167 | 4.23 | 6833 | 5 |
| 56 | 4949 | 2.34 | 1528 | | 3421 | 4.23 | 6579 | 4 |
| 57 | 5090 | 2.34 | 1414 | | 3675 | 4.23 | 6325 | 3 |
| 58 | 5230 | 2.34 | 1301 | | 3929 | 4.23 | 6071 | 2 |
| 59 | 5371 | 2.34 | 1187 | 1.89 | 4183 | 4.23 | 5817 | 1 |
| 60 | 9.825511 | | 9.871073 | | 9.954437 | | 10.045563 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 48° |

## 42° LOGARITHMIC

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.825511 | 2.34 | 9.871073 | 1.90 | 9.954437 | 4.23 | 10.045563 | 60 |
| 1 | 5651 | 2.33 | 0960 | | 4691 | 4.23 | 5309 | 59 |
| 2 | 5791 | 2.33 | 0846 | | 4946 | 4.23 | 5054 | 58 |
| 3 | 5931 | 2.33 | 0732 | | 5200 | 4.23 | 4800 | 57 |
| 4 | 6071 | 2.33 | 0618 | | 5454 | 4.23 | 4546 | 56 |
| 5 | 6211 | 2.33 | 0504 | | 5708 | 4.23 | 4292 | 55 |
| 6 | 6351 | 2.33 | 0390 | | 5961 | 4.23 | 4039 | 54 |
| 7 | 6491 | 2.33 | 0276 | | 6215 | 4.23 | 3785 | 53 |
| 8 | 6631 | 2.33 | 0161 | 1.90 | 6469 | 4.23 | 3531 | 52 |
| 9 | 6770 | 2.32 | 9.870047 | 1.91 | 6723 | 4.23 | 3277 | 51 |
| 10 | 6910 | 2.32 | 9.869933 | | 6977 | 4.23 | 3023 | 50 |
| 11 | 9.827049 | 2.32 | 9818 | | 9.957231 | 4.23 | 10.042769 | 49 |
| 12 | 7189 | 2.32 | 9704 | | 7485 | 4.23 | 2515 | 48 |
| 13 | 7328 | 2.32 | 9589 | | 7739 | 4.23 | 2261 | 47 |
| 14 | 7467 | 2.32 | 9474 | | 7993 | 4.23 | 2007 | 46 |
| 15 | 7606 | 2.32 | 9360 | | 8247 | 4.23 | 1753 | 45 |
| 16 | 7745 | 2.32 | 9245 | | 8500 | 4.23 | 1500 | 44 |
| 17 | 7884 | 2.31 | 9130 | 1.91 | 8754 | 4.23 | 1246 | 43 |
| 18 | 8023 | 2.31 | 9015 | 1.92 | 9008 | 4.23 | 0992 | 42 |
| 19 | 8162 | 2.31 | 8900 | | 9262 | 4.23 | 0738 | 41 |
| 20 | 8301 | 2.31 | 8785 | | 9516 | 4.23 | 0484 | 40 |
| 21 | 9.828439 | 2.31 | 9.868670 | | 9.959769 | 4.23 | 10.040231 | 39 |
| 22 | 8578 | 2.31 | 8555 | | 9.960023 | 4.23 | 10.039977 | 38 |
| 23 | 8716 | 2.31 | 8440 | | 0277 | 4.23 | 9723 | 37 |
| 24 | 8855 | 2.30 | 8324 | | 0530 | 4.23 | 9470 | 36 |
| 25 | 8993 | 2.30 | 8209 | | 0784 | 4.23 | 9216 | 35 |
| 26 | 9131 | 2.30 | 8093 | .92 | 1038 | 4.23 | 8962 | 34 |
| 27 | 9269 | 2.30 | 7978 | 1.93 | 1292 | 4.23 | 8708 | 33 |
| 28 | 9407 | 2.30 | 7862 | | 1545 | 4.23 | 8455 | 32 |
| 29 | 9545 | 2.30 | 7747 | | 1799 | 4.23 | 8201 | 31 |
| 30 | 9683 | 2.30 | 7631 | | 2052 | 4.23 | 7948 | 30 |
| 31 | 9821 | 2.29 | 9.867515 | | 9.962306 | 4.23 | 10.037694 | 29 |
| 32 | 9.829959 | 2.29 | 7399 | | 2560 | 4.23 | 7440 | 28 |
| 33 | 9.830097 | 2.29 | 7283 | | 2813 | 4.23 | 7187 | 27 |
| 34 | 0234 | 2.29 | 7167 | | 3067 | 4.23 | 6933 | 26 |
| 35 | 0372 | 2.29 | 7051 | 1.93 | 3320 | 4.23 | 6680 | 25 |
| 36 | 0509 | 2.29 | 6935 | 1.94 | 3574 | 4.23 | 6426 | 24 |
| 37 | 0646 | 2.29 | 6819 | | 3828 | 4.23 | 6172 | 23 |
| 38 | 0784 | 2.29 | 6703 | | 4081 | 4.23 | 5919 | 22 |
| 39 | 0921 | 2.28 | 6586 | | 4335 | 4.23 | 5665 | 21 |
| 40 | 1058 | 2.28 | 6470 | | 4588 | 4.22 | 5412 | 20 |
| 41 | 9.831195 | 2.28 | 9.866353 | | 9.964842 | 4.22 | 10.035158 | 19 |
| 42 | 1332 | 2.28 | 6237 | | 5095 | 4.22 | 4905 | 18 |
| 43 | 1469 | 2.28 | 6120 | 1.94 | 5349 | 4.22 | 4651 | 17 |
| 44 | 1606 | 2.28 | 6004 | 1.95 | 5602 | 4.22 | 4398 | 16 |
| 45 | 1742 | 2.28 | 5887 | | 5855 | 4.22 | 4145 | 15 |
| 46 | 1879 | 2.28 | 5770 | | 6109 | 4.22 | 3891 | 14 |
| 47 | 2015 | 2.27 | 5653 | | 6362 | 4.22 | 3638 | 13 |
| 48 | 2152 | 2.27 | 5536 | | 6616 | 4.22 | 3384 | 12 |
| 49 | 2288 | 2.27 | 5419 | | 6869 | 4.22 | 3131 | 11 |
| 50 | 2425 | 2.27 | 5302 | | 7123 | 4.22 | 2877 | 10 |
| 51 | 9.832561 | 2.27 | 9.865185 | | 9.967376 | 4.22 | 10.032624 | 9 |
| 52 | 2697 | 2.27 | 5068 | | 7629 | 4.22 | 2371 | 8 |
| 53 | 2833 | 2.27 | 4950 | 1.95 | 7883 | 4.22 | 2117 | 7 |
| 54 | 2969 | 2.26 | 4833 | 1.96 | 8136 | 4.22 | 1864 | 6 |
| 55 | 3105 | 2.26 | 4716 | | 8389 | 4.22 | 1611 | 5 |
| 56 | 3241 | 2.26 | 4598 | | 8643 | 4.22 | 1357 | 4 |
| 57 | 3377 | 2.26 | 4481 | | 8896 | 4.22 | 1104 | 3 |
| 58 | 3512 | 2.26 | 4363 | | 9149 | 4.22 | 0851 | 2 |
| 59 | 3648 | 2.26 | 4245 | 1.96 | 9403 | 4.22 | 0597 | 1 |
| 60 | 9.833783 | | 9.864127 | | 9.969656 | | 10.030344 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 47° |

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.833783 | 2.26 | 9.864127 | 1.96 | 9.969656 | 4.22 | 10.030344 | 60 |
| 1 | 3919 | 2.25 | 4010 | 1.96 | 9909 | 4.22 | 0091 | 59 |
| 2 | 4054 | 2.25 | 3892 | 1.97 | 9.970162 | 4.22 | 10.029838 | 58 |
| 3 | 4189 | 2.25 | 3774 | | 0416 | 4.22 | 9584 | 57 |
| 4 | 4325 | 2.25 | 3656 | | 0669 | 4.22 | 9331 | 56 |
| 5 | 4460 | 2.25 | 3538 | | 0922 | 4.22 | 9078 | 55 |
| 6 | 4595 | 2.25 | 3419 | | 1175 | 4.22 | 8825 | 54 |
| 7 | 4730 | 2.25 | 3301 | | 1429 | 4.22 | 8571 | 53 |
| 8 | 4865 | 2.25 | 3183 | | 1682 | 4.22 | 8318 | 52 |
| 9 | 4999 | 2.24 | 3064 | 1.97 | 1935 | 4.22 | 8065 | 51 |
| 10 | 5134 | 2.24 | 2946 | 1.98 | 2188 | 4.22 | 7812 | 50 |
| 11 | 9.835269 | 2.24 | 9.862827 | | 9.972441 | 4.22 | 10.027559 | 49 |
| 12 | 5403 | 2.24 | 2709 | | 2695 | 4.22 | 7305 | 48 |
| 13 | 5538 | 2.24 | 2590 | | 2948 | 4.22 | 7052 | 47 |
| 14 | 5672 | 2.24 | 2471 | | 3201 | 4.22 | 6799 | 46 |
| 15 | 5807 | 2.24 | 2353 | | 3454 | 4.22 | 6546 | 45 |
| 16 | 5941 | 2.24 | 2234 | | 3707 | 4.22 | 6293 | 44 |
| 17 | 6075 | 2.23 | 2115 | | 3960 | 4.22 | 6040 | 43 |
| 18 | 6209 | 2.23 | 1996 | | 4213 | 4.22 | 5787 | 42 |
| 19 | 6343 | 2.23 | 1877 | 1.98 | 4466 | 4.22 | 5534 | 41 |
| 20 | 6477 | 2.23 | 1758 | 1.99 | 4720 | 4.22 | 5280 | 40 |
| 21 | 9.836611 | 2.23 | 9.861638 | | 9.974973 | 4.22 | 10.025027 | 39 |
| 22 | 6745 | 2.23 | 1519 | | 5226 | 4.22 | 4774 | 38 |
| 23 | 6878 | 2.23 | 1400 | | 5479 | 4.22 | 4521 | 37 |
| 24 | 7012 | 2.22 | 1280 | | 5732 | 4.22 | 4268 | 36 |
| 25 | 7146 | 2.22 | 1161 | | 5985 | 4.22 | 4015 | 35 |
| 26 | 7279 | 2.22 | 1041 | | 6238 | 4.22 | 3762 | 34 |
| 27 | 7412 | 2.22 | 0922 | | 6491 | 4.22 | 3509 | 33 |
| 28 | 7546 | 2.22 | 0802 | 1.99 | 6744 | 4.22 | 3256 | 32 |
| 29 | 7679 | 2.22 | 0682 | 2.00 | 6997 | 4.22 | 3003 | 31 |
| 30 | 7812 | 2.22 | 0562 | | 7250 | 4.22 | 2750 | 30 |
| 31 | 9.837945 | 2.22 | 0442 | | 9.977503 | 4.22 | 10.022497 | 29 |
| 32 | 8078 | 2.21 | 0322 | | 7756 | 4.22 | 2244 | 28 |
| 33 | 8211 | 2.21 | 0202 | | 8009 | 4.22 | 1991 | 27 |
| 34 | 8344 | 2.21 | 9.860082 | | 8262 | 4.22 | 1738 | 26 |
| 35 | 8477 | 2.21 | 9.859962 | | 8515 | 4.22 | 1485 | 25 |
| 36 | 8610 | 2.21 | 9842 | 2.00 | 8768 | 4.22 | 1232 | 24 |
| 37 | 8742 | 2.21 | 9721 | 2.01 | 9021 | 4.22 | 0979 | 23 |
| 38 | 8875 | 2.21 | 9601 | | 9274 | 4.22 | 0726 | 22 |
| 39 | 9007 | 2.21 | 9480 | | 9527 | 4.22 | 0473 | 21 |
| 40 | 9140 | 2.20 | 9360 | | 9.979780 | 4.22 | 10.020220 | 20 |
| 41 | 9272 | 2.20 | 9239 | | 9.980033 | 4.22 | 10.019967 | 19 |
| 42 | 9404 | 2.20 | 9119 | | 0286 | 4.22 | 9714 | 18 |
| 43 | 9536 | 2.20 | 8998 | | 0538 | 4.22 | 9462 | 17 |
| 44 | 9668 | 2.20 | 8877 | 2.01 | 0791 | 4.21 | 9209 | 16 |
| 45 | 9800 | 2.20 | 8756 | 2.02 | 1044 | 4.21 | 8956 | 15 |
| 46 | 9.839932 | 2.20 | 8635 | | 1297 | 4.21 | 8703 | 14 |
| 47 | 9.840064 | 2.19 | 8514 | | 1550 | 4.21 | 8450 | 13 |
| 48 | 0196 | 2.19 | 8393 | | 1803 | 4.21 | 8197 | 12 |
| 49 | 0328 | 2.19 | 8272 | | 2056 | 4.21 | 7944 | 11 |
| 50 | 0459 | 2.19 | 8151 | | 2309 | 4.21 | 7691 | 10 |
| 51 | 0591 | 2.19 | 9.858029 | | 9.982562 | 4.21 | 10.017438 | 9 |
| 52 | 0722 | 2.19 | 7908 | | 2814 | 4.21 | 7186 | 8 |
| 53 | 0854 | 2.19 | 7786 | 2.02 | 3067 | 4.21 | 6933 | 7 |
| 54 | 0985 | 2.19 | 7665 | 2.03 | 3320 | 4.21 | 6680 | 6 |
| 55 | 1116 | 2.19 | 7543 | | 3573 | 4.21 | 6427 | 5 |
| 56 | 1247 | 2.18 | 7422 | | 3826 | 4.21 | 6174 | 4 |
| 57 | 1378 | 2.18 | 7300 | | 4079 | 4.21 | 5921 | 3 |
| 58 | 1509 | 2.18 | 7178 | | 4332 | 4.21 | 5668 | 2 |
| 59 | 1640 | 2.18 | 7056 | 2.03 | 4584 | 4.21 | 5416 | 1 |
| 60 | 9.841771 | | 9.856934 | | 9.984837 | | 10.015163 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 46° |

| M. | Sine. | Diff. 1″ | Cosine. | Diff. 1″ | Tang. | Diff. 1″ | Cotang. | M. |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.841771 | 2.18 | 9.856934 | 2.03 | 9.984837 | 4.21 | 10.015163 | 60 |
| 1 | 1902 | 2.18 | 6812 | 2.03 | 5090 | 4.21 | 4910 | 59 |
| 2 | 2033 | 2.18 | 6690 | 2.04 | 5343 | 4.21 | 4657 | 58 |
| 3 | 2163 | 2.17 | 6568 | | 5596 | 4.21 | 4404 | 57 |
| 4 | 2294 | 2.17 | 6446 | | 5848 | 4.21 | 4152 | 56 |
| 5 | 2424 | 2.17 | 6323 | | 6101 | 4.21 | 3899 | 55 |
| 6 | 2555 | 2.17 | 6201 | | 6354 | 4.21 | 3646 | 54 |
| 7 | 2685 | 2.17 | 6078 | | 6607 | 4.21 | 3393 | 53 |
| 8 | 2815 | 2.17 | 5956 | | 6860 | 4.21 | 3140 | 52 |
| 9 | 2946 | 2.17 | 5833 | 2.04 | 7112 | 4.21 | 2888 | 51 |
| 10 | 3076 | 2.17 | 5711 | 2.05 | 7365 | 4.21 | 2635 | 50 |
| 11 | 9.843206 | 2.16 | 9.855588 | | 9.987618 | 4.21 | 10.012382 | 49 |
| 12 | 3336 | 2.16 | 5465 | | 7871 | 4.21 | 2129 | 48 |
| 13 | 3466 | 2.16 | 5342 | | 8123 | 4.21 | 1877 | 47 |
| 14 | 3595 | 2.16 | 5219 | | 8376 | 4.21 | 1624 | 46 |
| 15 | 3725 | 2.16 | 5096 | | 8629 | 4.21 | 1371 | 45 |
| 16 | 3855 | 2.16 | 4973 | | 8882 | 4.21 | 1118 | 44 |
| 17 | 3984 | 2.16 | 4850 | | 9134 | 4.21 | 0866 | 43 |
| 18 | 4114 | 2.16 | 4727 | 2.05 | 9387 | 4.21 | 0613 | 42 |
| 19 | 4243 | 2.15 | 4603 | 2.06 | 9640 | 4.21 | 0360 | 41 |
| 20 | 4372 | 2.15 | 4480 | | 9.989893 | 4.21 | 10.010107 | 40 |
| 21 | 9.844502 | 2.15 | 9.854356 | | 9.990145 | 4.21 | 10.009855 | 39 |
| 22 | 4631 | 2.15 | 4233 | | 0398 | 4.21 | 9602 | 38 |
| 23 | 4760 | 2.15 | 4109 | | 0651 | 4.21 | 9349 | 37 |
| 24 | 4889 | 2.15 | 3986 | | 0903 | 4.21 | 9097 | 36 |
| 25 | 5018 | 2.15 | 3862 | | 1156 | 4.21 | 8844 | 35 |
| 26 | 5147 | 2.15 | 3738 | 2.06 | 1409 | 4.21 | 8591 | 34 |
| 27 | 5276 | 2.14 | 3614 | 2.07 | 1662 | 4.21 | 8338 | 33 |
| 28 | 5405 | 2.14 | 3490 | | 1914 | 4.21 | 8086 | 32 |
| 29 | 5533 | 2.14 | 3366 | | 2167 | 4.21 | 7833 | 31 |
| 30 | 5662 | 2.14 | 3242 | | 2420 | 4.21 | 7580 | 30 |
| 31 | 9.845790 | 2.14 | 9.853118 | | 9.992672 | 4.21 | 10.007328 | 29 |
| 32 | 5919 | 2.14 | 2994 | | 2925 | 4.21 | 7075 | 28 |
| 33 | 6047 | 2.14 | 2869 | | 3178 | 4.21 | 6822 | 27 |
| 34 | 6175 | 2.14 | 2745 | | 3431 | 4.21 | 6569 | 26 |
| 35 | 6304 | 2.14 | 2620 | 2.07 | 3683 | 4.21 | 6317 | 25 |
| 36 | 6432 | 2.13 | 2496 | 2.08 | 3936 | 4.21 | 6064 | 24 |
| 37 | 6560 | 2.13 | 2371 | | 4189 | 4.21 | 5811 | 23 |
| 38 | 6688 | 2.13 | 2247 | | 4441 | 4.21 | 5559 | 22 |
| 39 | 6816 | 2.13 | 2122 | | 4694 | 4.21 | 5306 | 21 |
| 40 | 6944 | 2.13 | 1997 | | 4947 | 4.21 | 5053 | 20 |
| 41 | 9.847071 | 2.13 | 9.851872 | | 9.995199 | 4.21 | 10.004801 | 19 |
| 42 | 7199 | 2.13 | 1747 | | 5452 | 4.21 | 4548 | 18 |
| 43 | 7327 | 2.13 | 1622 | 2.08 | 5705 | 4.21 | 4295 | 17 |
| 44 | 7454 | 2.12 | 1497 | 2.09 | 5957 | 4.21 | 4043 | 16 |
| 45 | 7582 | 2.12 | 1372 | | 6210 | 4.21 | 3790 | 15 |
| 46 | 7709 | 2.12 | 1246 | | 6463 | 4.21 | 3537 | 14 |
| 47 | 7836 | 2.12 | 1121 | | 6715 | 4.21 | 3285 | 13 |
| 48 | 7964 | 2.12 | 0996 | | 6968 | 4.21 | 3032 | 12 |
| 49 | 8091 | 2.12 | 0870 | | 7221 | 4.21 | 2779 | 11 |
| 50 | 8218 | 2.12 | 0745 | | 7473 | 4.21 | 2527 | 10 |
| 51 | 9.848345 | 2.12 | 9.850619 | 2.09 | 9.997726 | 4.21 | 10.002274 | 9 |
| 52 | 8472 | 2.11 | 0493 | 2.10 | 7979 | 4.21 | 2021 | 8 |
| 53 | 8599 | 2.11 | 0368 | | 8231 | 4.21 | 1769 | 7 |
| 54 | 8726 | 2.11 | 0242 | | 8484 | 4.21 | 1516 | 6 |
| 55 | 8852 | 2.11 | 9.850116 | | 8737 | 4.21 | 1263 | 5 |
| 56 | 8979 | 2.11 | 9.849990 | | 8989 | 4.21 | 1011 | 4 |
| 57 | 9106 | 2.11 | 9864 | | 9242 | 4.21 | 0758 | 3 |
| 58 | 9232 | 2.11 | 9738 | | 9495 | 4.21 | 0505 | 2 |
| 59 | 9359 | 2.11 | 9611 | 2.10 | 9.999747 | 4.21 | 0253 | 1 |
| 60 | 9.849485 | | 9.849485 | | 10.000000 | | 10.000000 | 0 |
| | Cosine. | Diff. 1″ | Sine. | Diff. 1″ | Cotang. | Diff. 1″ | Tang. | 45° |

## PROPORTIONAL

| D. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 44 | 4 | 9 | 13 | 18 | 22 | 26 | 31 | 35 | 40 |
| 45 | 5 | 9 | 14 | 18 | 23 | 27 | 32 | 36 | 41 |
| 46 | 5 | 9 | 14 | 18 | 23 | 28 | 32 | 37 | 41 |
| 47 | 5 | 9 | 14 | 19 | 24 | 28 | 33 | 38 | 42 |
| 48 | 5 | 10 | 14 | 19 | 24 | 29 | 34 | 38 | 43 |
| 49 | 5 | 10 | 15 | 20 | 25 | 29 | 34 | 39 | 44 |
| 50 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 |
| 51 | 5 | 10 | 15 | 20 | 26 | 31 | 36 | 41 | 46 |
| 52 | 5 | 10 | 16 | 21 | 26 | 31 | 36 | 42 | 47 |
| 53 | 5 | 11 | 16 | 21 | 27 | 32 | 37 | 42 | 48 |
| 54 | 5 | 11 | 16 | 22 | 27 | 32 | 38 | 43 | 49 |
| 55 | 6 | 11 | 17 | 22 | 28 | 33 | 39 | 44 | 50 |
| 56 | 6 | 11 | 17 | 22 | 28 | 34 | 39 | 45 | 50 |
| 57 | 6 | 11 | 17 | 23 | 29 | 34 | 40 | 46 | 51 |
| 58 | 6 | 12 | 17 | 23 | 29 | 35 | 41 | 46 | 52 |
| 59 | 6 | 12 | 18 | 24 | 30 | 35 | 41 | 47 | 53 |
| 60 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 |
| 61 | 6 | 12 | 18 | 24 | 31 | 37 | 43 | 49 | 55 |
| 62 | 6 | 12 | 19 | 25 | 31 | 37 | 43 | 50 | 56 |
| 63 | 6 | 13 | 19 | 25 | 32 | 38 | 44 | 50 | 57 |
| 64 | 6 | 13 | 19 | 26 | 32 | 38 | 45 | 51 | 58 |
| 65 | 7 | 13 | 20 | 26 | 33 | 39 | 46 | 52 | 59 |
| 66 | 7 | 13 | 20 | 26 | 33 | 40 | 46 | 53 | 59 |
| 67 | 7 | 13 | 20 | 27 | 34 | 40 | 47 | 54 | 60 |
| 68 | 7 | 14 | 20 | 27 | 34 | 41 | 48 | 54 | 61 |
| 69 | 7 | 14 | 21 | 28 | 35 | 41 | 48 | 55 | 62 |
| 70 | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 |
| 71 | 7 | 14 | 21 | 28 | 36 | 43 | 50 | 57 | 64 |
| 72 | 7 | 14 | 22 | 29 | 36 | 43 | 50 | 58 | 65 |
| 73 | 7 | 15 | 22 | 29 | 37 | 44 | 51 | 58 | 66 |
| 74 | 7 | 15 | 22 | 30 | 37 | 44 | 52 | 59 | 67 |
| 75 | 8 | 15 | 23 | 30 | 38 | 45 | 53 | 60 | 68 |
| 76 | 8 | 15 | 23 | 30 | 38 | 46 | 53 | 61 | 68 |
| 77 | 8 | 15 | 23 | 31 | 39 | 46 | 54 | 62 | 69 |
| 78 | 8 | 16 | 23 | 31 | 39 | 47 | 55 | 62 | 70 |
| 80 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 |
| 82 | 8 | 16 | 25 | 33 | 41 | 49 | 57 | 66 | 74 |
| 84 | 8 | 17 | 25 | 34 | 42 | 50 | 59 | 67 | 76 |
| 86 | 9 | 17 | 26 | 34 | 43 | 52 | 60 | 69 | 77 |
| 88 | 9 | 18 | 26 | 35 | 44 | 53 | 62 | 70 | 79 |
| 90 | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 |
| 92 | 9 | 18 | 28 | 37 | 46 | 55 | 64 | 74 | 83 |
| 94 | 9 | 19 | 28 | 38 | 47 | 56 | 66 | 75 | 85 |
| 96 | 10 | 19 | 29 | 38 | 48 | 58 | 67 | 77 | 86 |
| 98 | 10 | 20 | 29 | 39 | 49 | 59 | 69 | 78 | 88 |
| 100 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 |
| 104 | 10 | 21 | 31 | 42 | 52 | 62 | 73 | 83 | 94 |
| 108 | 11 | 22 | 32 | 43 | 54 | 65 | 76 | 86 | 97 |
| 112 | 11 | 22 | 34 | 45 | 56 | 67 | 78 | 90 | 101 |
| 116 | 12 | 23 | 35 | 46 | 58 | 70 | 81 | 93 | 104 |
| 120 | 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 |
| 124 | 12 | 25 | 37 | 50 | 62 | 74 | 87 | 99 | 112 |
| 128 | 13 | 26 | 38 | 51 | 64 | 77 | 90 | 102 | 115 |
| 132 | 13 | 26 | 40 | 53 | 66 | 79 | 92 | 106 | 119 |
| 136 | 14 | 27 | 41 | 54 | 68 | 82 | 95 | 109 | 122 |
| 140 | 14 | 28 | 42 | 56 | 70 | 84 | 98 | 112 | 126 |
| 144 | 14 | 29 | 43 | 58 | 72 | 86 | 101 | 115 | 130 |
| 148 | 15 | 30 | 44 | 59 | 74 | 89 | 104 | 118 | 133 |
| 152 | 15 | 30 | 46 | 61 | 76 | 91 | 106 | 122 | 137 |
| 156 | 16 | 31 | 47 | 62 | 78 | 94 | 109 | 125 | 140 |

| D. | PARTS. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 160 | 16 | 32 | 48 | 64 | 80 | 96 | 112 | 128 | 144 |
| 164 | 16 | 33 | 49 | 66 | 82 | 98 | 115 | 131 | 148 |
| 168 | 17 | 34 | 50 | 67 | 84 | 101 | 118 | 134 | 151 |
| 172 | 17 | 34 | 52 | 69 | 86 | 103 | 120 | 138 | 155 |
| 176 | 18 | 35 | 53 | 70 | 88 | 106 | 123 | 141 | 158 |
| 180 | 18 | 36 | 54 | 72 | 90 | 108 | 126 | 144 | 162 |
| 184 | 18 | 37 | 55 | 74 | 92 | 110 | 129 | 147 | 166 |
| 188 | 19 | 38 | 56 | 75 | 94 | 113 | 132 | 150 | 169 |
| 192 | 19 | 38 | 58 | 77 | 96 | 115 | 134 | 154 | 173 |
| 196 | 20 | 39 | 59 | 78 | 98 | 118 | 137 | 157 | 176 |
| 200 | 20 | 40 | 60 | 80 | 100 | 120 | 140 | 160 | 180 |
| 204 | 20 | 41 | 61 | 82 | 102 | 122 | 143 | 163 | 184 |
| 208 | 21 | 42 | 62 | 83 | 104 | 125 | 146 | 166 | 187 |
| 212 | 21 | 42 | 64 | 85 | 106 | 127 | 148 | 170 | 191 |
| 216 | 22 | 43 | 65 | 86 | 108 | 130 | 151 | 173 | 194 |
| 220 | 22 | 44 | 66 | 88 | 110 | 132 | 154 | 176 | 198 |
| 224 | 22 | 45 | 67 | 90 | 112 | 134 | 157 | 179 | 202 |
| 228 | 23 | 46 | 68 | 91 | 114 | 137 | 160 | 182 | 205 |
| 232 | 23 | 46 | 70 | 93 | 116 | 139 | 162 | 186 | 209 |
| 236 | 24 | 47 | 71 | 94 | 118 | 142 | 165 | 189 | 212 |
| 240 | 24 | 48 | 72 | 96 | 120 | 144 | 168 | 192 | 216 |
| 244 | 24 | 49 | 73 | 98 | 122 | 146 | 171 | 195 | 220 |
| 248 | 25 | 50 | 74 | 99 | 124 | 149 | 174 | 198 | 223 |
| 252 | 25 | 50 | 76 | 101 | 126 | 151 | 176 | 202 | 227 |
| 256 | 26 | 51 | 77 | 102 | 128 | 154 | 179 | 205 | 230 |
| 260 | 26 | 52 | 78 | 104 | 130 | 156 | 182 | 208 | 234 |
| 264 | 26 | 53 | 79 | 106 | 132 | 158 | 185 | 211 | 238 |
| 268 | 27 | 54 | 80 | 107 | 134 | 161 | 188 | 214 | 241 |
| 272 | 27 | 54 | 82 | 109 | 136 | 163 | 190 | 218 | 245 |
| 276 | 28 | 55 | 83 | 110 | 138 | 166 | 193 | 221 | 248 |
| 280 | 28 | 56 | 84 | 112 | 140 | 168 | 196 | 224 | 252 |
| 284 | 28 | 57 | 85 | 114 | 142 | 170 | 199 | 227 | 256 |
| 288 | 29 | 58 | 86 | 115 | 144 | 173 | 202 | 230 | 259 |
| 292 | 29 | 58 | 88 | 117 | 146 | 175 | 204 | 234 | 263 |
| 296 | 30 | 59 | 89 | 118 | 148 | 178 | 207 | 237 | 266 |
| 300 | 30 | 60 | 90 | 120 | 150 | 180 | 210 | 240 | 270 |
| 304 | 30 | 61 | 91 | 122 | 152 | 182 | 213 | 243 | 274 |
| 308 | 31 | 62 | 92 | 123 | 154 | 185 | 216 | 246 | 277 |
| 312 | 31 | 62 | 94 | 125 | 156 | 187 | 218 | 250 | 281 |
| 316 | 32 | 63 | 95 | 126 | 158 | 190 | 221 | 253 | 284 |
| 320 | 32 | 64 | 96 | 128 | 160 | 192 | 224 | 256 | 288 |
| 324 | 32 | 65 | 97 | 130 | 162 | 194 | 227 | 259 | 292 |
| 328 | 33 | 66 | 98 | 131 | 164 | 197 | 230 | 262 | 295 |
| 332 | 33 | 66 | 100 | 133 | 166 | 199 | 232 | 266 | 299 |
| 336 | 34 | 67 | 101 | 134 | 168 | 202 | 235 | 269 | 302 |
| 340 | 34 | 68 | 102 | 136 | 170 | 204 | 238 | 272 | 306 |
| 344 | 34 | 69 | 103 | 138 | 172 | 206 | 241 | 275 | 310 |
| 348 | 35 | 70 | 104 | 139 | 174 | 209 | 244 | 278 | 313 |
| 352 | 35 | 70 | 106 | 141 | 176 | 211 | 246 | 282 | 317 |
| 356 | 36 | 71 | 107 | 142 | 178 | 214 | 249 | 285 | 320 |
| 360 | 36 | 72 | 108 | 144 | 180 | 216 | 252 | 288 | 324 |
| 364 | 36 | 73 | 109 | 146 | 182 | 218 | 255 | 291 | 328 |
| 368 | 37 | 74 | 110 | 147 | 184 | 221 | 258 | 294 | 331 |
| 372 | 37 | 74 | 112 | 149 | 186 | 223 | 260 | 298 | 335 |
| 376 | 38 | 75 | 113 | 150 | 188 | 226 | 263 | 301 | 338 |
| 380 | 38 | 76 | 114 | 152 | 190 | 228 | 266 | 304 | 342 |
| 384 | 38 | 77 | 115 | 154 | 192 | 230 | 269 | 307 | 346 |
| 388 | 39 | 78 | 116 | 155 | 194 | 233 | 272 | 310 | 349 |
| 392 | 39 | 78 | 118 | 157 | 196 | 235 | 274 | 314 | 353 |
| 396 | 40 | 79 | 119 | 158 | 198 | 238 | 277 | 317 | 356 |

# TRAVERSE TABLE;

OR,

## TABLE OF

## Differences of Latitude and Departures.

| D. | ¼ Deg. | | ½ Deg. | | ¾ Deg. | | 1 Deg. | | D. |
|---|---|---|---|---|---|---|---|---|---|
| | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | |
| 1 | 1.0000 | .0044 | 1.0000 | .0087 | .9999 | .0131 | .9998 | .0175 | 1 |
| 2 | 2.0000 | .0087 | 1.9999 | .0175 | 1.9998 | .0262 | 1.9997 | .0349 | 2 |
| 3 | 3.0000 | .0131 | 2.9999 | .0262 | 2.9997 | .0393 | 2.9995 | .0524 | 3 |
| 4 | 4.0000 | .0175 | 3.9998 | .0349 | 3.9997 | .0524 | 3.9994 | .0698 | 4 |
| 5 | 5.0000 | .0218 | 4.9998 | .0436 | 4.9996 | .0654 | 4.9992 | .0873 | 5 |
| 6 | 5.9999 | .0262 | 5.9998 | .0524 | 5.9995 | .0785 | 5.9991 | .1047 | 6 |
| 7 | 6.9999 | .0305 | 6.9997 | .0611 | 6.9994 | .0916 | 6.9989 | .1222 | 7 |
| 8 | 7.9999 | .0349 | 7.9997 | .0698 | 7.9993 | .1047 | 7.9988 | .1396 | 8 |
| 9 | 8.9999 | .0393 | 8.9997 | .0785 | 8.9992 | .1178 | 8.9986 | .1571 | 9 |
| 10 | 9.9999 | .0436 | 9.9996 | .0873 | 9.9991 | .1309 | 9.9985 | .1745 | 10 |
| | 89¾ Deg. | | 89½ Deg. | | 89¼ Deg. | | 89 Deg. | | |
| | 1¼ Deg. | | 1½ Deg. | | 1¾ Deg. | | 2 Deg. | | |
| 1 | .9998 | .0218 | .9997 | .0262 | .9995 | .0305 | .9994 | .0349 | 1 |
| 2 | 1.9995 | .0436 | 1.9993 | .0524 | 1.9991 | .0611 | 1.9988 | .0698 | 2 |
| 3 | 2.9993 | .0654 | 2.9990 | .0785 | 2.9986 | .0916 | 2.9982 | .1047 | 3 |
| 4 | 3.9990 | .0873 | 3.9986 | .1047 | 3.9981 | .1222 | 3.9976 | .1396 | 4 |
| 5 | 4.9988 | .1091 | 4.9983 | .1309 | 4.9977 | .1527 | 4.9970 | .1745 | 5 |
| 6 | 5.9986 | .1309 | 5.9979 | .1571 | 5.9972 | .1832 | 5.9963 | .2094 | 6 |
| 7 | 6.9983 | .1527 | 6.9976 | .1832 | 6.9967 | .2138 | 6.9957 | .2443 | 7 |
| 8 | 7.9981 | .1745 | 7.9973 | .2094 | 7.9963 | .2443 | 7.9951 | .2792 | 8 |
| 9 | 8.9979 | .1963 | 8.9969 | .2356 | 8.9958 | .2748 | 8.9945 | .3141 | 9 |
| 10 | 9.9976 | .2181 | 9.9966 | .2618 | 9.9953 | .3054 | 9.9939 | .3490 | 10 |
| | 88¾ Deg. | | 88½ Deg. | | 88¼ Deg. | | 88 Deg. | | |
| | 2¼ Deg. | | 2½ Deg. | | 2¾ Deg. | | 3 Deg. | | |
| 1 | .9992 | .0393 | .9990 | .0436 | .9988 | .0480 | .9986 | .0523 | 1 |
| 2 | 1.9985 | .0785 | 1.9981 | .0872 | 1.9977 | .0960 | 1.9973 | .1047 | 2 |
| 3 | 2.9977 | .1178 | 2.9971 | .1308 | 2.9965 | .1439 | 2.9959 | .1570 | 3 |
| 4 | 3.9969 | .1570 | 3.9962 | .1745 | 3.9954 | .1919 | 3.9945 | .2093 | 4 |
| 5 | 4.9961 | .1963 | 4.9952 | .2181 | 4.9942 | .2399 | 4.9931 | .2617 | 5 |
| 6 | 5.9954 | .2356 | 5.9943 | .2617 | 5.9931 | .2879 | 5.9918 | .3140 | 6 |
| 7 | 6.9946 | .2748 | 6.9933 | .3053 | 6.9919 | .3358 | 6.9904 | .3664 | 7 |
| 8 | 7.9938 | .3140 | 7.9924 | .3490 | 7.9908 | .3838 | 7.9890 | .4187 | 8 |
| 9 | 8.9931 | .3533 | 8.9914 | .3926 | 8.9896 | .4318 | 8.9877 | .4710 | 9 |
| 10 | 9.9913 | .3926 | 9.9905 | .4362 | 9.9885 | .4798 | 9.9863 | .5234 | 10 |
| | 87¾ Deg. | | 87½ Deg. | | 87¼ Deg. | | 87 Deg. | | |
| | 3¼ Deg. | | 3½ Deg. | | 3¾ Deg. | | 4 Deg. | | |
| 1 | .9984 | .0567 | .9981 | .0610 | .9979 | .0654 | .9976 | .0698 | 1 |
| 2 | 1.9968 | .1134 | 1.9963 | .1221 | 1.9957 | .1308 | 1.9951 | .1395 | 2 |
| 3 | 2.9952 | .1701 | 2.9944 | .1831 | 2.9936 | .1962 | 2.9927 | .2093 | 3 |
| 4 | 3.9936 | .2268 | 3.9925 | .2442 | 3.9914 | .2616 | 3.9903 | .2790 | 4 |
| 5 | 4.9920 | .2835 | 4.9907 | .3052 | 4.9893 | .3270 | 4.9878 | .3488 | 5 |
| 6 | 5.9904 | .3402 | 5.9888 | .3663 | 5.9872 | .3924 | 5.9854 | .4185 | 6 |
| 7 | 6.9887 | .3968 | 6.9869 | .4273 | 6.9850 | .4578 | 6.9829 | .4883 | 7 |
| 8 | 7.9871 | .4535 | 7.9851 | .4884 | 7.9829 | .5232 | 7.9805 | .5581 | 8 |
| 9 | 8.9855 | .5102 | 8.9832 | .5494 | 8.9807 | .5886 | 8.9781 | .6278 | 9 |
| 10 | 9.9839 | .5669 | 9.9813 | .6105 | 9.9786 | .6540 | 9.9756 | .6976 | 10 |
| D. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | D. |
| | 86¾ Deg. | | 86½ Deg. | | 86¼ Deg. | | 86 Deg. | | |

| D. | 4¼ Deg. | | 4½ Deg. | | 4¾ Deg. | | 5 Deg. | | D. |
|---|---|---|---|---|---|---|---|---|---|
| | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | |
| 1 | .9973 | .0741 | .9969 | .0785 | .9966 | .0828 | .9962 | .0872 | 1 |
| 2 | 1.9945 | .1482 | 1.9938 | .1569 | 1.9931 | .1656 | 1.9924 | .1743 | 2 |
| 3 | 2.9918 | .2223 | 2.9908 | .2354 | 2.9897 | .2484 | 2.9886 | .2615 | 3 |
| 4 | 3.9890 | .2964 | 3.9877 | .3138 | 3.9863 | .3312 | 3.9848 | .3486 | 4 |
| 5 | 4.9863 | .3705 | 4.9846 | .3923 | 4.9828 | .4140 | 4.9810 | .4358 | 5 |
| 6 | 5.9835 | .4447 | 5.9815 | .4708 | 5.9794 | .4968 | 5.9772 | .5229 | 6 |
| 7 | 6.9808 | .5188 | 6.9784 | .5492 | 6.9760 | .5797 | 6.9734 | .6101 | 7 |
| 8 | 7.9780 | .5929 | 7.9753 | .6277 | 7.9725 | .6625 | 7.9696 | .6972 | 8 |
| 9 | 8.9753 | .6670 | 8.9723 | .7061 | 8.9691 | .7453 | 8.9658 | .7844 | 9 |
| 10 | 9.9725 | .7411 | 9.9692 | .7846 | 9.9657 | .8281 | 9.9619 | .8716 | 10 |
| | 85¾ Deg. | | 85½ Deg. | | 85¼ Deg. | | 85 Deg. | | |
| | 5¼ Deg. | | 5½ Deg. | | 5¾ Deg. | | 6 Deg. | | |
| 1 | .9958 | .0915 | .9954 | .0958 | .9950 | .1002 | .9945 | .1045 | 1 |
| 2 | 1.9916 | .1830 | 1.9908 | .1917 | 1.9899 | .2004 | 1.9890 | .2091 | 2 |
| 3 | 2.9874 | .2745 | 2.9862 | .2875 | 2.9849 | .3006 | 2.9836 | .3136 | 3 |
| 4 | 3.9832 | .3660 | 3.9816 | .3834 | 3.9799 | .4008 | 3.9781 | .4181 | 4 |
| 5 | 4.9790 | .4575 | 4.9770 | .4792 | 4.9748 | .5009 | 4.9726 | .5226 | 5 |
| 6 | 5.9748 | .5490 | 5.9724 | .5751 | 5.9698 | .6011 | 5.9671 | .6272 | 6 |
| 7 | 6.9706 | .6405 | 6.9678 | .6709 | 6.9648 | .7013 | 6.9617 | .7317 | 7 |
| 8 | 7.9664 | .7320 | 7.9632 | .7668 | 7.9597 | .8015 | 7.9562 | .8362 | 8 |
| 9 | 8.9622 | .8235 | 8.9586 | .8626 | 8.9547 | .9017 | 8.9507 | .9408 | 9 |
| 10 | 9.9580 | .9150 | 9.9540 | .9585 | 9.9497 | 1.0019 | 9.9452 | 1.0453 | 10 |
| | 84¾ Deg. | | 84½ Deg. | | 84¼ Deg. | | 84 Deg. | | |
| | 6¼ Deg. | | 6½ Deg. | | 6¾ Deg. | | 7 Deg. | | |
| 1 | .9941 | .1089 | .9936 | .1132 | .9931 | .1175 | .9925 | .1219 | 1 |
| 2 | 1.9881 | .2177 | 1.9871 | .2264 | 1.9861 | .2351 | 1.9851 | .2437 | 2 |
| 3 | 2.9822 | .3266 | 2.9807 | .3396 | 2.9792 | .3526 | 2.9776 | .3656 | 3 |
| 4 | 3.9762 | .4355 | 3.9743 | .4528 | 3.9723 | .4701 | 3.9702 | .4875 | 4 |
| 5 | 4.9703 | .5443 | 4.9679 | .5660 | 4.9653 | .5877 | 4.9627 | .6093 | 5 |
| 6 | 5.9643 | .6532 | 5.9614 | .6792 | 5.9584 | .7052 | 5.9553 | .7312 | 6 |
| 7 | 6.9584 | .7621 | 6.9550 | .7924 | 6.9515 | .8228 | 6.9478 | .8531 | 7 |
| 8 | 7.9524 | .8709 | 7.9486 | .9056 | 7.9445 | .9403 | 7.9404 | .9750 | 8 |
| 9 | 8.9465 | .9798 | 8.9421 | 1.0188 | 8.9376 | 1.0578 | 8.9329 | 1.0968 | 9 |
| 10 | 9.9406 | 1.0887 | 9.9357 | 1.1320 | 9.9307 | 1.1754 | 9.9255 | 1.2187 | 10 |
| | 83¾ Deg. | | 83½ Deg. | | 83¼ Deg. | | 83 Deg. | | |
| | 7¼ Deg. | | 7½ Deg. | | 7¾ Deg. | | 8 Deg. | | |
| 1 | .9920 | .1262 | .9914 | .1305 | .9909 | .1349 | .9903 | .1392 | 1 |
| 2 | 1.9840 | .2524 | 1.9829 | .2611 | 1.9817 | .2697 | 1.9805 | .2783 | 2 |
| 3 | 2.9760 | .3786 | 2.9743 | .3916 | 2.9726 | .4046 | 2.9708 | .4175 | 3 |
| 4 | 3.9680 | .5048 | 3.9658 | .5221 | 3.9635 | .5394 | 3.9611 | .5567 | 4 |
| 5 | 4.9600 | .6310 | 4.9572 | .6526 | 4.9543 | .6743 | 4.9513 | .6959 | 5 |
| 6 | 5.9520 | .7572 | 5.9487 | .7832 | 5.9452 | .8091 | 5.9416 | .8350 | 6 |
| 7 | 6.9440 | .8834 | 6.9401 | .9137 | 6.9361 | .9440 | 6.9319 | .9742 | 7 |
| 8 | 7.9360 | 1.0096 | 7.9316 | 1.0442 | 7.9269 | 1.0788 | 7.9221 | 1.1134 | 8 |
| 9 | 8.9280 | 1.1358 | 8.9230 | 1.1747 | 8.9178 | 1.2137 | 8.9124 | 1.2526 | 9 |
| 10 | 9.9200 | 1.2620 | 9.9144 | 1.3053 | 9.9087 | 1.3485 | 9.9027 | 1.3917 | 10 |
| D. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | D. |
| | 82¾ Deg. | | 82½ Deg. | | 82¼ Deg. | | 82 Deg. | | |

| D. | 8¼ Deg. | | 8½ Deg. | | 8¾ Deg. | | 9 Deg. | | D. |
|---|---|---|---|---|---|---|---|---|---|
| | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | |
| 1 | .9897 | .1435 | .9890 | .1478 | .9884 | .1521 | .9877 | .1564 | 1 |
| 2 | 1.9793 | .2870 | 1.9780 | .2956 | 1.9767 | .3042 | 1.9754 | .3129 | 2 |
| 3 | 2.9690 | .4305 | 2.9670 | .4434 | 2.9651 | .4564 | 2.9631 | .4693 | 3 |
| 4 | 3.9586 | .5740 | 3.9561 | .5912 | 3.9534 | .6085 | 3.9508 | .6257 | 4 |
| 5 | 4.9483 | .7175 | 4.9451 | .7390 | 4.9418 | .7606 | 4.9384 | .7822 | 5 |
| 6 | 5.9379 | .8610 | 5.9341 | .8869 | 5.9302 | .9127 | 5.9261 | .9386 | 6 |
| 7 | 6.9276 | 1.0044 | 6.9231 | 1.0347 | 6.9185 | 1.0649 | 6.9138 | 1.0950 | 7 |
| 8 | 7.9172 | 1.1479 | 7.9121 | 1.1825 | 7.9069 | 1.2170 | 7.9015 | 1.2515 | 8 |
| 9 | 8.9069 | 1.2914 | 8.9011 | 1.3303 | 8.8953 | 1.3691 | 8.8892 | 1.4079 | 9 |
| 10 | 9.8965 | 1.4349 | 9.8902 | 1.4781 | 9.8836 | 1.5212 | 9.8769 | 1.5643 | 10 |
| | 81¾ Deg. | | 81½ Deg. | | 81¼ Deg. | | 81 Deg. | | |
| | 9¼ Deg. | | 9½ Deg. | | 9¾ Deg. | | 10 Deg. | | |
| 1 | .9870 | .1607 | .9863 | .1650 | .9856 | .1693 | .9848 | .1736 | 1 |
| 2 | 1.9740 | .3215 | 1.9726 | .3301 | 1.9711 | .3387 | 1.9696 | .3473 | 2 |
| 3 | 2.9610 | .4822 | 2.9589 | .4951 | 2.9567 | .5080 | 2.9544 | .5209 | 3 |
| 4 | 3.9480 | .6430 | 3.9451 | .6602 | 3.9422 | .6774 | 3.9392 | .6946 | 4 |
| 5 | 4.9350 | .8037 | 4.9314 | .8252 | 4.9278 | .8467 | 4.9240 | .8682 | 5 |
| 6 | 5.9220 | .9645 | 5.9177 | .9903 | 5.9133 | 1.0161 | 5.9088 | 1.0419 | 6 |
| 7 | 6.9090 | 1.1252 | 6.9040 | 1.1553 | 6.8989 | 1.1854 | 6.8937 | 1.2155 | 7 |
| 8 | 7.8960 | 1.2859 | 7.8903 | 1.3204 | 7.8844 | 1.3548 | 7.8785 | 1.3892 | 8 |
| 9 | 8.8830 | 1.4467 | 8.8766 | 1.4854 | 8.8700 | 1.5241 | 8.8633 | 1.5628 | 9 |
| 10 | 9.8700 | 1.6074 | 9.8629 | 1.6505 | 9.8556 | 1.6935 | 9.8481 | 1.7365 | 10 |
| | 80¾ Deg. | | 80½ Deg. | | 80¼ Deg. | | 80 Deg. | | |
| | 10¼ Deg. | | 10½ Deg. | | 10¾ Deg. | | 11 Deg. | | |
| 1 | .9840 | .1779 | .9833 | .1822 | .9825 | .1865 | .9816 | .1908 | 1 |
| 2 | 1.9681 | .3559 | 1.9665 | .3645 | 1.9649 | .3730 | 1.9633 | .3816 | 2 |
| 3 | 2.9521 | .5338 | 2.9498 | .5467 | 2.9474 | .5596 | 2.9449 | .5724 | 3 |
| 4 | 3.9362 | .7118 | 3.9330 | .7289 | 3.9298 | .7461 | 3.9265 | .7632 | 4 |
| 5 | 4.9202 | .8897 | 4.9163 | .9112 | 4.9123 | .9326 | 4.9081 | .9540 | 5 |
| 6 | 5.9042 | 1.0677 | 5.8995 | 1.0934 | 5.8947 | 1.1191 | 5.8898 | 1.1449 | 6 |
| 7 | 6.8883 | 1.2456 | 6.8828 | 1.2756 | 6.8772 | 1.3057 | 6.8714 | 1.3357 | 7 |
| 8 | 7.8723 | 1.4235 | 7.8660 | 1.4579 | 7.8596 | 1.4922 | 7.8530 | 1.5265 | 8 |
| 9 | 8.8564 | 1.6015 | 8.8493 | 1.6401 | 8.8421 | 1.6787 | 8.8346 | 1.7173 | 9 |
| 10 | 9.8404 | 1.7794 | 9.8325 | 1.8224 | 9.8245 | 1.8652 | 9.8163 | 1.9081 | 10 |
| | 79¾ Deg. | | 79½ Deg. | | 79¼ Deg. | | 79 Deg. | | |
| | 11¼ Deg. | | 11½ Deg. | | 11¾ Deg. | | 12 Deg. | | |
| 1 | .9808 | .1951 | .9799 | .1994 | .9790 | .2036 | .9781 | .2079 | 1 |
| 2 | 1.9616 | .3902 | 1.9598 | .3987 | 1.9581 | .4073 | 1.9563 | .4158 | 2 |
| 3 | 2.9424 | .5853 | 2.9398 | .5981 | 2.9371 | .6109 | 2.9344 | .6237 | 3 |
| 4 | 3.9231 | .7804 | 3.9197 | .7975 | 3.9162 | .8146 | 3.9126 | .8316 | 4 |
| 5 | 4.9039 | .9755 | 4.8996 | .9968 | 4.8952 | 1.0182 | 4.8907 | 1.0396 | 5 |
| 6 | 5.8847 | 1.1705 | 5.8795 | 1.1962 | 5.8743 | 1.2219 | 5.8689 | 1.2475 | 6 |
| 7 | 6.8655 | 1.3656 | 6.8595 | 1.3956 | 6.8533 | 1.4255 | 6.8470 | 1.4554 | 7 |
| 8 | 7.8463 | 1.5607 | 7.8394 | 1.5949 | 7.8324 | 1.6291 | 7.8252 | 1.6633 | 8 |
| 9 | 8.8271 | 1.7558 | 8.8193 | 1.7943 | 8.8114 | 1.8328 | 8.8033 | 1.8712 | 9 |
| 10 | 9.8079 | 1.9509 | 9.7992 | 1.9937 | 9.7905 | 2.0364 | 9.7815 | 2.0791 | 10 |
| D. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | D. |
| | 78¾ Deg. | | 78½ Deg. | | 78¼ Deg. | | 78 Deg. | | |

| D. | 12¼ Deg. | | 12½ Deg. | | 12¾ Deg. | | 13 Deg. | | D. |
|---|---|---|---|---|---|---|---|---|---|
| | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | |
| 1 | .9772 | .2122 | .9763 | .2164 | .9753 | .2207 | .9744 | .2250 | 1 |
| 2 | 1.9545 | .4244 | 1.9526 | .4329 | 1.9507 | .4414 | 1.9487 | .4499 | 2 |
| 3 | 2.9317 | .6365 | 2.9289 | .6493 | 2.9260 | .6621 | 2.9231 | .6749 | 3 |
| 4 | 3.9089 | .8487 | 3.9052 | .8658 | 3.9014 | .8828 | 3.8975 | .8998 | 4 |
| 5 | 4.8862 | 1.0609 | 4.8815 | 1.0822 | 4.8767 | 1.1035 | 4.8719 | 1.1248 | 5 |
| 6 | 5.8634 | 1.2731 | 5.8578 | 1.2986 | 5.8521 | 1.3242 | 5.8462 | 1.3497 | 6 |
| 7 | 6.8406 | 1.4852 | 6.8341 | 1.5151 | 6.8274 | 1.5449 | 6.8206 | 1.5747 | 7 |
| 8 | 7.8178 | 1.6974 | 7.8104 | 1.7315 | 7.8027 | 1.7656 | 7.7950 | 1.7996 | 8 |
| 9 | 8.7951 | 1.9096 | 8.7867 | 1.9480 | 8.7781 | 1.9863 | 8.7693 | 2.0246 | 9 |
| 10 | 9.7723 | 2.1218 | 9.7630 | 2.1644 | 9.7534 | 2.2070 | 9.7437 | 2.2495 | 10 |
| | 77¾ Deg. | | 77½ Deg. | | 77¼ Deg. | | 77 Deg. | | |
| | 13¼ Deg. | | 13½ Deg. | | 13¾ Deg. | | 14 Deg. | | |
| 1 | .9734 | .2292 | .9724 | .2334 | .9713 | .2377 | .9703 | .2419 | 1 |
| 2 | 1.9468 | .4584 | 1.9447 | .4669 | 1.9427 | .4754 | 1.9406 | .4838 | 2 |
| 3 | 2.9201 | .6876 | 2.9171 | .7003 | 2.9140 | .7131 | 2.9109 | .7258 | 3 |
| 4 | 3.8935 | .9168 | 3.8895 | .9338 | 3.8854 | .9507 | 3.8812 | .9677 | 4 |
| 5 | 4.8669 | 1.1460 | 4.8618 | 1.1672 | 4.8567 | 1.1884 | 4.8515 | 1.2096 | 5 |
| 6 | 5.8403 | 1.3752 | 5.8342 | 1.4007 | 5.8281 | 1.4261 | 5.8218 | 1.4515 | 6 |
| 7 | 6.8137 | 1.6044 | 6.8066 | 1.6341 | 6.7994 | 1.6638 | 6.7921 | 1.6935 | 7 |
| 8 | 7.7870 | 1.8336 | 7.7790 | 1.8676 | 7.7707 | 1.9015 | 7.7624 | 1.9354 | 8 |
| 9 | 8.7604 | 2.0628 | 8.7513 | 2.1010 | 8.7421 | 2.1392 | 8.7327 | 2.1773 | 9 |
| 10 | 9.7338 | 2.2920 | 9.7237 | 2.3345 | 9.7134 | 2.3769 | 9.7030 | 2.4192 | 10 |
| | 76¾ Deg. | | 76½ Deg. | | 76¼ Deg. | | 76 Deg. | | |
| | 14¼ Deg. | | 14½ Deg. | | 14¾ Deg. | | 15 Deg. | | |
| 1 | .9692 | .2462 | .9681 | .2504 | .9670 | .2546 | .9659 | .2588 | 1 |
| 2 | 1.9385 | .4923 | 1.9363 | .5008 | 1.9341 | .5092 | 1.9319 | .5176 | 2 |
| 3 | 2.9077 | .7385 | 2.9044 | .7511 | 2.9011 | .7638 | 2.8978 | .7765 | 3 |
| 4 | 3.8769 | .9846 | 3.8726 | 1.0015 | 3.8682 | 1.0184 | 3.8637 | 1.0353 | 4 |
| 5 | 4.8462 | 1.2308 | 4.8407 | 1.2519 | 4.8352 | 1.2730 | 4.8296 | 1.2941 | 5 |
| 6 | 5.8154 | 1.4769 | 5.8089 | 1.5023 | 5.8023 | 1.5276 | 5.7956 | 1.5529 | 6 |
| 7 | 6.7846 | 1.7231 | 6.7770 | 1.7527 | 6.7693 | 1.7822 | 6.7615 | 1.8117 | 7 |
| 8 | 7.7538 | 1.9692 | 7.7452 | 2.0030 | 7.7364 | 2.0368 | 7.7274 | 2.0706 | 8 |
| 9 | 8.7231 | 2.2154 | 8.7133 | 2.2534 | 8.7034 | 2.2914 | 8.6933 | 2.3294 | 9 |
| 10 | 9.6923 | 2.4615 | 9.6815 | 2.5038 | 9.6705 | 2.5460 | 9.6593 | 2.5882 | 10 |
| | 75¾ Deg. | | 75½ Deg. | | 75¼ Deg. | | 75 Deg. | | |
| | 15¼ Deg. | | 15½ Deg. | | 15¾ Deg. | | 16 Deg. | | |
| 1 | .9648 | .2630 | .9636 | .2672 | .9625 | .2714 | .9613 | .2756 | 1 |
| 2 | 1.9296 | .5261 | 1.9273 | .5345 | 1.9249 | .5429 | 1.9225 | .5513 | 2 |
| 3 | 2.8944 | .7891 | 2.8909 | .8017 | 2.8874 | .8143 | 2.8838 | .8269 | 3 |
| 4 | 3.8591 | 1.0521 | 3.8545 | 1.0690 | 3.8498 | 1.0858 | 3.8450 | 1.1025 | 4 |
| 5 | 4.8239 | 1.3152 | 4.8182 | 1.3362 | 4.8123 | 1.3572 | 4.8063 | 1.3782 | 5 |
| 6 | 5.7887 | 1.5782 | 5.7818 | 1.6034 | 5.7747 | 1.6286 | 5.7676 | 1.6538 | 6 |
| 7 | 6.7535 | 1.8412 | 6.7454 | 1.8707 | 6.7372 | 1.9001 | 6.7288 | 1.9295 | 7 |
| 8 | 7.7183 | 2.1042 | 7.7090 | 2.1379 | 7.6996 | 2.1715 | 7.6901 | 2.2051 | 8 |
| 9 | 8.6831 | 2.3673 | 8.6727 | 2.4051 | 8.6621 | 2.4430 | 8.6514 | 2.4807 | 9 |
| 10 | 9.6479 | 2.6303 | 9.6363 | 2.6724 | 9.6246 | 2.7144 | 9.6126 | 2.7564 | 10 |
| | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | |
| D. | 74¾ Deg. | | 74½ Deg. | | 74¼ Deg. | | 74 Deg. | | D. |

| D. | 16¼ Deg. | | 16½ Deg. | | 16¾ Deg. | | 17 Deg. | | D. |
|---|---|---|---|---|---|---|---|---|---|
| | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | |
| 1 | .9600 | .2798 | .9588 | .2840 | .9576 | .2882 | .9563 | .2924 | 1 |
| 2 | 1.9201 | .5597 | 1.9176 | .5680 | 1.9151 | .5764 | 1.9126 | .5847 | 2 |
| 3 | 2.8801 | .8395 | 2.8765 | .8520 | 2.8727 | .8646 | 2.8689 | .8771 | 3 |
| 4 | 3.8402 | 1.1193 | 3.8353 | 1.1361 | 3.8303 | 1.1528 | 3.8252 | 1.1695 | 4 |
| 5 | 4.8002 | 1.3991 | 4.7941 | 1.4201 | 4.7879 | 1.4410 | 4.7815 | 1.4619 | 5 |
| 6 | 5.7603 | 1.6790 | 5.7529 | 1.7041 | 5.7454 | 1.7292 | 5.7378 | 1.7542 | 6 |
| 7 | 6.7203 | 1.9588 | 6.7117 | 1.9881 | 6.7030 | 2.0174 | 6.6941 | 2.0466 | 7 |
| 8 | 7.6804 | 2.2386 | 7.6706 | 2.2721 | 7.6606 | 2.3056 | 7.6504 | 2.3390 | 8 |
| 9 | 8.6404 | 2.5185 | 8.6294 | 2.5561 | 8.6181 | 2.5938 | 8.6067 | 2.6313 | 9 |
| 10 | 9.6005 | 2.7983 | 9.5882 | 2.8402 | 9.5757 | 2.8820 | 9.5630 | 2.9237 | 10 |
| | 73¾ Deg. | | 73½ Deg. | | 73¼ Deg. | | 73 Deg. | | |
| | 17¼ Deg. | | 17½ Deg. | | 17¾ Deg. | | 18 Deg. | | |
| 1 | .9550 | .2965 | .9537 | .3007 | .9524 | .3049 | .9511 | .3090 | 1 |
| 2 | 1.9100 | .5931 | 1.9074 | .6014 | 1.9048 | .6097 | 1.9021 | .6180 | 2 |
| 3 | 2.8651 | .8896 | 2.8612 | .9021 | 2.8572 | .9146 | 2.8532 | .9271 | 3 |
| 4 | 3.8201 | 1.1862 | 3.8149 | 1.2028 | 3.8096 | 1.2195 | 3.8042 | 1.2361 | 4 |
| 5 | 4.7751 | 1.4827 | 4.7686 | 1.5035 | 4.7620 | 1.5243 | 4.7553 | 1.5451 | 5 |
| 6 | 5.7301 | 1.7792 | 5.7223 | 1.8042 | 5.7144 | 1.8292 | 5.7063 | 1.8541 | 6 |
| 7 | 6.6851 | 2.0758 | 6.6760 | 2.1049 | 6.6668 | 2.1341 | 6.6574 | 2.1631 | 7 |
| 8 | 7.6402 | 2.3723 | 7.6297 | 2.4056 | 7.6192 | 2.4389 | 7.6085 | 2.4721 | 8 |
| 9 | 8.5952 | 2.6689 | 8.5835 | 2.7064 | 8.5716 | 2.7438 | 8.5595 | 2.7812 | 9 |
| 10 | 9.5502 | 2.9654 | 9.5372 | 3.0071 | 9.5240 | 3.0486 | 9.5106 | 3.0902 | 10 |
| | 72¾ Deg. | | 72½ Deg. | | 72¼ Deg. | | 72 Deg. | | |
| | 18¼ Deg. | | 18½ Deg. | | 18¾ Deg. | | 19 Deg. | | |
| 1 | .9497 | .3132 | .9483 | .3173 | .9469 | .3214 | .9455 | .3256 | 1 |
| 2 | 1.8994 | .6263 | 1.8966 | .6346 | 1.8939 | .6429 | 1.8910 | .6511 | 2 |
| 3 | 2.8491 | .9395 | 2.8450 | .9519 | 2.8408 | .9643 | 2.8366 | .9767 | 3 |
| 4 | 3.7988 | 1.2527 | 3.7933 | 1.2692 | 3.7877 | 1.2858 | 3.7821 | 1.3023 | 4 |
| 5 | 4.7485 | 1.5658 | 4.7416 | 1.5865 | 4.7347 | 1.6072 | 4.7276 | 1.6278 | 5 |
| 6 | 5.6982 | 1.8790 | 5.6899 | 1.9038 | 5.6816 | 1.9286 | 5.6731 | 1.9534 | 6 |
| 7 | 6.6479 | 2.1921 | 6.6383 | 2.2211 | 6.6285 | 2.2501 | 6.6186 | 2.2790 | 7 |
| 8 | 7.5976 | 2.5053 | 7.5866 | 2.5384 | 7.5754 | 2.5715 | 7.5641 | 2.6045 | 8 |
| 9 | 8.5473 | 2.8185 | 8.5349 | 2.8557 | 8.5224 | 2.8930 | 8.5097 | 2.9301 | 9 |
| 10 | 9.4970 | 3.1316 | 9.4832 | 3.1730 | 9.4693 | 3.2144 | 9.4552 | 3.2557 | 10 |
| | 71¾ Deg. | | 71½ Deg. | | 71¼ Deg. | | 71 Deg. | | |
| | 19¼ Deg. | | 19½ Deg. | | 19¾ Deg. | | 20 Deg. | | |
| 1 | .9441 | .3297 | .9426 | .3338 | .9412 | .3379 | .9397 | .3420 | 1 |
| 2 | 1.8882 | .6594 | 1.8853 | .6676 | 1.8824 | .6758 | 1.8794 | .6840 | 2 |
| 3 | 2.8323 | .9891 | 2.8279 | 1.0014 | 2.8235 | 1.0138 | 2.8191 | 1.0261 | 3 |
| 4 | 3.7764 | 1.3188 | 3.7706 | 1.3352 | 3.7647 | 1.3517 | 3.7588 | 1.3681 | 4 |
| 5 | 4.7204 | 1.6485 | 4.7132 | 1.6690 | 4.7059 | 1.6896 | 4.6985 | 1.7101 | 5 |
| 6 | 5.6645 | 1.9781 | 5.6558 | 2.0028 | 5.6471 | 2.0275 | 5.6382 | 2.0521 | 6 |
| 7 | 6.6086 | 2.3078 | 6.5985 | 2.3366 | 6.5882 | 2.3654 | 6.5778 | 2.3941 | 7 |
| 8 | 7.5527 | 2.6375 | 7.5411 | 2.6705 | 7.5294 | 2.7033 | 7.5175 | 2.7362 | 8 |
| 9 | 8.4968 | 2.9672 | 8.4838 | 3.0043 | 8.4706 | 3.0413 | 8.4572 | 3.0782 | 9 |
| 10 | 9.4409 | 3.2969 | 9.4264 | 3.3381 | 9.4118 | 3.3792 | 9.3969 | 3.4202 | 10 |
| D. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | D. |
| | 70¾ Deg. | | 70½ Deg. | | 70¼ Deg. | | 70 Deg. | | |

| D. | 20¼ Deg. | | 20½ Deg. | | 20¾ Deg. | | 21 Deg. | | D. |
|---|---|---|---|---|---|---|---|---|---|
| | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | |
| 1 | .9382 | .3461 | .9367 | .3502 | .9351 | .3543 | .9336 | .3584 | 1 |
| 2 | 1.8764 | .6922 | 1.8733 | .7004 | 1.8703 | .7086 | 1.8672 | .7167 | 2 |
| 3 | 2.8146 | 1.0384 | 2.8100 | 1.0506 | 2.8054 | 1.0629 | 2.8007 | 1.0751 | 3 |
| 4 | 3.7528 | 1.3845 | 3.7467 | 1.4008 | 3.7405 | 1.4172 | 3.7343 | 1.4335 | 4 |
| 5 | 4.6910 | 1.7306 | 4.6834 | 1.7510 | 4.6757 | 1.7715 | 4.6679 | 1.7918 | 5 |
| 6 | 5.6291 | 2.0767 | 5.6200 | 2.1012 | 5.6108 | 2.1257 | 5.6015 | 2.1502 | 6 |
| 7 | 6.5673 | 2.4228 | 6.5567 | 2.4515 | 6.5459 | 2.4800 | 6.5351 | 2.5086 | 7 |
| 8 | 7.5055 | 2.7689 | 7.4934 | 2.8017 | 7.4811 | 2.8343 | 7.4686 | 2.8669 | 8 |
| 9 | 8.4437 | 3.1151 | 8.4300 | 3.1519 | 8.4162 | 3.1886 | 8.4022 | 3.2253 | 9 |
| 10 | 9.3819 | 3.4612 | 9.3667 | 3.5021 | 9.3514 | 3.5429 | 9.3358 | 3.5837 | 10 |
| | 69¾ Deg. | | 69½ Deg. | | 69¼ Deg. | | 69 Deg. | | |
| | 21¼ Deg. | | 21½ Deg. | | 21¾ Deg. | | 22 Deg. | | |
| 1 | .9320 | .3624 | .9304 | .3665 | .9288 | .3706 | .9272 | .3746 | 1 |
| 2 | 1.8640 | .7249 | 1.8608 | .7330 | 1.8576 | .7411 | 1.8544 | .7492 | 2 |
| 3 | 2.7960 | 1.0873 | 2.7913 | 1.0995 | 2.7864 | 1.1117 | 2.7816 | 1.1238 | 3 |
| 4 | 3.7280 | 1.4498 | 3.7217 | 1.4660 | 3.7152 | 1.4822 | 3.7087 | 1.4984 | 4 |
| 5 | 4.6600 | 1.8122 | 4.6521 | 1.8325 | 4.6440 | 1.8528 | 4.6359 | 1.8730 | 5 |
| 6 | 5.5920 | 2.1746 | 5.5825 | 2.1990 | 5.5729 | 2.2233 | 5.5631 | 2.2476 | 6 |
| 7 | 6.5241 | 2.5371 | 6.5129 | 2.5655 | 6.5017 | 2.5939 | 6.4903 | 2.6222 | 7 |
| 8 | 7.4561 | 2.8995 | 7.4433 | 2.9320 | 7.4305 | 2.9645 | 7.4175 | 2.9969 | 8 |
| 9 | 8.3881 | 3.2619 | 8.3738 | 3.2985 | 8.3593 | 3.3350 | 8.3447 | 3.3715 | 9 |
| 10 | 9.3201 | 3.6244 | 9.3042 | 3.6650 | 9.2881 | 3.7056 | 9.2718 | 3.7461 | 10 |
| | 68¾ Deg. | | 68½ Deg. | | 68¼ Deg. | | 68 Deg. | | |
| | 22¼ Deg. | | 22½ Deg. | | 22¾ Deg. | | 23 Deg. | | |
| 1 | .9255 | .3786 | .9239 | .3827 | .9222 | .3867 | .9205 | .3907 | 1 |
| 2 | 1.8511 | .7573 | 1.8478 | .7654 | 1.8444 | .7734 | 1.8410 | .7815 | 2 |
| 3 | 2.7766 | 1.1359 | 2.7716 | 1.1481 | 2.7666 | 1.1601 | 2.7615 | 1.1722 | 3 |
| 4 | 3.7022 | 1.5146 | 3.6955 | 1.5307 | 3.6888 | 1.5468 | 3.6820 | 1.5629 | 4 |
| 5 | 4.6277 | 1.8932 | 4.6194 | 1.9134 | 4.6110 | 1.9336 | 4.6025 | 1.9537 | 5 |
| 6 | 5.5532 | 2.2719 | 5.5433 | 2.2961 | 5.5332 | 2.3203 | 5.5230 | 2.3444 | 6 |
| 7 | 6.4788 | 2.6505 | 6.4672 | 2.6788 | 6.4554 | 2.7070 | 6.4435 | 2.7351 | 7 |
| 8 | 7.4043 | 3.0292 | 7.3910 | 3.0615 | 7.3776 | 3.0937 | 7.3640 | 3.1258 | 8 |
| 9 | 8.3299 | 3.4078 | 8.3149 | 3.4442 | 8.2998 | 3.4804 | 8.2845 | 3.5166 | 9 |
| 10 | 9.2554 | 3.7865 | 9.2388 | 3.8268 | 9.2220 | 3.8671 | 9.2050 | 3.9073 | 10 |
| | 67¾ Deg. | | 67½ Deg. | | 67¼ Deg. | | 67 Deg. | | |
| | 23¼ Deg. | | 23½ Deg. | | 23¾ Deg. | | 24 Deg. | | |
| 1 | .9188 | .3947 | .9171 | .3987 | .9153 | .4027 | .9135 | .4067 | 1 |
| 2 | 1.8376 | .7895 | 1.8341 | .7975 | 1.8306 | .8055 | 1.8271 | .8135 | 2 |
| 3 | 2.7564 | 1.1842 | 2.7512 | 1.1962 | 2.7459 | 1.2082 | 2.7406 | 1.2202 | 3 |
| 4 | 3.6752 | 1.5790 | 3.6682 | 1.5950 | 3.6612 | 1.6110 | 3.6542 | 1.6269 | 4 |
| 5 | 4.5940 | 1.9737 | 4.5853 | 1.9937 | 4.5766 | 2.0137 | 4.5677 | 2.0337 | 5 |
| 6 | 5.5127 | 2.3685 | 5.5024 | 2.3925 | 5.4919 | 2.4165 | 5.4813 | 2.4404 | 6 |
| 7 | 6.4315 | 2.7632 | 6.4194 | 2.7912 | 6.4072 | 2.8192 | 6.3948 | 2.8472 | 7 |
| 8 | 7.3503 | 3.1580 | 7.3365 | 3.1900 | 7.3225 | 3.2220 | 7.3084 | 3.2539 | 8 |
| 9 | 8.2691 | 3.5527 | 8.2535 | 3.5887 | 8.2378 | 3.6247 | 8.2219 | 3.6606 | 9 |
| 10 | 9.1879 | 3.9474 | 9.1706 | 3.9875 | 9.1531 | 4.0275 | 9.1355 | 4.0674 | 10 |
| D. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | D. |
| | 66¾ Deg. | | 66½ Deg. | | 66¼ Deg. | | 66 Deg. | | |

| D. | 24¼ Deg. | | 24½ Deg. | | 24¾ Deg. | | 25 Deg. | | D. |
|---|---|---|---|---|---|---|---|---|---|
| | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | |
| 1 | .9118 | .4107 | .9100 | .4147 | .9081 | .4187 | .9063 | .4226 | 1 |
| 2 | 1.8235 | .8214 | 1.8199 | .8294 | 1.8163 | .8373 | 1.8126 | .8452 | 2 |
| 3 | 2.7353 | 1.2322 | 2.7299 | 1.2441 | 2.7244 | 1.2560 | 2.7189 | 1.2679 | 3 |
| 4 | 3.6470 | 1.6429 | 3.6398 | 1.6588 | 3.6326 | 1.6746 | 3.6252 | 1.6905 | 4 |
| 5 | 4.5588 | 2.0536 | 4.5498 | 2.0735 | 4.5407 | 2.0933 | 4.5315 | 2.1131 | 5 |
| 6 | 5.4706 | 2.4643 | 5.4598 | 2.4882 | 5.4489 | 2.5120 | 5.4378 | 2.5357 | 6 |
| 7 | 6.3823 | 2.8750 | 6.3697 | 2.9029 | 6.3570 | 2.9306 | 6.3442 | 2.9583 | 7 |
| 8 | 7.2941 | 3.2858 | 7.2797 | 3.3175 | 7.2651 | 3.3493 | 7.2505 | 3.3809 | 8 |
| 9 | 8.2059 | 3.6965 | 8.1897 | 3.7322 | 8.1733 | 3.7679 | 8.1568 | 3.8036 | 9 |
| 10 | 9.1176 | 4.1072 | 9.0996 | 4.1469 | 9.0814 | 4.1866 | 9.0631 | 4.2262 | 10 |
| | 65¾ Deg. | | 65½ Deg. | | 65¼ Deg. | | 65 Deg. | | |
| | 25¼ Deg. | | 25½ Deg. | | 25¾ Deg. | | 26 Deg. | | |
| 1 | .9045 | .4266 | .9026 | .4305 | .9007 | .4344 | .8988 | .4384 | 1 |
| 2 | 1.8089 | .8531 | 1.8052 | .8610 | 1.8014 | .8689 | 1.7976 | .8767 | 2 |
| 3 | 2.7134 | 1.2797 | 2.7078 | 1.2915 | 2.7021 | 1.3033 | 2.6964 | 1.3151 | 3 |
| 4 | 3.6178 | 1.7063 | 3.6103 | 1.7220 | 3.6028 | 1.7378 | 3.5952 | 1.7535 | 4 |
| 5 | 4.5223 | 2.1328 | 4.5129 | 2.1526 | 4.5035 | 2.1722 | 4.4940 | 2.1919 | 5 |
| 6 | 5.4267 | 2.5594 | 5.4155 | 2.5831 | 5.4042 | 2.6067 | 5.3928 | 2.6302 | 6 |
| 7 | 6.3312 | 2.9860 | 6.3181 | 3.0136 | 6.3049 | 3.0411 | 6.2916 | 3.0686 | 7 |
| 8 | 7.2356 | 3.4125 | 7.2207 | 3.4441 | 7.2056 | 3.4756 | 7.1904 | 3.5070 | 8 |
| 9 | 8.1401 | 3.8391 | 8.1233 | 3.8746 | 8.1063 | 3.9100 | 8.0891 | 3.9453 | 9 |
| 10 | 9.0446 | 4.2657 | 9.0259 | 4.3051 | 9.0070 | 4.3445 | 8.9879 | 4.3837 | 10 |
| | 64¾ Deg. | | 64½ Deg. | | 64¼ Deg. | | 64 Deg. | | |
| | 26¼ Deg. | | 26½ Deg. | | 26¾ Deg. | | 27 Deg. | | |
| 1 | .8969 | .4423 | .8949 | .4462 | .8930 | .4501 | .8910 | .4540 | 1 |
| 2 | 1.7937 | .8846 | 1.7899 | .8924 | 1.7860 | .9002 | 1.7820 | .9080 | 2 |
| 3 | 2.6906 | 1.3269 | 2.6848 | 1.3386 | 2.6789 | 1.3503 | 2.6730 | 1.3620 | 3 |
| 4 | 3.5875 | 1.7692 | 3.5797 | 1.7848 | 3.5719 | 1.8004 | 3.5640 | 1.8160 | 4 |
| 5 | 4.4844 | 2.2114 | 4.4747 | 2.2310 | 4.4649 | 2.2505 | 4.4550 | 2.2700 | 5 |
| 6 | 5.3812 | 2.6537 | 5.3696 | 2.6772 | 5.3579 | 2.7006 | 5.3460 | 2.7239 | 6 |
| 7 | 6.2781 | 3.0960 | 6.2645 | 3.1234 | 6.2509 | 3.1507 | 6.2370 | 3.1779 | 7 |
| 8 | 7.1750 | 3.5383 | 7.1595 | 3.5696 | 7.1438 | 3.6008 | 7.1281 | 3.6319 | 8 |
| 9 | 8.0719 | 3.9806 | 8.0544 | 4.0158 | 8.0368 | 4.0509 | 8.0191 | 4.0859 | 9 |
| 10 | 8.9687 | 4.4229 | 8.9493 | 4.4620 | 8.9298 | 4.5010 | 8.9101 | 4.5399 | 10 |
| | 63¾ Deg. | | 63½ Deg. | | 63¼ Deg. | | 63 Deg. | | |
| | 27¼ Deg. | | 27½ Deg. | | 27¾ Deg. | | 28 Deg. | | |
| 1 | .8890 | .4579 | .8870 | .4617 | .8850 | .4656 | .8829 | .4695 | 1 |
| 2 | 1.7780 | .9157 | 1.7740 | .9235 | 1.7700 | .9312 | 1.7659 | .9389 | 2 |
| 3 | 2.6671 | 1.3736 | 2.6610 | 1.3852 | 2.6550 | 1.3968 | 2.6488 | 1.4084 | 3 |
| 4 | 3.5561 | 1.8315 | 3.5480 | 1.8470 | 3.5400 | 1.8625 | 3.5318 | 1.8779 | 4 |
| 5 | 4.4451 | 2.2894 | 4.4351 | 2.3087 | 4.4249 | 2.3281 | 4.4147 | 2.3474 | 5 |
| 6 | 5.3341 | 2.7472 | 5.3221 | 2.7705 | 5.3099 | 2.7937 | 5.2977 | 2.8168 | 6 |
| 7 | 6.2231 | 3.2051 | 6.2091 | 3.2322 | 6.1949 | 3.2593 | 6.1806 | 3.2863 | 7 |
| 8 | 7.1121 | 3.6630 | 7.0961 | 3.6940 | 7.0799 | 3.7249 | 7.0636 | 3.7558 | 8 |
| 9 | 8.0012 | 4.1209 | 7.9831 | 4.1557 | 7.9649 | 4.1905 | 7.9465 | 4.2252 | 9 |
| 10 | 8.8902 | 4.5787 | 8.8701 | 4.6175 | 8.8499 | 4.6561 | 8.8295 | 4.6947 | 10 |
| D. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | D. |
| | 62¾ Deg. | | 62½ Deg. | | 62¼ Deg. | | 62 Deg. | | |

# LATITUDES AND DEPARTURES.

| D. | 28¼ Deg. | | 28½ Deg. | | 28¾ Deg. | | 29 Deg. | | D. |
|---|---|---|---|---|---|---|---|---|---|
| | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | |
| 1 | .8809 | .4733 | .8788 | .4772 | .8767 | .4810 | .8746 | .4848 | 1 |
| 2 | 1.7618 | .9466 | 1.7576 | .9543 | 1.7535 | .9620 | 1.7492 | .9696 | 2 |
| 3 | 2.6427 | 1.4200 | 2.6365 | 1.4315 | 2.6302 | 1.4430 | 2.6239 | 1.4544 | 3 |
| 4 | 3.5236 | 1.8933 | 3.5153 | 1.9086 | 3.5069 | 1.9240 | 3.4985 | 1.9392 | 4 |
| 5 | 4.4045 | 2.3666 | 4.3941 | 2.3858 | 4.3836 | 2.4049 | 4.3731 | 2.4240 | 5 |
| 6 | 5.2853 | 2.8399 | 5.2729 | 2.8630 | 5.2604 | 2.8859 | 5.2477 | 2.9089 | 6 |
| 7 | 6.1662 | 3.3132 | 6.1517 | 3.3401 | 6.1371 | 3.3669 | 6.1223 | 3.3937 | 7 |
| 8 | 7.0471 | 3.7866 | 7.0305 | 3.8173 | 7.0138 | 3.8479 | 6.9970 | 3.8785 | 8 |
| 9 | 7.9280 | 4.2599 | 7.9094 | 4.2944 | 7.8905 | 4.3289 | 7.8716 | 4.3633 | 9 |
| 10 | 8.8089 | 4.7332 | 8.7882 | 4.7716 | 8.7673 | 4.8099 | 8.7462 | 4.8481 | 10 |
| | 61¾ Deg. | | 61½ Deg. | | 61¼ Deg. | | 61 Deg. | | |
| | 29¼ Deg. | | 29½ Deg. | | 29¾ Deg. | | 30 Deg. | | |
| 1 | .8725 | .4886 | .8704 | .4924 | .8682 | .4962 | .8660 | .5000 | 1 |
| 2 | 1.7450 | .9772 | 1.7407 | .9848 | 1.7364 | .9924 | 1.7321 | 1.0000 | 2 |
| 3 | 2.6175 | 1.4659 | 2.6111 | 1.4773 | 2.6046 | 1.4886 | 2.5981 | 1.5000 | 3 |
| 4 | 3.4900 | 1.9545 | 3.4814 | 1.9697 | 3.4728 | 1.9849 | 3.4641 | 2.0000 | 4 |
| 5 | 4.3625 | 2.4431 | 4.3518 | 2.4621 | 4.3410 | 2.4811 | 4.3301 | 2.5000 | 5 |
| 6 | 5.2350 | 2.9317 | 5.2221 | 2.9545 | 5.2092 | 2.9773 | 5.1962 | 3.0000 | 6 |
| 7 | 6.1075 | 3.4203 | 6.0925 | 3.4470 | 6.0774 | 3.4735 | 6.0622 | 3.5000 | 7 |
| 8 | 6.9800 | 3.9090 | 6.9628 | 3.9394 | 6.9456 | 3.9697 | 6.9282 | 4.0000 | 8 |
| 9 | 7.8525 | 4.3976 | 7.8332 | 4.4318 | 7.8138 | 4.4659 | 7.7942 | 4.5000 | 9 |
| 10 | 8.7250 | 4.8862 | 8.7036 | 4.9242 | 8.6820 | 4.9622 | 8.6603 | 5.0000 | 10 |
| | 60¾ Deg. | | 60½ Deg. | | 60¼ Deg. | | 60 Deg. | | |
| | 30¼ Deg. | | 30½ Deg. | | 30¾ Deg. | | 31 Deg. | | |
| 1 | .8638 | .5038 | .8616 | .5075 | .8594 | .5113 | .8572 | .5150 | 1 |
| 2 | 1.7277 | 1.0075 | 1.7233 | 1.0151 | 1.7188 | 1.0226 | 1.7143 | 1.0301 | 2 |
| 3 | 2.5915 | 1.5113 | 2.5849 | 1.5226 | 2.5782 | 1.5339 | 2.5715 | 1.5451 | 3 |
| 4 | 3.4553 | 2.0151 | 3.4465 | 2.0302 | 3.4376 | 2.0452 | 3.4287 | 2.0602 | 4 |
| 5 | 4.3192 | 2.5189 | 4.3081 | 2.5377 | 4.2970 | 2.5565 | 4.2858 | 2.5752 | 5 |
| 6 | 5.1830 | 3.0226 | 5.1698 | 3.0452 | 5.1564 | 3.0678 | 5.1430 | 3.0902 | 6 |
| 7 | 6.0468 | 3.5264 | 6.0314 | 3.5528 | 6.0158 | 3.5791 | 6.0002 | 3.6053 | 7 |
| 8 | 6.9107 | 4.0302 | 6.8930 | 4.0603 | 6.8753 | 4.0903 | 6.8573 | 4.1203 | 8 |
| 9 | 7.7745 | 4.5340 | 7.7547 | 4.5678 | 7.7347 | 4.6016 | 7.7145 | 4.6353 | 9 |
| 10 | 8.6384 | 5.0377 | 8.6163 | 5.0754 | 8.5941 | 5.1129 | 8.5717 | 5.1504 | 10 |
| | 59¾ Deg. | | 59½ Deg. | | 59¼ Deg. | | 59 Deg. | | |
| | 31¼ Deg. | | 31½ Deg. | | 31¾ Deg. | | 32 Deg. | | |
| 1 | .8549 | .5188 | .8526 | .5225 | .8504 | .5262 | .8480 | .5299 | 1 |
| 2 | 1.7098 | 1.0375 | 1.7053 | 1.0450 | 1.7007 | 1.0524 | 1.6961 | 1.0598 | 2 |
| 3 | 2.5647 | 1.5563 | 2.5579 | 1.5675 | 2.5511 | 1.5786 | 2.5441 | 1.5898 | 3 |
| 4 | 3.4196 | 2.0751 | 3.4106 | 2.0900 | 3.4014 | 2.1049 | 3.3922 | 2.1197 | 4 |
| 5 | 4.2746 | 2.5939 | 4.2632 | 2.6125 | 4.2518 | 2.6311 | 4.2402 | 2.6496 | 5 |
| 6 | 5.1295 | 3.1126 | 5.1158 | 3.1350 | 5.1021 | 3.1573 | 5.0883 | 3.1795 | 6 |
| 7 | 5.9844 | 3.6314 | 5.9685 | 3.6575 | 5.9525 | 3.6835 | 5.9363 | 3.7094 | 7 |
| 8 | 6.8393 | 4.1502 | 6.8211 | 4.1800 | 6.8028 | 4.2097 | 6.7844 | 4.2394 | 8 |
| 9 | 7.6942 | 4.6690 | 7.6738 | 4.7025 | 7.6532 | 4.7359 | 7.6324 | 4.7693 | 9 |
| 10 | 8.5491 | 5.1877 | 8.5264 | 5.2250 | 8.5035 | 5.2621 | 8.4805 | 5.2992 | 10 |
| D. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | D. |
| | 58¾ Deg. | | 58½ Deg. | | 58¼ Deg. | | 58 Deg. | | |

# LATITUDES AND DEPARTURES.

| D. | 32¼ Deg. | | 32½ Deg. | | 32¾ Deg. | | 33 Deg. | | D. |
|---|---|---|---|---|---|---|---|---|---|
| | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | |
| 1 | .8457 | .5336 | .8434 | .5373 | .8410 | .5410 | .8387 | .5446 | 1 |
| 2 | 1.6915 | 1.0672 | 1.6868 | 1.0746 | 1.6821 | 1.0819 | 1.6773 | 1.0893 | 2 |
| 3 | 2.5372 | 1.6008 | 2.5302 | 1.6119 | 2.5231 | 1.6229 | 2.5160 | 1.6339 | 3 |
| 4 | 3.3829 | 2.1345 | 3.3736 | 2.1492 | 3.3642 | 2.1639 | 3.3547 | 2.1786 | 4 |
| 5 | 4.2286 | 2.6681 | 4.2170 | 2.6865 | 4.2052 | 2.7049 | 4.1934 | 2.7232 | 5 |
| 6 | 5.0744 | 3.2017 | 5.0603 | 3.2238 | 5.0462 | 3.2458 | 5.0320 | 3.2678 | 6 |
| 7 | 5.9201 | 3.7353 | 5.9037 | 3.7611 | 5.8873 | 3.7868 | 5.8707 | 3.8125 | 7 |
| 8 | 6.7658 | 4.2689 | 6.7471 | 4.2984 | 6.7283 | 4.3278 | 6.7094 | 4.3571 | 8 |
| 9 | 7.6116 | 4.8025 | 7.5905 | 4.8357 | 7.5694 | 4.8688 | 7.5480 | 4.9018 | 9 |
| 10 | 8.4573 | 5.3361 | 8.4339 | 5.3730 | 8.4104 | 5.4097 | 8.3867 | 5.4464 | 10 |
| | 57¾ Deg. | | 57½ Deg. | | 57¼ Deg. | | 57 Deg. | | |
| | 33¼ Deg. | | 33½ Deg. | | 33¾ Deg. | | 34 Deg. | | |
| 1 | .8363 | .5483 | .8339 | .5519 | .8315 | .5556 | .8290 | .5592 | 1 |
| 2 | 1.6726 | 1.0966 | 1.6678 | 1.1039 | 1.6629 | 1.1111 | 1.6581 | 1.1184 | 2 |
| 3 | 2.5089 | 1.6449 | 2.5017 | 1.6558 | 2.4944 | 1.6667 | 2.4871 | 1.6776 | 3 |
| 4 | 3.3451 | 2.1932 | 3.3355 | 2.2077 | 3.3259 | 2.2223 | 3.3162 | 2.2368 | 4 |
| 5 | 4.1814 | 2.7415 | 4.1694 | 2.7597 | 4.1573 | 2.7779 | 4.1452 | 2.7960 | 5 |
| 6 | 5.0177 | 3.2898 | 5.0033 | 3.3116 | 4.9888 | 3.3334 | 4.9742 | 3.3552 | 6 |
| 7 | 5.8540 | 3.8381 | 5.8372 | 3.8636 | 5.8203 | 3.8890 | 5.8033 | 3.9144 | 7 |
| 8 | 6.6903 | 4.3863 | 6.6711 | 4.4155 | 6.6518 | 4.4446 | 6.6323 | 4.4735 | 8 |
| 9 | 7.5266 | 4.9346 | 7.5050 | 4.9674 | 7.4832 | 5.0001 | 7.4613 | 5.0327 | 9 |
| 10 | 8.3629 | 5.4829 | 8.3389 | 5.5194 | 8.3147 | 5.5557 | 8.2904 | 5.5919 | 10 |
| | 56¾ Deg. | | 56½ Deg. | | 56¼ Deg. | | 56 Deg. | | |
| | 34¼ Deg. | | 34½ Deg. | | 34¾ Deg. | | 35 Deg. | | |
| 1 | .8266 | .5628 | .8241 | .5664 | .8216 | .5700 | .8192 | .5736 | 1 |
| 2 | 1.6532 | 1.1256 | 1.6483 | 1.1328 | 1.6433 | 1.1400 | 1.6383 | 1.1472 | 2 |
| 3 | 2.4798 | 1.6884 | 2.4724 | 1.6992 | 2.4649 | 1.7100 | 2.4575 | 1.7207 | 3 |
| 4 | 3.3064 | 2.2512 | 3.2965 | 2.2656 | 3.2866 | 2.2800 | 3.2766 | 2.2943 | 4 |
| 5 | 4.1329 | 2.8140 | 4.1206 | 2.8320 | 4.1082 | 2.8500 | 4.0958 | 2.8679 | 5 |
| 6 | 4.9595 | 3.3768 | 4.9448 | 3.3984 | 4.9299 | 3.4200 | 4.9149 | 3.4415 | 6 |
| 7 | 5.7861 | 3.9396 | 5.7689 | 3.9648 | 5.7515 | 3.9900 | 5.7341 | 4.0150 | 7 |
| 8 | 6.6127 | 4.5024 | 6.5930 | 4.5312 | 6.5732 | 4.5600 | 6.5532 | 4.5886 | 8 |
| 9 | 7.4393 | 5.0652 | 7.4171 | 5.0977 | 7.3948 | 5.1300 | 7.3724 | 5.1622 | 9 |
| 10 | 8.2659 | 5.6280 | 8.2413 | 5.6641 | 8.2165 | 5.7000 | 8.1915 | 5.7358 | 10 |
| | 55¾ Deg. | | 55½ Deg. | | 55¼ Deg. | | 55 Deg. | | |
| | 35¼ Deg. | | 35½ Deg. | | 35¾ Deg. | | 36 Deg. | | |
| 1 | .8166 | .5771 | .8141 | .5807 | .8116 | .5842 | .8090 | .5878 | 1 |
| 2 | 1.6333 | 1.1543 | 1.6282 | 1.1614 | 1.6231 | 1.1685 | 1.6180 | 1.1756 | 2 |
| 3 | 2.4499 | 1.7314 | 2.4423 | 1.7421 | 2.4347 | 1.7527 | 2.4271 | 1.7634 | 3 |
| 4 | 3.2666 | 2.3086 | 3.2565 | 2.3228 | 3.2463 | 2.3370 | 3.2361 | 2.3511 | 4 |
| 5 | 4.0832 | 2.8857 | 4.0706 | 2.9035 | 4.0579 | 2.9212 | 4.0451 | 2.9389 | 5 |
| 6 | 4.8998 | 3.4629 | 4.8847 | 3.4842 | 4.8694 | 3.5055 | 4.8541 | 3.5267 | 6 |
| 7 | 5.7165 | 4.0400 | 5.6988 | 4.0649 | 5.6810 | 4.0897 | 5.6631 | 4.1145 | 7 |
| 8 | 6.5331 | 4.6172 | 6.5129 | 4.6456 | 6.4926 | 4.6740 | 6.4721 | 4.7023 | 8 |
| 9 | 7.3498 | 5.1943 | 7.3270 | 5.2263 | 7.3042 | 5.2582 | 7.2812 | 5.2901 | 9 |
| 10 | 8.1664 | 5.7715 | 8.1412 | 5.8070 | 8.1157 | 5.8425 | 8.0902 | 5.8779 | 10 |
| | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | |
| D. | 54¾ Deg. | | 54½ Deg. | | 54¼ Deg. | | 54 Deg. | | D. |

| D. | 36¼ Deg. | | 36½ Deg. | | 36¾ Deg. | | 37 Deg. | | D. |
|---|---|---|---|---|---|---|---|---|---|
| | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | |
| 1 | .8064 | .5913 | .8039 | .5948 | .8013 | .5983 | .7986 | .6018 | 1 |
| 2 | 1.6129 | 1.1826 | 1.6077 | 1.1896 | 1.6025 | 1.1966 | 1.5973 | 1.2036 | 2 |
| 3 | 2.4193 | 1.7739 | 2.4116 | 1.7845 | 2.4038 | 1.7950 | 2.3959 | 1.8054 | 3 |
| 4 | 3.2258 | 2.3652 | 3.2154 | 2.3793 | 3.2050 | 2.3933 | 3.1945 | 2.4073 | 4 |
| 5 | 4.0322 | 2.9565 | 4.0193 | 2.9741 | 4.0063 | 2.9916 | 3.9932 | 3.0091 | 5 |
| 6 | 4.8387 | 3.5479 | 4.8231 | 3.5689 | 4.8075 | 3.5899 | 4.7918 | 3.6109 | 6 |
| 7 | 5.6451 | 4.1392 | 5.6270 | 4.1638 | 5.6088 | 4.1883 | 5.5904 | 4.2127 | 7 |
| 8 | 6.4516 | 4.7305 | 6.4309 | 4.7586 | 6.4100 | 4.7866 | 6.3891 | 4.8145 | 8 |
| 9 | 7.2580 | 5.3218 | 7.2347 | 5.3534 | 7.2113 | 5.3849 | 7.1877 | 5.4163 | 9 |
| 10 | 8.0644 | 5.9131 | 8.0386 | 5.9482 | 8.0125 | 5.9832 | 7.9864 | 6.0181 | 10 |
| | 53¾ Deg. | | 53½ Deg. | | 53¼ Deg. | | 53 Deg. | | |
| | 37¼ Deg. | | 37½ Deg. | | 37¾ Deg. | | 38 Deg. | | |
| 1 | .7960 | .6053 | .7934 | .6088 | .7907 | .6122 | .7880 | .6157 | 1 |
| 2 | 1.5920 | 1.2106 | 1.5867 | 1.2175 | 1.5814 | 1.2244 | 1.5760 | 1.2313 | 2 |
| 3 | 2.3880 | 1.8159 | 2.3801 | 1.8263 | 2.3721 | 1.8367 | 2.3640 | 1.8470 | 3 |
| 4 | 3.1840 | 2.4212 | 3.1734 | 2.4350 | 3.1628 | 2.4489 | 3.1520 | 2.4626 | 4 |
| 5 | 3.9800 | 3.0265 | 3.9668 | 3.0438 | 3.9534 | 3.0611 | 3.9401 | 3.0783 | 5 |
| 6 | 4.7760 | 3.6318 | 4.7601 | 3.6526 | 4.7441 | 3.6733 | 4.7281 | 3.6940 | 6 |
| 7 | 5.5720 | 4.2371 | 5.5535 | 4.2613 | 5.5348 | 4.2855 | 5.5161 | 4.3096 | 7 |
| 8 | 6.3680 | 4.8424 | 6.3468 | 4.8701 | 6.3255 | 4.8977 | 6.3041 | 4.9253 | 8 |
| 9 | 7.1640 | 5.4476 | 7.1402 | 5.4789 | 7.1162 | 5.5100 | 7.0921 | 5.5410 | 9 |
| 10 | 7.9600 | 6.0529 | 7.9335 | 6.0876 | 7.9069 | 6.1222 | 7.8801 | 6.1566 | 10 |
| | 52¾ Deg. | | 52½ Deg. | | 52¼ Deg. | | 52 Deg. | | |
| | 38¼ Deg. | | 38½ Deg. | | 38¾ Deg. | | 39 Deg. | | |
| 1 | .7853 | .6191 | .7826 | .6225 | .7799 | .6259 | .7771 | .6293 | 1 |
| 2 | 1.5706 | 1.2382 | 1.5652 | 1.2450 | 1.5598 | 1.2518 | 1.5543 | 1.2586 | 2 |
| 3 | 2.3560 | 1.8573 | 2.3478 | 1.8675 | 2.3397 | 1.8778 | 2.3314 | 1.8880 | 3 |
| 4 | 3.1413 | 2.4764 | 3.1304 | 2.4901 | 3.1195 | 2.5037 | 3.1086 | 2.5173 | 4 |
| 5 | 3.9266 | 3.0955 | 3.9130 | 3.1126 | 3.8994 | 3.1296 | 3.8857 | 3.1466 | 5 |
| 6 | 4.7119 | 3.7146 | 4.6956 | 3.7351 | 4.6793 | 3.7555 | 4.6629 | 3.7759 | 6 |
| 7 | 5.4972 | 4.3337 | 5.4783 | 4.3576 | 5.4592 | 4.3815 | 5.4400 | 4.4052 | 7 |
| 8 | 6.2825 | 4.9528 | 6.2609 | 4.9801 | 6.2391 | 5.0074 | 6.2172 | 5.0346 | 8 |
| 9 | 7.0679 | 5.5718 | 7.0435 | 5.6026 | 7.0190 | 5.6333 | 6.9943 | 5.6639 | 9 |
| 10 | 7.8532 | 6.1909 | 7.8261 | 6.2251 | 7.7988 | 6.2592 | 7.7715 | 6.2932 | 10 |
| | 51¾ Deg. | | 51½ Deg. | | 51¼ Deg. | | 51 Deg. | | |
| | 39¼ Deg. | | 39½ Deg. | | 39¾ Deg. | | 40 Deg. | | |
| 1 | .7744 | .6327 | .7716 | .6361 | .7688 | .6394 | .7660 | .6428 | 1 |
| 2 | 1.5488 | 1.2654 | 1.5432 | 1.2722 | 1.5377 | 1.2789 | 1.5321 | 1.2856 | 2 |
| 3 | 2.3232 | 1.8981 | 2.3149 | 1.9082 | 2.3065 | 1.9183 | 2.2981 | 1.9284 | 3 |
| 4 | 3.0976 | 2.5308 | 3.0865 | 2.5443 | 3.0754 | 2.5578 | 3.0642 | 2.5712 | 4 |
| 5 | 3.8720 | 3.1635 | 3.8581 | 3.1804 | 3.8442 | 3.1972 | 3.8302 | 3.2139 | 5 |
| 6 | 4.6464 | 3.7962 | 4.6297 | 3.8165 | 4.6131 | 3.8366 | 4.5963 | 3.8567 | 6 |
| 7 | 5.4207 | 4.4289 | 5.4014 | 4.4525 | 5.3819 | 4.4761 | 5.3623 | 4.4995 | 7 |
| 8 | 6.1951 | 5.0616 | 6.1730 | 5.0886 | 6.1507 | 5.1155 | 6.1284 | 5.1423 | 8 |
| 9 | 6.9695 | 5.6943 | 6.9446 | 5.7247 | 6.9196 | 5.7550 | 6.8944 | 5.7851 | 9 |
| 10 | 7.7439 | 6.3271 | 7.7162 | 6.3608 | 7.6884 | 6.3944 | 7.6604 | 6.4279 | 10 |
| D. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | D. |
| | 50¾ Deg. | | 50½ Deg. | | 50¼ Deg. | | 50 Deg. | | |

# LATITUDES AND DEPARTURES.

| D. | 40¼ Deg. | | 40½ Deg. | | 40¾ Deg. | | 41 Deg. | | D. |
|---|---|---|---|---|---|---|---|---|---|
| | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | |
| 1 | .7632 | .6461 | .7604 | .6494 | .7576 | .6528 | .7547 | .6561 | 1 |
| 2 | 1.5265 | 1.2922 | 1.5208 | 1.2989 | 1.5151 | 1.3055 | 1.5094 | 1.3121 | 2 |
| 3 | 2.2897 | 1.9384 | 2.2812 | 1.9483 | 2.2727 | 1.9583 | 2.2641 | 1.9682 | 3 |
| 4 | 3.0529 | 2.5845 | 3.0416 | 2.5978 | 3.0303 | 2.6110 | 3.0188 | 2.6242 | 4 |
| 5 | 3.8162 | 3.2306 | 3.8020 | 3.2472 | 3.7878 | 3.2638 | 3.7735 | 3.2803 | 5 |
| 6 | 4.5794 | 3.8767 | 4.5624 | 3.8967 | 4.5454 | 3.9166 | 4.5283 | 3.9364 | 6 |
| 7 | 5.3426 | 4.5229 | 5.3228 | 4.5461 | 5.3030 | 4.5693 | 5.2830 | 4.5924 | 7 |
| 8 | 6.1059 | 5.1690 | 6.0832 | 5.1956 | 6.0605 | 5.2221 | 6.0377 | 5.2485 | 8 |
| 9 | 6.8691 | 5.8151 | 6.8437 | 5.8450 | 6.8181 | 5.8748 | 6.7924 | 5.9045 | 9 |
| 10 | 7.6323 | 6.4612 | 7.6041 | 6.4945 | 7.5756 | 6.5276 | 7.5471 | 6.5606 | 10 |
| | 49¾ Deg. | | 49½ Deg. | | 49¼ Deg. | | 49 Deg. | | |
| | 41¼ Deg. | | 41½ Deg. | | 41¾ Deg. | | 42 Deg. | | |
| 1 | .7518 | .6593 | .7490 | .6626 | .7461 | .6659 | .7431 | .6691 | 1 |
| 2 | 1.5037 | 1.3187 | 1.4979 | 1.3252 | 1.4921 | 1.3318 | 1.4863 | 1.3383 | 2 |
| 3 | 2.2555 | 1.9780 | 2.2469 | 1.9879 | 2.2382 | 1.9976 | 2.2294 | 2.0074 | 3 |
| 4 | 3.0074 | 2.6374 | 2.9958 | 2.6505 | 2.9842 | 2.6635 | 2.9726 | 2.6765 | 4 |
| 5 | 3.7592 | 3.2967 | 3.7448 | 3.3131 | 3.7303 | 3.3294 | 3.7157 | 3.3457 | 5 |
| 6 | 4.5110 | 3.9561 | 4.4937 | 3.9757 | 4.4763 | 3.9953 | 4.4589 | 4.0148 | 6 |
| 7 | 5.2629 | 4.6154 | 5.2427 | 4.6383 | 5.2224 | 4.6612 | 5.2020 | 4.6839 | 7 |
| 8 | 6.0147 | 5.2748 | 5.9916 | 5.3010 | 5.9685 | 5.3271 | 5.9452 | 5.3530 | 8 |
| 9 | 6.7666 | 5.9341 | 6.7406 | 5.9636 | 6.7145 | 5.9929 | 6.6883 | 6.0222 | 9 |
| 10 | 7.5184 | 6.5935 | 7.4896 | 6.6262 | 7.4606 | 6.6588 | 7.4314 | 6.6913 | 10 |
| | 48¾ Deg. | | 48½ Deg. | | 48¼ Deg. | | 48 Deg. | | |
| | 42¼ Deg. | | 42½ Deg. | | 42¾ Deg. | | 43 Deg. | | |
| 1 | .7402 | .6724 | .7373 | .6756 | .7343 | .6788 | .7314 | .6820 | 1 |
| 2 | 1.4804 | 1.3447 | 1.4746 | 1.3512 | 1.4686 | 1.3576 | 1.4627 | 1.3640 | 2 |
| 3 | 2.2207 | 2.0171 | 2.2118 | 2.0268 | 2.2030 | 2.0364 | 2.1941 | 2.0460 | 3 |
| 4 | 2.9609 | 2.6895 | 2.9491 | 2.7024 | 2.9373 | 2.7152 | 2.9254 | 2.7280 | 4 |
| 5 | 3.7011 | 3.3618 | 3.6864 | 3.3780 | 3.6716 | 3.3940 | 3.6568 | 3.4100 | 5 |
| 6 | 4.4413 | 4.0342 | 4.4237 | 4.0535 | 4.4059 | 4.0728 | 4.3881 | 4.0920 | 6 |
| 7 | 5.1815 | 4.7066 | 5.1609 | 4.7291 | 5.1403 | 4.7516 | 5.1195 | 4.7740 | 7 |
| 8 | 5.9217 | 5.3789 | 5.8982 | 5.4047 | 5.8746 | 5.4304 | 5.8508 | 5.4560 | 8 |
| 9 | 6.6620 | 6.0513 | 6.6355 | 6.0803 | 6.6089 | 6.1092 | 6.5822 | 6.1380 | 9 |
| 10 | 7.4022 | 6.7237 | 7.3728 | 6.7559 | 7.3432 | 6.7880 | 7.3135 | 6.8200 | 10 |
| | 47¾ Deg. | | 47½ Deg. | | 47¼ Deg. | | 47 Deg. | | |
| | 43¼ Deg. | | 43½ Deg. | | 43¾ Deg. | | 44 Deg. | | |
| 1 | .7284 | .6852 | .7254 | .6884 | .7224 | .6915 | .7193 | .6947 | 1 |
| 2 | 1.4567 | 1.3704 | 1.4507 | 1.3767 | 1.4447 | 1.3830 | 1.4387 | 1.3893 | 2 |
| 3 | 2.1851 | 2.0555 | 2.1761 | 2.0651 | 2.1671 | 2.0745 | 2.1580 | 2.0840 | 3 |
| 4 | 2.9135 | 2.7407 | 2.9015 | 2.7534 | 2.8895 | 2.7661 | 2.8774 | 2.7786 | 4 |
| 5 | 3.6419 | 3.4259 | 3.6269 | 3.4418 | 3.6118 | 3.4576 | 3.5967 | 3.4733 | 5 |
| 6 | 4.3702 | 4.1111 | 4.3522 | 4.1301 | 4.3342 | 4.1491 | 4.3160 | 4.1680 | 6 |
| 7 | 5.0986 | 4.7963 | 5.0776 | 4.8185 | 5.0565 | 4.8406 | 5.0354 | 4.8626 | 7 |
| 8 | 5.8270 | 5.4815 | 5.8030 | 5.5068 | 5.7789 | 5.5321 | 5.7547 | 5.5573 | 8 |
| 9 | 6.5553 | 6.1666 | 6.5284 | 6.1952 | 6.5013 | 6.2236 | 6.4741 | 6.2519 | 9 |
| 10 | 7.2837 | 6.8518 | 7.2537 | 6.8835 | 7.2236 | 6.9151 | 7.1934 | 6.9466 | 10 |
| | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | |
| D. | 46¾ Deg. | | 46½ Deg. | | 46¼ Deg. | | 46 Deg. | | D. |

## LATITUDES AND DEPARTURES.

| D. | 44¼ Deg. | | 44½ Deg. | | 44¾ Deg. | | 45 Deg. | | D. |
|---|---|---|---|---|---|---|---|---|---|
| | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | |
| 1 | .7163 | .6978 | .7133 | .7009 | .7102 | .7040 | .7071 | .7071 | 1 |
| 2 | 1.4326 | 1.3956 | 1.4265 | 1.4018 | 1.4204 | 1.4080 | 1.4142 | 1.4142 | 2 |
| 3 | 2.1489 | 2.0934 | 2.1398 | 2.1027 | 2.1306 | 2.1120 | 2.1213 | 2.1213 | 3 |
| 4 | 2.8652 | 2.7912 | 2.8530 | 2.8036 | 2.8407 | 2.8161 | 2.8284 | 2.8284 | 4 |
| 5 | 3.5815 | 3.4890 | 3.5663 | 3.5045 | 3.5509 | 3.5201 | 3.5355 | 3.5355 | 5 |
| 6 | 4.2978 | 4.1867 | 4.2795 | 4.2055 | 4.2611 | 4.2241 | 4.2426 | 4.2426 | 6 |
| 7 | 5.0141 | 4.8845 | 4.9928 | 4.9064 | 4.9713 | 4.9281 | 4.9497 | 4.9497 | 7 |
| 8 | 5.7304 | 5.5823 | 5.7060 | 5.6073 | 5.6815 | 5.6321 | 5.6569 | 5.6569 | 8 |
| 9 | 6.4467 | 6.2801 | 6.4193 | 6.3082 | 6.3917 | 6.3361 | 6.3640 | 6.3640 | 9 |
| 10 | 7.1630 | 6.9779 | 7.1325 | 7.0091 | 7.1019 | 7.0401 | 7.0711 | 7.0711 | 10 |
| D. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | D. |
| | 45¾ Deg. | | 45½ Deg. | | 45¼ Deg. | | 45 Deg. | | |

## TABLE OF USEFUL NUMBERS.

Logarithms.

Ratio of circumference to diameter $\pi$ = 3.1415926536 ..... 0.4971499
Area of circle to radius 1 = ..... " ..... "
Surface of sphere to diameter 1 = ..... " ..... "
Area of circle to diameter 1 = ..... .7853981634 ..... — 1.8950899
Base of Napierian Logarithms = .....2.7182818285 ..... .4342945
Modulus of common " = ..... .4342944819 ..... — 1.6377843
Equatorial radius of the earth, in feet = 20923599.98 ..... 7.3206364
Polar " " " = 20853657.16 ..... 7.3191823
Length of seconds pendulum, in London, in inches = 39.13929.
" " " Paris " = 39.1285.
" " " New York " = 39.1012.

U. S. standard gallon contains 231 c. in., or 58372.175 grains = 8.338882 lbs. avoirdupois of water at 39.8° Fahr.

U. S. standard bushel contains 2150.42 c. in., or 77.627413 lbs. av. of water at 39.8° Fahr.

British imperial gallon contains 277.274 c. in., = 1.2003 wine gallons of 231 c. in.

French metre = 39.37079 in. = 3.28089917 feet.
" toise = 6.39459252 feet.
" are = 100 sq. metres = 1076.4299 sq. ft.
" hectare = 100 ares = 2.471143 acres = 107642.9936 sq. ft.
" litre = 1 cubic decimeter = 61.02705 c. in. = .26418637 gallons of 231 c. in.
" hectolitre = 100 litres = 26.418637 gallons.

1 pound avoirdupois = 7000 grs. = 1.215277 pounds Troy.
1 " Troy = 5760 grs. = .822857 pounds avoir.
1 gramme = 15.442 grains.
1 kilogramme = 1000 grammes = 15442 grs. = 2.20607 lbs. avoir.
Tropical year = 365 d. 5 h. 45 m. 47.588 sec.

# TABLE

OF

# NATURAL SINES AND COSINES.

# NATURAL SINES AND COSINES.

| ′ | 0° | | 1° | | 2° | | 3° | | 4° | | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | |
| 0 | 00000 | Unit. | 01745 | 99985 | 03490 | 99939 | 05234 | 99863 | 06976 | 99756 | 60 |
| 1 | 00029 | Unit. | 01774 | 99984 | 03519 | 99938 | 05263 | 99861 | 07005 | 99754 | 59 |
| 2 | 00058 | Unit. | 01803 | 99984 | 03548 | 99937 | 05292 | 99860 | 07034 | 99752 | 58 |
| 3 | 00087 | Unit. | 01832 | 99983 | 03577 | 99936 | 05321 | 99858 | 07063 | 99750 | 57 |
| 4 | 00116 | Unit. | 01862 | 99983 | 03606 | 99935 | 05350 | 99857 | 07092 | 99748 | 56 |
| 5 | 00145 | Unit. | 01891 | 99982 | 03635 | 99934 | 05379 | 99855 | 07121 | 99746 | 55 |
| 6 | 00175 | Unit. | 01920 | 99982 | 03664 | 99933 | 05408 | 99854 | 07150 | 99744 | 54 |
| 7 | 00204 | Unit. | 01949 | 99981 | 03693 | 99932 | 05437 | 99852 | 07179 | 99742 | 53 |
| 8 | 00233 | Unit. | 01978 | 99980 | 03723 | 99931 | 05466 | 99851 | 07208 | 99740 | 52 |
| 9 | 00262 | Unit. | 02007 | 99980 | 03752 | 99930 | 05495 | 99849 | 07237 | 99738 | 51 |
| 10 | 00291 | Unit. | 02036 | 99979 | 03781 | 99929 | 05524 | 99847 | 07266 | 99736 | 50 |
| 11 | 00320 | 99999 | 02065 | 99979 | 03810 | 99927 | 05553 | 99846 | 07295 | 99734 | 49 |
| 12 | 00349 | 99999 | 02094 | 99978 | 03839 | 99926 | 05582 | 99844 | 07324 | 99731 | 48 |
| 13 | 00378 | 99999 | 02123 | 99977 | 03868 | 99925 | 05611 | 99842 | 07353 | 99729 | 47 |
| 14 | 00407 | 99999 | 02152 | 99977 | 03897 | 99924 | 05640 | 99841 | 07382 | 99727 | 46 |
| 15 | 00436 | 99999 | 02181 | 99976 | 03926 | 99923 | 05669 | 99839 | 07411 | 99725 | 45 |
| 16 | 00465 | 99999 | 02211 | 99976 | 03955 | 99922 | 05698 | 99838 | 07440 | 99723 | 44 |
| 17 | 00495 | 99999 | 02240 | 99975 | 03984 | 99921 | 05727 | 99836 | 07469 | 99721 | 43 |
| 18 | 00524 | 99999 | 02269 | 99974 | 04013 | 99919 | 05756 | 99834 | 07498 | 99719 | 42 |
| 19 | 00553 | 99998 | 02298 | 99974 | 04042 | 99918 | 05785 | 99833 | 07527 | 99716 | 41 |
| 20 | 00582 | 99998 | 02327 | 99973 | 04071 | 99917 | 05814 | 99831 | 07556 | 99714 | 40 |
| 21 | 00611 | 99998 | 02356 | 99972 | 04100 | 99916 | 05844 | 99829 | 07585 | 99712 | 39 |
| 22 | 00640 | 99998 | 02385 | 99972 | 04129 | 99915 | 05873 | 99827 | 07614 | 99710 | 38 |
| 23 | 00669 | 99998 | 02414 | 99971 | 04159 | 99913 | 05902 | 99826 | 07643 | 99708 | 37 |
| 24 | 00698 | 99998 | 02443 | 99970 | 04188 | 99912 | 05931 | 99824 | 07672 | 99705 | 36 |
| 25 | 00727 | 99997 | 02472 | 99969 | 04217 | 99911 | 05960 | 99822 | 07701 | 99703 | 35 |
| 26 | 00756 | 99997 | 02501 | 99969 | 04246 | 99910 | 05989 | 99821 | 07730 | 99701 | 34 |
| 27 | 00785 | 99997 | 02530 | 99968 | 04275 | 99909 | 06018 | 99819 | 07759 | 99699 | 33 |
| 28 | 00814 | 99997 | 02560 | 99967 | 04304 | 99907 | 06047 | 99817 | 07788 | 99696 | 32 |
| 29 | 00844 | 99996 | 02589 | 99966 | 04333 | 99906 | 06076 | 99815 | 07817 | 99694 | 31 |
| 30 | 00873 | 99996 | 02618 | 99966 | 04362 | 99905 | 06105 | 99813 | 07846 | 99692 | 30 |
| 31 | 00902 | 99996 | 02647 | 99965 | 04391 | 99904 | 06134 | 99812 | 07875 | 99689 | 29 |
| 32 | 00931 | 99996 | 02676 | 99964 | 04420 | 99902 | 06163 | 99810 | 07904 | 99687 | 28 |
| 33 | 00960 | 99995 | 02705 | 99963 | 04449 | 99901 | 06192 | 99808 | 07933 | 99685 | 27 |
| 34 | 00989 | 99995 | 02734 | 99963 | 04478 | 99900 | 06221 | 99806 | 07962 | 99683 | 26 |
| 35 | 01018 | 99995 | 02763 | 99962 | 04507 | 99898 | 06250 | 99804 | 07991 | 99680 | 25 |
| 36 | 01047 | 99995 | 02792 | 99961 | 04536 | 99897 | 06279 | 99803 | 08020 | 99678 | 24 |
| 37 | 01076 | 99994 | 02821 | 99960 | 04565 | 99896 | 06308 | 99801 | 08049 | 99676 | 23 |
| 38 | 01105 | 99994 | 02850 | 99959 | 04594 | 99894 | 06337 | 99799 | 08078 | 99673 | 22 |
| 39 | 01134 | 99994 | 02879 | 99959 | 04623 | 99893 | 06366 | 99797 | 08107 | 99671 | 21 |
| 40 | 01164 | 99993 | 02908 | 99958 | 04653 | 99892 | 06395 | 99795 | 08136 | 99668 | 20 |
| 41 | 01193 | 99993 | 02938 | 99957 | 04682 | 99890 | 06424 | 99793 | 08165 | 99666 | 19 |
| 42 | 01222 | 99993 | 02967 | 99956 | 04711 | 99889 | 06453 | 99792 | 08194 | 99664 | 18 |
| 43 | 01251 | 99992 | 02996 | 99955 | 04740 | 99888 | 06482 | 99790 | 08223 | 99661 | 17 |
| 44 | 01280 | 99992 | 03025 | 99954 | 04769 | 99886 | 06511 | 99788 | 08252 | 99659 | 16 |
| 45 | 01309 | 99991 | 03054 | 99953 | 04798 | 99885 | 06540 | 99786 | 08281 | 99657 | 15 |
| 46 | 01338 | 99991 | 03083 | 99952 | 04827 | 99883 | 06569 | 99784 | 08310 | 99654 | 14 |
| 47 | 01367 | 99991 | 03112 | 99952 | 04856 | 99882 | 06598 | 99782 | 08339 | 99652 | 13 |
| 48 | 01396 | 99990 | 03141 | 99951 | 04885 | 99881 | 06627 | 99780 | 08368 | 99649 | 12 |
| 49 | 01425 | 99990 | 03170 | 99950 | 04914 | 99879 | 06656 | 99778 | 08397 | 99647 | 11 |
| 50 | 01454 | 99989 | 03199 | 99949 | 04943 | 99878 | 06685 | 99776 | 08426 | 99644 | 10 |
| 51 | 01483 | 99989 | 03228 | 99948 | 04972 | 99876 | 06714 | 99774 | 08455 | 99642 | 9 |
| 52 | 01513 | 99989 | 03257 | 99947 | 05001 | 99875 | 06743 | 99772 | 08484 | 99639 | 8 |
| 53 | 01542 | 99988 | 03286 | 99946 | 05030 | 99873 | 06773 | 99770 | 08513 | 99637 | 7 |
| 54 | 01571 | 99988 | 03316 | 99945 | 05059 | 99872 | 06802 | 99768 | 08542 | 99635 | 6 |
| 55 | 01600 | 99987 | 03345 | 99944 | 05088 | 99870 | 06831 | 99766 | 08571 | 99632 | 5 |
| 56 | 01629 | 99987 | 03374 | 99943 | 05117 | 99869 | 06860 | 99764 | 08600 | 99630 | 4 |
| 57 | 01658 | 99986 | 03403 | 99942 | 05146 | 99867 | 06889 | 99762 | 08629 | 99627 | 3 |
| 58 | 01687 | 99986 | 03432 | 99941 | 05175 | 99866 | 06918 | 99760 | 08658 | 99625 | 2 |
| 59 | 01716 | 99985 | 03461 | 99940 | 05205 | 99864 | 06947 | 99758 | 08687 | 99622 | 1 |
| 60 | 01745 | 99985 | 03490 | 99939 | 05234 | 99863 | 06976 | 99756 | 08716 | 99619 | 0 |
| ′ | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | ′ |
| | 89° | | 88° | | 87° | | 86° | | 85° | | |

# NATURAL SINES AND COSINES.

| ′ | 5° | | 6° | | 7° | | 8° | | 9° | | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | |
| 0 | 08716 | 99619 | 10453 | 99452 | 12187 | 99255 | 13917 | 99027 | 15643 | 98769 | 60 |
| 1 | 08745 | 99617 | 10482 | 99449 | 12216 | 99251 | 13946 | 99023 | 15672 | 98764 | 59 |
| 2 | 08774 | 99614 | 10511 | 99446 | 12245 | 99248 | 13975 | 99019 | 15701 | 98760 | 58 |
| 3 | 08803 | 99612 | 10540 | 99443 | 12274 | 99244 | 14004 | 99015 | 15730 | 98755 | 57 |
| 4 | 08831 | 99609 | 10569 | 99440 | 12302 | 99240 | 14033 | 99011 | 15758 | 98751 | 56 |
| 5 | 08860 | 99607 | 10597 | 99437 | 12331 | 99237 | 14061 | 99006 | 15787 | 98746 | 55 |
| 6 | 08889 | 99604 | 10626 | 99434 | 12360 | 99233 | 14090 | 99002 | 15816 | 98741 | 54 |
| 7 | 08918 | 99602 | 10655 | 99431 | 12389 | 99230 | 14119 | 98998 | 15845 | 98737 | 53 |
| 8 | 08947 | 99599 | 10684 | 99428 | 12418 | 99226 | 14148 | 98994 | 15873 | 98732 | 52 |
| 9 | 08976 | 99596 | 10713 | 99424 | 12447 | 99222 | 14177 | 98990 | 15902 | 98728 | 51 |
| 10 | 09005 | 99594 | 10742 | 99421 | 12476 | 99219 | 14205 | 98986 | 15931 | 98723 | 50 |
| 11 | 09034 | 99591 | 10771 | 99418 | 12504 | 99215 | 14234 | 98982 | 15959 | 98718 | 49 |
| 12 | 09063 | 99588 | 10800 | 99415 | 12533 | 99211 | 14263 | 98978 | 15988 | 98714 | 48 |
| 13 | 09092 | 99586 | 10829 | 99412 | 12562 | 99208 | 14292 | 98973 | 16017 | 98709 | 47 |
| 14 | 09121 | 99583 | 10858 | 99409 | 12591 | 99204 | 14320 | 98969 | 16046 | 98704 | 46 |
| 15 | 09150 | 99580 | 10887 | 99406 | 12620 | 99200 | 14349 | 98965 | 16074 | 98700 | 45 |
| 16 | 09179 | 99578 | 10916 | 99402 | 12649 | 99197 | 14378 | 98961 | 16103 | 98695 | 44 |
| 17 | 09208 | 99575 | 10945 | 99399 | 12678 | 99193 | 14407 | 98957 | 16132 | 98690 | 43 |
| 18 | 09237 | 99572 | 10973 | 99396 | 12706 | 99189 | 14436 | 98953 | 16160 | 98686 | 42 |
| 19 | 09266 | 99570 | 11002 | 99393 | 12735 | 99186 | 14464 | 98948 | 16189 | 98681 | 41 |
| 20 | 09295 | 99567 | 11031 | 99390 | 12764 | 99182 | 14493 | 98944 | 16218 | 98676 | 40 |
| 21 | 09324 | 99564 | 11060 | 99386 | 12793 | 99178 | 14522 | 98940 | 16246 | 98671 | 39 |
| 22 | 09353 | 99562 | 11089 | 99383 | 12822 | 99175 | 14551 | 98936 | 16275 | 98667 | 38 |
| 23 | 09382 | 99559 | 11118 | 99380 | 12851 | 99171 | 14580 | 98931 | 16304 | 98662 | 37 |
| 24 | 09411 | 99556 | 11147 | 99377 | 12880 | 99167 | 14608 | 98927 | 16333 | 98657 | 36 |
| 25 | 09440 | 99553 | 11176 | 99374 | 12908 | 99163 | 14637 | 98923 | 16361 | 98652 | 35 |
| 26 | 09469 | 99551 | 11205 | 99370 | 12937 | 99160 | 14666 | 98919 | 16390 | 98648 | 34 |
| 27 | 09498 | 99548 | 11234 | 99367 | 12966 | 99156 | 14695 | 98914 | 16419 | 98643 | 33 |
| 28 | 09527 | 99545 | 11263 | 99364 | 12995 | 99152 | 14723 | 98910 | 16447 | 98638 | 32 |
| 29 | 09556 | 99542 | 11291 | 99360 | 13024 | 99148 | 14752 | 98906 | 16476 | 98633 | 31 |
| 30 | 09585 | 99540 | 11320 | 99357 | 13053 | 99144 | 14781 | 98902 | 16505 | 98629 | 30 |
| 31 | 09614 | 99537 | 11349 | 99354 | 13081 | 99141 | 14810 | 98897 | 16533 | 98624 | 29 |
| 32 | 09642 | 99534 | 11378 | 99351 | 13110 | 99137 | 14838 | 98893 | 16562 | 98619 | 28 |
| 33 | 09671 | 99531 | 11407 | 99347 | 13139 | 99133 | 14867 | 98889 | 16591 | 98614 | 27 |
| 34 | 09700 | 99528 | 11436 | 99344 | 13168 | 99129 | 14896 | 98884 | 16620 | 98609 | 26 |
| 35 | 09729 | 99526 | 11465 | 99341 | 13197 | 99125 | 14925 | 98880 | 16648 | 98604 | 25 |
| 36 | 09758 | 99523 | 11494 | 99337 | 13226 | 99122 | 14954 | 98876 | 16677 | 98600 | 24 |
| 37 | 09787 | 99520 | 11523 | 99334 | 13254 | 99118 | 14982 | 98871 | 16706 | 98595 | 23 |
| 38 | 09816 | 99517 | 11552 | 99331 | 13283 | 99114 | 15011 | 98867 | 16734 | 98590 | 22 |
| 39 | 09845 | 99514 | 11580 | 99327 | 13312 | 99110 | 15040 | 98863 | 16763 | 98585 | 21 |
| 40 | 09874 | 99511 | 11609 | 99324 | 13341 | 99106 | 15069 | 98858 | 16792 | 98580 | 20 |
| 41 | 09903 | 99508 | 11638 | 99320 | 13370 | 99102 | 15097 | 98854 | 16820 | 98575 | 19 |
| 42 | 09932 | 99506 | 11667 | 99317 | 13399 | 99098 | 15126 | 98849 | 16849 | 98570 | 18 |
| 43 | 09961 | 99503 | 11696 | 99314 | 13427 | 99094 | 15155 | 98845 | 16878 | 98565 | 17 |
| 44 | 09990 | 99500 | 11725 | 99310 | 13456 | 99091 | 15184 | 98841 | 16906 | 98561 | 16 |
| 45 | 10019 | 99497 | 11754 | 99307 | 13485 | 99087 | 15212 | 98836 | 16935 | 98556 | 15 |
| 46 | 10048 | 99494 | 11783 | 99303 | 13514 | 99083 | 15241 | 98832 | 16964 | 98551 | 14 |
| 47 | 10077 | 99491 | 11812 | 99300 | 13543 | 99079 | 15270 | 98827 | 16992 | 98546 | 13 |
| 48 | 10106 | 99488 | 11840 | 99297 | 13572 | 99075 | 15299 | 98823 | 17021 | 98541 | 12 |
| 49 | 10135 | 99485 | 11869 | 99293 | 13600 | 99071 | 15327 | 98818 | 17050 | 98536 | 11 |
| 50 | 10164 | 99482 | 11898 | 99290 | 13629 | 99067 | 15356 | 98814 | 17078 | 98531 | 10 |
| 51 | 10192 | 99479 | 11927 | 99286 | 13658 | 99063 | 15385 | 98809 | 17107 | 98526 | 9 |
| 52 | 10221 | 99476 | 11956 | 99283 | 13687 | 99059 | 15414 | 98805 | 17136 | 98521 | 8 |
| 53 | 10250 | 99473 | 11985 | 99279 | 13716 | 99055 | 15442 | 98800 | 17164 | 98516 | 7 |
| 54 | 10279 | 99470 | 12014 | 99276 | 13744 | 99051 | 15471 | 98796 | 17193 | 98511 | 6 |
| 55 | 10308 | 99467 | 12043 | 99272 | 13773 | 99047 | 15500 | 98791 | 17222 | 98506 | 5 |
| 56 | 10337 | 99464 | 12071 | 99269 | 13802 | 99043 | 15529 | 98787 | 17250 | 98501 | 4 |
| 57 | 10306 | 99461 | 12100 | 99265 | 13831 | 99039 | 15557 | 98782 | 17279 | 98496 | 3 |
| 58 | 10395 | 99458 | 12129 | 99262 | 13860 | 99035 | 15586 | 98778 | 17308 | 98491 | 2 |
| 59 | 10424 | 99455 | 12158 | 99258 | 13889 | 99031 | 15615 | 98773 | 17336 | 98486 | 1 |
| 60 | 10453 | 99452 | 12187 | 99255 | 13917 | 99027 | 15643 | 98769 | 17365 | 98481 | 0 |
| ′ | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | ′ |
| | 84° | | 83° | | 82° | | 81° | | 80° | | |

# NATURAL SINES AND COSINES.

| ′ | 10° | | 11° | | 12° | | 13° | | 14° | | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | |
| 0 | 17365 | 98481 | 19081 | 98163 | 20791 | 97815 | 22495 | 97437 | 24192 | 97030 | 60 |
| 1 | 17393 | 98476 | 19109 | 98157 | 20820 | 97809 | 22523 | 97430 | 24220 | 97023 | 59 |
| 2 | 17422 | 98471 | 19138 | 98152 | 20848 | 97803 | 22552 | 97424 | 24249 | 97015 | 58 |
| 3 | 17451 | 98466 | 19167 | 98146 | 20877 | 97797 | 22580 | 97417 | 24277 | 97008 | 57 |
| 4 | 17479 | 98461 | 19195 | 98140 | 20905 | 97791 | 22608 | 97411 | 24305 | 97001 | 56 |
| 5 | 17508 | 98455 | 19224 | 98135 | 20933 | 97784 | 22637 | 97404 | 24333 | 96994 | 55 |
| 6 | 17537 | 98450 | 19252 | 98129 | 20962 | 97778 | 22665 | 97398 | 24362 | 96987 | 54 |
| 7 | 17565 | 98445 | 19281 | 98124 | 20990 | 97772 | 22693 | 97391 | 24390 | 96980 | 53 |
| 8 | 17594 | 98440 | 19309 | 98118 | 21019 | 97766 | 22722 | 97384 | 24418 | 96973 | 52 |
| 9 | 17623 | 98435 | 19338 | 98112 | 21047 | 97760 | 22750 | 97378 | 24446 | 96966 | 51 |
| 10 | 17651 | 98430 | 19366 | 98107 | 21076 | 97754 | 22778 | 97371 | 24474 | 96959 | 50 |
| 11 | 17680 | 98425 | 19395 | 98101 | 21104 | 97748 | 22807 | 97365 | 24503 | 96952 | 49 |
| 12 | 17708 | 98420 | 19423 | 98096 | 21132 | 97742 | 22835 | 97358 | 24531 | 96945 | 48 |
| 13 | 17737 | 98414 | 19452 | 98090 | 21161 | 97735 | 22863 | 97351 | 24559 | 96937 | 47 |
| 14 | 17766 | 98409 | 19481 | 98084 | 21189 | 97729 | 22892 | 97345 | 24587 | 96930 | 46 |
| 15 | 17794 | 98404 | 19509 | 98079 | 21218 | 97723 | 22920 | 97338 | 24615 | 96923 | 45 |
| 16 | 17823 | 98399 | 19538 | 98073 | 21246 | 97717 | 22948 | 97331 | 24644 | 96916 | 44 |
| 17 | 17852 | 98394 | 19566 | 98067 | 21275 | 97711 | 22977 | 97325 | 24672 | 96909 | 43 |
| 18 | 17880 | 98389 | 19595 | 98061 | 21303 | 97705 | 23005 | 97318 | 24700 | 96902 | 42 |
| 19 | 17909 | 98383 | 19623 | 98056 | 21331 | 97698 | 23033 | 97311 | 24728 | 96894 | 41 |
| 20 | 17937 | 98378 | 19652 | 98050 | 21360 | 97692 | 23062 | 97304 | 24756 | 96887 | 40 |
| 21 | 17966 | 98373 | 19680 | 98044 | 21388 | 97686 | 23090 | 97298 | 24784 | 96880 | 39 |
| 22 | 17995 | 98368 | 19709 | 98039 | 21417 | 97680 | 23118 | 97291 | 24813 | 96873 | 38 |
| 23 | 18023 | 98362 | 19737 | 98033 | 21445 | 97673 | 23146 | 97284 | 24841 | 96866 | 37 |
| 24 | 18052 | 98357 | 19766 | 98027 | 21474 | 97667 | 23175 | 97278 | 24869 | 96858 | 36 |
| 25 | 18081 | 98352 | 19794 | 98021 | 21502 | 97661 | 23203 | 97271 | 24897 | 96851 | 35 |
| 26 | 18109 | 98347 | 19823 | 98016 | 21530 | 97655 | 23231 | 97264 | 24925 | 96844 | 34 |
| 27 | 18138 | 98341 | 19851 | 98010 | 21559 | 97648 | 23260 | 97257 | 24954 | 96837 | 33 |
| 28 | 18166 | 98336 | 19880 | 98004 | 21587 | 97642 | 23288 | 97251 | 24982 | 96829 | 32 |
| 29 | 18195 | 98331 | 19908 | 97998 | 21616 | 97636 | 23316 | 97244 | 25010 | 96822 | 31 |
| 30 | 18224 | 98325 | 19937 | 97992 | 21644 | 97630 | 23345 | 97237 | 25038 | 96815 | 30 |
| 31 | 18252 | 98320 | 19965 | 97987 | 21672 | 97623 | 23373 | 97230 | 25066 | 96807 | 29 |
| 32 | 18281 | 98315 | 19994 | 97981 | 21701 | 97617 | 23401 | 97223 | 25094 | 96800 | 28 |
| 33 | 18309 | 98310 | 20022 | 97975 | 21729 | 97611 | 23429 | 97217 | 25122 | 96793 | 27 |
| 34 | 18338 | 98304 | 20051 | 97969 | 21758 | 97604 | 23458 | 97210 | 25151 | 96786 | 26 |
| 35 | 18367 | 98299 | 20079 | 97963 | 21786 | 97598 | 23486 | 97203 | 25179 | 96778 | 25 |
| 36 | 18395 | 98294 | 20108 | 97958 | 21814 | 97592 | 23514 | 97196 | 25207 | 96771 | 24 |
| 37 | 18424 | 98288 | 20136 | 97952 | 21843 | 97585 | 23542 | 97189 | 25235 | 96764 | 23 |
| 38 | 18452 | 98283 | 20165 | 97946 | 21871 | 97579 | 23571 | 97182 | 25263 | 96756 | 22 |
| 39 | 18481 | 98277 | 20193 | 97940 | 21899 | 97573 | 23599 | 97176 | 25291 | 96749 | 21 |
| 40 | 18509 | 98272 | 20222 | 97934 | 21928 | 97566 | 23627 | 97169 | 25320 | 96742 | 20 |
| 41 | 18538 | 98267 | 20250 | 97928 | 21956 | 97560 | 23656 | 97162 | 25348 | 96734 | 19 |
| 42 | 18567 | 98261 | 20279 | 97922 | 21985 | 97553 | 23684 | 97155 | 25376 | 96727 | 18 |
| 43 | 18595 | 98256 | 20307 | 97916 | 22013 | 97547 | 23712 | 97148 | 25404 | 96719 | 17 |
| 44 | 18624 | 98250 | 20336 | 97910 | 22041 | 97541 | 23740 | 97141 | 25432 | 96712 | 16 |
| 45 | 18652 | 98245 | 20364 | 97905 | 22070 | 97534 | 23769 | 97134 | 25460 | 96705 | 15 |
| 46 | 18681 | 98240 | 20393 | 97899 | 22098 | 97528 | 23797 | 97127 | 25488 | 96697 | 14 |
| 47 | 18710 | 98234 | 20421 | 97893 | 22126 | 97521 | 23825 | 97120 | 25516 | 96690 | 13 |
| 48 | 18738 | 98229 | 20450 | 97887 | 22155 | 97515 | 23853 | 97113 | 25545 | 96682 | 12 |
| 49 | 18767 | 98223 | 20478 | 97881 | 22183 | 97508 | 23882 | 97106 | 25573 | 96675 | 11 |
| 50 | 18795 | 98218 | 20507 | 97875 | 22212 | 97502 | 23910 | 97100 | 25601 | 96667 | 10 |
| 51 | 18824 | 98212 | 20535 | 97869 | 22240 | 97496 | 23938 | 97093 | 25629 | 96660 | 9 |
| 52 | 18852 | 98207 | 20563 | 97863 | 22268 | 97489 | 23966 | 97086 | 25657 | 96653 | 8 |
| 53 | 18881 | 98201 | 20592 | 97857 | 22297 | 97483 | 23995 | 97079 | 25685 | 96645 | 7 |
| 54 | 18910 | 98196 | 20620 | 97851 | 22325 | 97476 | 24023 | 97072 | 25713 | 96638 | 6 |
| 55 | 18938 | 98190 | 20649 | 97845 | 22353 | 97470 | 24051 | 97065 | 25741 | 96630 | 5 |
| 56 | 18967 | 98185 | 20677 | 97839 | 22382 | 97463 | 24079 | 97058 | 25769 | 96623 | 4 |
| 57 | 18995 | 98179 | 20706 | 97833 | 22410 | 97457 | 24108 | 97051 | 25798 | 96615 | 3 |
| 58 | 19024 | 98174 | 20734 | 97827 | 22438 | 97450 | 24136 | 97044 | 25826 | 96608 | 2 |
| 59 | 19052 | 98168 | 20763 | 97821 | 22467 | 97444 | 24164 | 97037 | 25854 | 96600 | 1 |
| 60 | 19081 | 98163 | 20791 | 97815 | 22495 | 97437 | 24192 | 97030 | 25882 | 96593 | 0 |
| ′ | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine | Sine. | Cosine. | Sine. | ′ |
| | 79° | | 78° | | 77° | | 76° | | 75° | | |

| ′ | 15° | | 16° | | 17° | | 18° | | 19° | | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | |
| 0 | 25882 | 96593 | 27564 | 96126 | 29237 | 95630 | 30902 | 95106 | 32557 | 94552 | 60 |
| 1 | 25910 | 96585 | 27592 | 96118 | 29265 | 95622 | 30929 | 95097 | 32584 | 94542 | 59 |
| 2 | 25938 | 96578 | 27620 | 96110 | 29293 | 95613 | 30957 | 95088 | 32612 | 94533 | 58 |
| 3 | 25966 | 96570 | 27648 | 96102 | 29321 | 95605 | 30985 | 95079 | 32639 | 94523 | 57 |
| 4 | 25994 | 96562 | 27676 | 96094 | 29348 | 95596 | 31012 | 95070 | 32667 | 94514 | 56 |
| 5 | 26022 | 96555 | 27704 | 96086 | 29376 | 95588 | 31040 | 95061 | 32694 | 94504 | 55 |
| 6 | 26050 | 96547 | 27731 | 96078 | 29404 | 95579 | 31068 | 95052 | 32722 | 94495 | 54 |
| 7 | 26079 | 96540 | 27759 | 96070 | 29432 | 95571 | 31095 | 95043 | 32749 | 94485 | 53 |
| 8 | 26107 | 96532 | 27787 | 96062 | 29460 | 95562 | 31123 | 95033 | 32777 | 94476 | 52 |
| 9 | 26135 | 96524 | 27815 | 96054 | 29487 | 95554 | 31151 | 95024 | 32804 | 94466 | 51 |
| 10 | 26163 | 96517 | 27843 | 96046 | 29515 | 95545 | 31178 | 95015 | 32832 | 94457 | 50 |
| 11 | 26191 | 96509 | 27871 | 96037 | 29543 | 95536 | 31206 | 95006 | 32859 | 94447 | 49 |
| 12 | 26219 | 96502 | 27899 | 96029 | 29571 | 95528 | 31233 | 94997 | 32887 | 94438 | 48 |
| 13 | 26247 | 96494 | 27927 | 96021 | 29599 | 95519 | 31261 | 94988 | 32914 | 94428 | 47 |
| 14 | 26275 | 96486 | 27955 | 96013 | 29626 | 95511 | 31289 | 94979 | 32942 | 94418 | 46 |
| 15 | 26303 | 96479 | 27983 | 96005 | 29654 | 95502 | 31316 | 94970 | 32969 | 94409 | 45 |
| 16 | 26331 | 96471 | 28011 | 95997 | 29682 | 95493 | 31344 | 94961 | 32997 | 94399 | 44 |
| 17 | 26359 | 96463 | 28039 | 95989 | 29710 | 95485 | 31372 | 94952 | 33024 | 94390 | 43 |
| 18 | 26387 | 96456 | 28067 | 95981 | 29737 | 95476 | 31399 | 94943 | 33051 | 94380 | 42 |
| 19 | 26415 | 96448 | 28095 | 95972 | 29765 | 95467 | 31427 | 94933 | 33079 | 94370 | 41 |
| 20 | 26443 | 96440 | 28123 | 95964 | 29793 | 95459 | 31454 | 94924 | 33106 | 94361 | 40 |
| 21 | 26471 | 96433 | 28150 | 95956 | 29821 | 95450 | 31482 | 94915 | 33134 | 94351 | 39 |
| 22 | 26500 | 96425 | 28178 | 95948 | 29849 | 95441 | 31510 | 94906 | 33161 | 94342 | 38 |
| 23 | 26528 | 96417 | 28206 | 95940 | 29876 | 95433 | 31537 | 94897 | 33189 | 94332 | 37 |
| 24 | 26556 | 96410 | 28234 | 95931 | 29904 | 95424 | 31565 | 94888 | 33216 | 94322 | 36 |
| 25 | 26584 | 96402 | 28262 | 95923 | 29932 | 95415 | 31593 | 94878 | 33244 | 94313 | 35 |
| 26 | 26612 | 96394 | 28290 | 95915 | 29960 | 95407 | 31620 | 94869 | 33271 | 94303 | 34 |
| 27 | 26640 | 96386 | 28318 | 95907 | 29987 | 95398 | 31648 | 94860 | 33298 | 94293 | 33 |
| 28 | 26668 | 96379 | 28346 | 95898 | 30015 | 95389 | 31675 | 94851 | 33326 | 94284 | 32 |
| 29 | 26696 | 96371 | 28374 | 95890 | 30043 | 95380 | 31703 | 94842 | 33353 | 94274 | 31 |
| 30 | 26724 | 96363 | 28402 | 95882 | 30071 | 95372 | 31730 | 94832 | 33381 | 94264 | 30 |
| 31 | 26752 | 96355 | 28429 | 95874 | 30098 | 95363 | 31758 | 94823 | 33408 | 94254 | 29 |
| 32 | 26780 | 96347 | 28457 | 95865 | 30126 | 95354 | 31786 | 94814 | 33436 | 94245 | 28 |
| 33 | 26808 | 96340 | 28485 | 95857 | 30154 | 95345 | 31813 | 94805 | 33463 | 94235 | 27 |
| 34 | 26836 | 96332 | 28513 | 95849 | 30182 | 95337 | 31841 | 94795 | 33490 | 94225 | 26 |
| 35 | 26864 | 96324 | 28541 | 95841 | 30209 | 95328 | 31868 | 94786 | 33518 | 94215 | 25 |
| 36 | 26892 | 96316 | 28569 | 95832 | 30237 | 95319 | 31896 | 94777 | 33545 | 94206 | 24 |
| 37 | 26920 | 96308 | 28597 | 95824 | 30265 | 95310 | 31923 | 94768 | 33573 | 94196 | 23 |
| 38 | 26948 | 96301 | 28625 | 95816 | 30292 | 95301 | 31951 | 94758 | 33600 | 94186 | 22 |
| 39 | 26976 | 96293 | 28652 | 95807 | 30320 | 95293 | 31979 | 94749 | 33627 | 94176 | 21 |
| 40 | 27004 | 96285 | 28680 | 95799 | 30348 | 95284 | 32006 | 94740 | 33655 | 94167 | 20 |
| 41 | 27032 | 96277 | 28708 | 95791 | 30376 | 95275 | 32034 | 94730 | 33682 | 94157 | 19 |
| 42 | 27060 | 96269 | 28736 | 95782 | 30403 | 95266 | 32061 | 94721 | 33710 | 94147 | 18 |
| 43 | 27088 | 96261 | 28764 | 95774 | 30431 | 95257 | 32089 | 94712 | 33737 | 94137 | 17 |
| 44 | 27116 | 96253 | 28792 | 95766 | 30459 | 95248 | 32116 | 94702 | 33764 | 94127 | 16 |
| 45 | 27144 | 96246 | 28820 | 95757 | 30486 | 95240 | 32144 | 94693 | 33792 | 94118 | 15 |
| 46 | 27172 | 96238 | 28847 | 95749 | 30514 | 95231 | 32171 | 94684 | 33819 | 94108 | 14 |
| 47 | 27200 | 96230 | 28875 | 95740 | 30542 | 95222 | 32199 | 94674 | 33846 | 94098 | 13 |
| 48 | 27228 | 96222 | 28903 | 95732 | 30570 | 95213 | 32227 | 94665 | 33874 | 94088 | 12 |
| 49 | 27256 | 96214 | 28931 | 95724 | 30597 | 95204 | 32254 | 94656 | 33901 | 94078 | 11 |
| 50 | 27284 | 96206 | 28959 | 95715 | 30625 | 95195 | 32282 | 94646 | 33929 | 94068 | 10 |
| 51 | 27312 | 96198 | 28987 | 95707 | 30653 | 95186 | 32309 | 94637 | 33956 | 94058 | 9 |
| 52 | 27340 | 96190 | 29015 | 95698 | 30680 | 95177 | 32337 | 94627 | 33983 | 94049 | 8 |
| 53 | 27368 | 96182 | 29042 | 95690 | 30708 | 95168 | 32364 | 94618 | 34011 | 94039 | 7 |
| 54 | 27396 | 96174 | 29070 | 95681 | 30736 | 95159 | 32392 | 94609 | 34038 | 94029 | 6 |
| 55 | 27424 | 96166 | 29098 | 95673 | 30763 | 95150 | 32419 | 94599 | 34065 | 94019 | 5 |
| 56 | 27452 | 96158 | 29126 | 95664 | 30791 | 95142 | 32447 | 94590 | 34093 | 94009 | 4 |
| 57 | 27480 | 96150 | 29154 | 95656 | 30819 | 95133 | 32474 | 94580 | 34120 | 93999 | 3 |
| 58 | 27508 | 96142 | 29182 | 95647 | 30846 | 95124 | 32502 | 94571 | 34147 | 93989 | 2 |
| 59 | 27536 | 96134 | 29209 | 95639 | 30874 | 95115 | 32529 | 94561 | 34175 | 93979 | 1 |
| 60 | 27564 | 96126 | 29237 | 95630 | 30902 | 95106 | 32557 | 94552 | 34202 | 93969 | 0 |
| ′ | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine | Cosine. | Sine. | ′ |
| | 74° | | 73° | | 72° | | 71° | | 70° | | |

# NATURAL SINES AND COSINES.

| ′ | 20° | | 21° | | 22° | | 23° | | 24° | | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | |
| 0 | 34202 | 93969 | 35837 | 93358 | 37461 | 92718 | 39073 | 92050 | 40674 | 91355 | 60 |
| 1 | 34229 | 93959 | 35864 | 93348 | 37488 | 92707 | 39100 | 92039 | 40700 | 91343 | 59 |
| 2 | 34257 | 93949 | 35891 | 93337 | 37515 | 92697 | 39127 | 92028 | 40727 | 91331 | 58 |
| 3 | 34284 | 93939 | 35918 | 93327 | 37542 | 92686 | 39153 | 92016 | 40753 | 91319 | 57 |
| 4 | 34311 | 93929 | 35945 | 93316 | 37569 | 92675 | 39180 | 92005 | 40780 | 91307 | 56 |
| 5 | 34339 | 93919 | 35973 | 93306 | 37595 | 92664 | 39207 | 91994 | 40806 | 91295 | 55 |
| 6 | 34366 | 93909 | 36000 | 93295 | 37622 | 92653 | 39234 | 91982 | 40833 | 91283 | 54 |
| 7 | 34393 | 93899 | 36027 | 93285 | 37649 | 92642 | 39260 | 91971 | 40860 | 91272 | 53 |
| 8 | 34421 | 93889 | 36054 | 93274 | 37676 | 92631 | 39287 | 91959 | 40886 | 91260 | 52 |
| 9 | 34448 | 93879 | 36081 | 93264 | 37703 | 92620 | 39314 | 91948 | 40913 | 91248 | 51 |
| 10 | 34475 | 93869 | 36108 | 93253 | 37730 | 92609 | 39341 | 91936 | 40939 | 91236 | 50 |
| 11 | 34503 | 93859 | 36135 | 93243 | 37757 | 92598 | 39367 | 91925 | 40966 | 91224 | 49 |
| 12 | 34530 | 93849 | 36162 | 93232 | 37784 | 92587 | 39394 | 91914 | 40992 | 91212 | 48 |
| 13 | 34557 | 93839 | 36190 | 93222 | 37811 | 92576 | 39421 | 91902 | 41019 | 91200 | 47 |
| 14 | 34584 | 93829 | 36217 | 93211 | 37838 | 92565 | 39448 | 91891 | 41045 | 91188 | 46 |
| 15 | 34612 | 93819 | 36244 | 93201 | 37865 | 92554 | 39474 | 91879 | 41072 | 91176 | 45 |
| 16 | 34639 | 93809 | 36271 | 93190 | 37892 | 92543 | 39501 | 91868 | 41098 | 91164 | 44 |
| 17 | 34666 | 93799 | 36298 | 93180 | 37919 | 92532 | 39528 | 91856 | 41125 | 91152 | 43 |
| 18 | 34694 | 93789 | 36325 | 93169 | 37946 | 92521 | 39555 | 91845 | 41151 | 91140 | 42 |
| 19 | 34721 | 93779 | 36352 | 93159 | 37973 | 92510 | 39581 | 91833 | 41178 | 91128 | 41 |
| 20 | 34748 | 93769 | 36379 | 93148 | 37999 | 92499 | 39608 | 91822 | 41204 | 91116 | 40 |
| 21 | 34775 | 93759 | 36406 | 93137 | 38026 | 92488 | 39635 | 91810 | 41231 | 91104 | 39 |
| 22 | 34803 | 93748 | 36434 | 93127 | 38053 | 92477 | 39661 | 91799 | 41257 | 91092 | 38 |
| 23 | 34830 | 93738 | 36461 | 93116 | 38080 | 92466 | 39688 | 91787 | 41284 | 91080 | 37 |
| 24 | 34857 | 93728 | 36488 | 93106 | 38107 | 92455 | 39715 | 91775 | 41310 | 91068 | 36 |
| 25 | 34884 | 93718 | 36515 | 93095 | 38134 | 92444 | 39741 | 91764 | 41337 | 91056 | 35 |
| 26 | 34912 | 93708 | 36542 | 93084 | 38161 | 92432 | 39768 | 91752 | 41363 | 91044 | 34 |
| 27 | 34939 | 93698 | 36569 | 93074 | 38188 | 92421 | 39795 | 91741 | 41390 | 91032 | 33 |
| 28 | 34966 | 93688 | 36596 | 93063 | 38215 | 92410 | 39822 | 91729 | 41416 | 91020 | 32 |
| 29 | 34993 | 93677 | 36623 | 93052 | 38241 | 92399 | 39848 | 91718 | 41443 | 91008 | 31 |
| 30 | 35021 | 93667 | 36650 | 93042 | 38268 | 92388 | 39875 | 91706 | 41469 | 90996 | 30 |
| 31 | 35048 | 93657 | 36677 | 93031 | 38295 | 92377 | 39902 | 91694 | 41496 | 90984 | 29 |
| 32 | 35075 | 93647 | 36704 | 93020 | 38322 | 92366 | 39928 | 91683 | 41522 | 90972 | 28 |
| 33 | 35102 | 93637 | 36731 | 93010 | 38349 | 92355 | 39955 | 91671 | 41549 | 90960 | 27 |
| 34 | 35130 | 93626 | 36758 | 92999 | 38376 | 92343 | 39982 | 91660 | 41575 | 90948 | 26 |
| 35 | 35157 | 93616 | 36785 | 92988 | 38403 | 92332 | 40008 | 91648 | 41602 | 90936 | 25 |
| 36 | 35184 | 93606 | 36812 | 92978 | 38430 | 92321 | 40035 | 91636 | 41628 | 90924 | 24 |
| 37 | 35211 | 93596 | 36839 | 92967 | 38456 | 92310 | 40062 | 91625 | 41655 | 90911 | 23 |
| 38 | 35239 | 93585 | 36867 | 92956 | 38483 | 92299 | 40088 | 91613 | 41681 | 90899 | 22 |
| 39 | 35266 | 93575 | 36894 | 92945 | 38510 | 92287 | 40115 | 91601 | 41707 | 90887 | 21 |
| 40 | 35293 | 93565 | 36921 | 92935 | 38537 | 92276 | 40141 | 91590 | 41734 | 90875 | 20 |
| 41 | 35320 | 93555 | 36948 | 92924 | 38564 | 92265 | 40168 | 91578 | 41760 | 90863 | 19 |
| 42 | 35347 | 93544 | 36975 | 92913 | 38591 | 92254 | 40195 | 91566 | 41787 | 90851 | 18 |
| 43 | 35375 | 93534 | 37002 | 92902 | 38617 | 92243 | 40221 | 91555 | 41813 | 90839 | 17 |
| 44 | 35402 | 93524 | 37029 | 92892 | 38644 | 92231 | 40248 | 91543 | 41840 | 90826 | 16 |
| 45 | 35429 | 93514 | 37056 | 92881 | 38671 | 92220 | 40275 | 91531 | 41866 | 90814 | 15 |
| 46 | 35456 | 93503 | 37083 | 92870 | 38698 | 92209 | 40301 | 91519 | 41892 | 90802 | 14 |
| 47 | 35484 | 93493 | 37110 | 92859 | 38725 | 92198 | 40328 | 91508 | 41919 | 90790 | 13 |
| 48 | 35511 | 93483 | 37137 | 92849 | 38752 | 92186 | 40355 | 91496 | 41945 | 90778 | 12 |
| 49 | 35538 | 93472 | 37164 | 92838 | 38778 | 92175 | 40381 | 91484 | 41972 | 90766 | 11 |
| 50 | 35565 | 93462 | 37191 | 92827 | 38805 | 92164 | 40408 | 91472 | 41998 | 90753 | 10 |
| 51 | 35592 | 93452 | 37218 | 92816 | 38832 | 92152 | 40434 | 91461 | 42024 | 90741 | 9 |
| 52 | 35619 | 93441 | 37245 | 92805 | 38859 | 92141 | 40461 | 91449 | 42051 | 90729 | 8 |
| 53 | 35647 | 93431 | 37272 | 92794 | 38886 | 92130 | 40488 | 91437 | 42077 | 90717 | 7 |
| 54 | 35674 | 93420 | 37299 | 92784 | 38912 | 92119 | 40514 | 91425 | 42104 | 90704 | 6 |
| 55 | 35701 | 93410 | 37326 | 92773 | 38939 | 92107 | 40541 | 91414 | 42130 | 90692 | 5 |
| 56 | 35728 | 93400 | 37353 | 92762 | 38966 | 92096 | 40567 | 91402 | 42156 | 90680 | 4 |
| 57 | 35755 | 93389 | 37380 | 92751 | 38993 | 92085 | 40594 | 91390 | 42183 | 90668 | 3 |
| 58 | 35782 | 93379 | 37407 | 92740 | 39020 | 92073 | 40621 | 91378 | 42209 | 90655 | 2 |
| 59 | 35810 | 93368 | 37434 | 92729 | 39046 | 92062 | 40647 | 91366 | 42235 | 90643 | 1 |
| 60 | 35837 | 93358 | 37461 | 92718 | 39073 | 92050 | 40674 | 91355 | 42262 | 90631 | 0 |
| | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine | Sine. | Cosine. | Sine. | |
| ′ | 69° | | 68° | | 67° | | 66° | | 65° | | ′ |

# NATURAL SINES AND COSINES.

| ′ | 25° | | 26° | | 27° | | 28° | | 29° | | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | |
| 0 | 42262 | 90631 | 43837 | 89879 | 45399 | 89101 | 46947 | 88295 | 48481 | 87462 | 60 |
| 1 | 42288 | 90618 | 43863 | 89867 | 45425 | 89087 | 46973 | 88281 | 48506 | 87448 | 59 |
| 2 | 42315 | 90606 | 43889 | 89854 | 45451 | 89074 | 46999 | 88267 | 48532 | 87434 | 58 |
| 3 | 42341 | 90594 | 43916 | 89841 | 45477 | 89061 | 47024 | 88254 | 48557 | 87420 | 57 |
| 4 | 42367 | 90582 | 43942 | 89828 | 45503 | 89048 | 47050 | 88240 | 48583 | 87406 | 56 |
| 5 | 42394 | 90569 | 43968 | 89816 | 45529 | 89035 | 47076 | 88226 | 48608 | 87391 | 55 |
| 6 | 42420 | 90557 | 43994 | 89803 | 45554 | 89021 | 47101 | 88213 | 48634 | 87377 | 54 |
| 7 | 42446 | 90545 | 44020 | 89790 | 45580 | 89008 | 47127 | 88199 | 48659 | 87363 | 53 |
| 8 | 42473 | 90532 | 44046 | 89777 | 45606 | 88995 | 47153 | 88185 | 48684 | 87349 | 52 |
| 9 | 42499 | 90520 | 44072 | 89764 | 45632 | 88981 | 47178 | 88172 | 48710 | 87335 | 51 |
| 10 | 42525 | 90507 | 44098 | 89752 | 45658 | 88968 | 47204 | 88158 | 48735 | 87321 | 50 |
| 11 | 42552 | 90495 | 44124 | 89739 | 45684 | 88955 | 47229 | 88144 | 48761 | 87306 | 49 |
| 12 | 42578 | 90483 | 44151 | 89726 | 45710 | 88942 | 47255 | 88130 | 48786 | 87292 | 48 |
| 13 | 42604 | 90470 | 44177 | 89713 | 45736 | 88928 | 47281 | 88117 | 48811 | 87278 | 47 |
| 14 | 42631 | 90458 | 44203 | 89700 | 45762 | 88915 | 47306 | 88103 | 48837 | 87264 | 46 |
| 15 | 42657 | 90446 | 44229 | 89687 | 45787 | 88902 | 47332 | 88089 | 48862 | 87250 | 45 |
| 16 | 42683 | 90433 | 44255 | 89674 | 45813 | 88888 | 47358 | 88075 | 48888 | 87235 | 44 |
| 17 | 42709 | 90421 | 44281 | 89662 | 45839 | 88875 | 47383 | 88062 | 48913 | 87221 | 43 |
| 18 | 42736 | 90408 | 44307 | 89649 | 45865 | 88862 | 47409 | 88048 | 48938 | 87207 | 42 |
| 19 | 42762 | 90396 | 44333 | 89636 | 45891 | 88848 | 47434 | 88034 | 48964 | 87193 | 41 |
| 20 | 42788 | 90383 | 44359 | 89623 | 45917 | 88835 | 47460 | 88020 | 48989 | 87178 | 40 |
| 21 | 42815 | 90371 | 44385 | 89610 | 45942 | 88822 | 47486 | 88006 | 49014 | 87164 | 39 |
| 22 | 42841 | 90358 | 44411 | 89597 | 45968 | 88808 | 47511 | 87993 | 49040 | 87150 | 38 |
| 23 | 42867 | 90346 | 44437 | 89584 | 45994 | 88795 | 47537 | 87979 | 49065 | 87136 | 37 |
| 24 | 42894 | 90334 | 44464 | 89571 | 46020 | 88782 | 47562 | 87965 | 49090 | 87121 | 36 |
| 25 | 42920 | 90321 | 44490 | 89558 | 46046 | 88768 | 47588 | 87951 | 49116 | 87107 | 35 |
| 26 | 42946 | 90309 | 44516 | 89545 | 46072 | 88755 | 47614 | 87937 | 49141 | 87093 | 34 |
| 27 | 42972 | 90296 | 44542 | 89532 | 46097 | 88741 | 47639 | 87923 | 49166 | 87079 | 33 |
| 28 | 42999 | 90284 | 44568 | 89519 | 46123 | 88728 | 47665 | 87909 | 49192 | 87064 | 32 |
| 29 | 43025 | 90271 | 44594 | 89506 | 46149 | 88715 | 47690 | 87896 | 49217 | 87050 | 31 |
| 30 | 43051 | 90259 | 44620 | 89493 | 46175 | 88701 | 47716 | 87882 | 49242 | 87036 | 30 |
| 31 | 43077 | 90246 | 44646 | 89480 | 46201 | 88688 | 47741 | 87868 | 49268 | 87021 | 29 |
| 32 | 43104 | 90233 | 44672 | 89467 | 46226 | 88674 | 47767 | 87854 | 49293 | 87007 | 28 |
| 33 | 43130 | 90221 | 44698 | 89454 | 46252 | 88661 | 47793 | 87840 | 49318 | 86993 | 27 |
| 34 | 43156 | 90208 | 44724 | 89441 | 46278 | 88647 | 47818 | 87826 | 49344 | 86978 | 26 |
| 35 | 43182 | 90196 | 44750 | 89428 | 46304 | 88634 | 47844 | 87812 | 49369 | 86964 | 25 |
| 36 | 43209 | 90183 | 44776 | 89415 | 46330 | 88620 | 47869 | 87798 | 49394 | 86949 | 24 |
| 37 | 43235 | 90171 | 44802 | 89402 | 46355 | 88607 | 47895 | 87784 | 49419 | 86935 | 23 |
| 38 | 43261 | 90158 | 44828 | 89389 | 46381 | 88593 | 47920 | 87770 | 49445 | 86921 | 22 |
| 39 | 43287 | 90146 | 44854 | 89376 | 46407 | 88580 | 47946 | 87756 | 49470 | 86906 | 21 |
| 40 | 43313 | 90133 | 44880 | 89363 | 46433 | 88566 | 47971 | 87743 | 49495 | 86892 | 20 |
| 41 | 43340 | 90120 | 44906 | 89350 | 46458 | 88553 | 47997 | 87729 | 49521 | 86878 | 19 |
| 42 | 43366 | 90108 | 44932 | 89337 | 46484 | 88539 | 48022 | 87715 | 49546 | 86863 | 18 |
| 43 | 43392 | 90095 | 44958 | 89324 | 46510 | 88526 | 48048 | 87701 | 49571 | 86849 | 17 |
| 44 | 43418 | 90082 | 44984 | 89311 | 46536 | 88512 | 48073 | 87687 | 49596 | 86834 | 16 |
| 45 | 43445 | 90070 | 45010 | 89298 | 46561 | 88499 | 48099 | 87673 | 49622 | 86820 | 15 |
| 46 | 43471 | 90057 | 45036 | 89285 | 46587 | 88485 | 48124 | 87659 | 49647 | 86805 | 14 |
| 47 | 43497 | 90045 | 45062 | 89272 | 46613 | 88472 | 48150 | 87645 | 49672 | 86791 | 13 |
| 48 | 43523 | 90032 | 45088 | 89259 | 46639 | 88458 | 48175 | 87631 | 49697 | 86777 | 12 |
| 49 | 43549 | 90019 | 45114 | 89245 | 46664 | 88445 | 48201 | 87617 | 49723 | 86762 | 11 |
| 50 | 43575 | 90007 | 45140 | 89232 | 46690 | 88431 | 48226 | 87603 | 49748 | 86748 | 10 |
| 51 | 43602 | 89994 | 45166 | 89219 | 46716 | 88417 | 48252 | 87589 | 49773 | 86733 | 9 |
| 52 | 43628 | 89981 | 45192 | 89206 | 46742 | 88404 | 48277 | 87575 | 49798 | 86719 | 8 |
| 53 | 43654 | 89968 | 45218 | 89193 | 46767 | 88390 | 48303 | 87561 | 49824 | 86704 | 7 |
| 54 | 43680 | 89956 | 45243 | 89180 | 46793 | 88377 | 48328 | 87546 | 49849 | 86690 | 6 |
| 55 | 43706 | 89943 | 45269 | 89167 | 46819 | 88363 | 48354 | 87532 | 49874 | 86675 | 5 |
| 56 | 43733 | 89930 | 45295 | 89153 | 46844 | 88349 | 48379 | 87518 | 49899 | 86661 | 4 |
| 57 | 43759 | 89918 | 45321 | 89140 | 46870 | 88336 | 48405 | 87504 | 49924 | 86646 | 3 |
| 58 | 43785 | 89905 | 45347 | 89127 | 46896 | 88322 | 48430 | 87490 | 49950 | 86632 | 2 |
| 59 | 43811 | 89892 | 45373 | 89114 | 46921 | 88308 | 48456 | 87476 | 49975 | 86617 | 1 |
| 60 | 43837 | 89879 | 45399 | 89101 | 46947 | 88295 | 48481 | 87462 | 50000 | 86603 | 0 |
| ′ | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | ′ |
| | 64° | | 63° | | 62° | | 61° | | 60° | | |

# NATURAL SINES AND COSINES.

| ′ | 30° | | 31° | | 32° | | 33° | | 34° | | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | |
| 0 | 50000 | 86603 | 51504 | 85717 | 52992 | 84805 | 54464 | 83867 | 55919 | 82904 | 60 |
| 1 | 50025 | 86588 | 51529 | 85702 | 53017 | 84789 | 54488 | 83851 | 55943 | 82887 | 59 |
| 2 | 50050 | 86573 | 51554 | 85687 | 53041 | 84774 | 54513 | 83835 | 55968 | 82871 | 58 |
| 3 | 50076 | 86559 | 51579 | 85672 | 53066 | 84759 | 54537 | 83819 | 55992 | 82855 | 57 |
| 4 | 50101 | 86544 | 51604 | 85657 | 53091 | 84743 | 54561 | 83804 | 56016 | 82839 | 56 |
| 5 | 50126 | 86530 | 51628 | 85642 | 53115 | 84728 | 54586 | 83788 | 56040 | 82822 | 55 |
| 6 | 50151 | 86515 | 51653 | 85627 | 53140 | 84712 | 54610 | 83772 | 56064 | 82806 | 54 |
| 7 | 50176 | 86501 | 51678 | 85612 | 53164 | 84697 | 54635 | 83756 | 56088 | 82790 | 53 |
| 8 | 50201 | 86486 | 51703 | 85597 | 53189 | 84681 | 54659 | 83740 | 56112 | 82773 | 52 |
| 9 | 50227 | 86471 | 51728 | 85582 | 53214 | 84666 | 54683 | 83724 | 56136 | 82757 | 51 |
| 10 | 50252 | 86457 | 51753 | 85567 | 53238 | 84650 | 54708 | 83708 | 56160 | 82741 | 50 |
| 11 | 50277 | 86442 | 51778 | 85551 | 53263 | 84635 | 54732 | 83692 | 56184 | 82724 | 49 |
| 12 | 50302 | 86427 | 51803 | 85536 | 53288 | 84619 | 54756 | 83676 | 56208 | 82708 | 48 |
| 13 | 50327 | 86413 | 51828 | 85521 | 53312 | 84604 | 54781 | 83660 | 56232 | 82692 | 47 |
| 14 | 50352 | 86398 | 51852 | 85506 | 53337 | 84588 | 54805 | 83645 | 56256 | 82675 | 46 |
| 15 | 50377 | 86384 | 51877 | 85491 | 53361 | 84573 | 54829 | 83629 | 56280 | 82659 | 45 |
| 16 | 50403 | 86369 | 51902 | 85476 | 53386 | 84557 | 54854 | 83613 | 56305 | 82643 | 44 |
| 17 | 50428 | 86354 | 51927 | 85461 | 53411 | 84542 | 54878 | 83597 | 56329 | 82626 | 43 |
| 18 | 50453 | 86340 | 51952 | 85446 | 53435 | 84526 | 54902 | 83581 | 56353 | 82610 | 42 |
| 19 | 50478 | 86325 | 51977 | 85431 | 53460 | 84511 | 54927 | 83565 | 56377 | 82593 | 41 |
| 20 | 50503 | 86310 | 52002 | 85416 | 53484 | 84495 | 54951 | 83549 | 56401 | 82577 | 40 |
| 21 | 50528 | 86295 | 52026 | 85401 | 53509 | 84480 | 54975 | 83533 | 56425 | 82561 | 39 |
| 22 | 50553 | 86281 | 52051 | 85385 | 53534 | 84464 | 54999 | 83517 | 56449 | 82544 | 38 |
| 23 | 50578 | 86266 | 52076 | 85370 | 53558 | 84448 | 55024 | 83501 | 56473 | 82528 | 37 |
| 24 | 50603 | 86251 | 52101 | 85355 | 53583 | 84433 | 55048 | 83485 | 56497 | 82511 | 36 |
| 25 | 50628 | 86237 | 52126 | 85340 | 53607 | 84417 | 55072 | 83469 | 56521 | 82495 | 35 |
| 26 | 50654 | 86222 | 52151 | 85325 | 53632 | 84402 | 55097 | 83453 | 56545 | 82478 | 34 |
| 27 | 50679 | 86207 | 52175 | 85310 | 53656 | 84386 | 55121 | 83437 | 56569 | 82462 | 33 |
| 28 | 50704 | 86192 | 52200 | 85294 | 53681 | 84370 | 55145 | 83421 | 56593 | 82446 | 32 |
| 29 | 50729 | 86178 | 52225 | 85279 | 53705 | 84355 | 55169 | 83405 | 56617 | 82429 | 31 |
| 30 | 50754 | 86163 | 52250 | 85264 | 53730 | 84339 | 55194 | 83389 | 56641 | 82413 | 30 |
| 31 | 50779 | 86148 | 52275 | 85249 | 53754 | 84324 | 55218 | 83373 | 56665 | 82396 | 29 |
| 32 | 50804 | 86133 | 52299 | 85234 | 53779 | 84308 | 55242 | 83356 | 56689 | 82380 | 28 |
| 33 | 50829 | 86119 | 52324 | 85218 | 53804 | 84292 | 55266 | 83340 | 56713 | 82363 | 27 |
| 34 | 50854 | 86104 | 52349 | 85203 | 53828 | 84277 | 55291 | 83324 | 56736 | 82347 | 26 |
| 35 | 50879 | 86089 | 52374 | 85188 | 53853 | 84261 | 55315 | 83308 | 56760 | 82330 | 25 |
| 36 | 50904 | 86074 | 52399 | 85173 | 53877 | 84245 | 55339 | 83292 | 56784 | 82314 | 24 |
| 37 | 50929 | 86059 | 52423 | 85157 | 53902 | 84230 | 55363 | 83276 | 56808 | 82297 | 23 |
| 38 | 50954 | 86045 | 52448 | 85142 | 53926 | 84214 | 55388 | 83260 | 56832 | 82281 | 22 |
| 39 | 50979 | 86030 | 52473 | 85127 | 53951 | 84198 | 55412 | 83244 | 56856 | 82264 | 21 |
| 40 | 51004 | 86015 | 52498 | 85112 | 53975 | 84182 | 55436 | 83228 | 56880 | 82248 | 20 |
| 41 | 51029 | 86000 | 52522 | 85096 | 54000 | 84167 | 55460 | 83212 | 56904 | 82231 | 19 |
| 42 | 51054 | 85985 | 52547 | 85081 | 54024 | 84151 | 55484 | 83195 | 56928 | 82214 | 18 |
| 43 | 51079 | 85970 | 52572 | 85066 | 54049 | 84135 | 55509 | 83179 | 56952 | 82198 | 17 |
| 44 | 51104 | 85956 | 52597 | 85051 | 54073 | 84120 | 55533 | 83163 | 56976 | 82181 | 16 |
| 45 | 51129 | 85941 | 52621 | 85035 | 54097 | 84104 | 55557 | 83147 | 57000 | 82165 | 15 |
| 46 | 51154 | 85926 | 52646 | 85020 | 54122 | 84088 | 55581 | 83131 | 57024 | 82148 | 14 |
| 47 | 51179 | 85911 | 52671 | 85005 | 54146 | 84072 | 55605 | 83115 | 57047 | 82132 | 13 |
| 48 | 51204 | 85896 | 52696 | 84989 | 54171 | 84057 | 55630 | 83098 | 57071 | 82115 | 12 |
| 49 | 51229 | 85881 | 52720 | 84974 | 54195 | 84041 | 55654 | 83082 | 57095 | 82098 | 11 |
| 50 | 51254 | 85866 | 52745 | 84959 | 54220 | 84025 | 55678 | 83066 | 57119 | 82082 | 10 |
| 51 | 51279 | 85851 | 52770 | 84943 | 54244 | 84009 | 55702 | 83050 | 57143 | 82065 | 9 |
| 52 | 51304 | 85836 | 52794 | 84928 | 54269 | 83994 | 55726 | 83034 | 57167 | 82048 | 8 |
| 53 | 51329 | 85821 | 52819 | 84913 | 54293 | 83978 | 55750 | 83017 | 57191 | 82032 | 7 |
| 54 | 51354 | 85806 | 52844 | 84897 | 54317 | 83962 | 55775 | 83001 | 57215 | 82015 | 6 |
| 55 | 51379 | 85792 | 52869 | 84882 | 54342 | 83946 | 55799 | 82985 | 57238 | 81999 | 5 |
| 56 | 51404 | 85777 | 52893 | 84866 | 54366 | 83930 | 55823 | 82969 | 57262 | 81982 | 4 |
| 57 | 51429 | 85762 | 52918 | 84851 | 54391 | 83915 | 55847 | 82953 | 57286 | 81965 | 3 |
| 58 | 51454 | 85747 | 52943 | 84836 | 54415 | 83899 | 55871 | 82936 | 57310 | 81949 | 2 |
| 59 | 51479 | 85732 | 52967 | 84820 | 54440 | 83883 | 55895 | 82920 | 57334 | 81932 | 1 |
| 60 | 51504 | 85717 | 52992 | 84805 | 54464 | 83867 | 55919 | 82904 | 57358 | 81915 | 0 |
| ′ | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | ′ |
| | 59° | | 58° | | 57° | | 56° | | 55° | | |

# NATURAL SINES AND COSINES.

| ′ | 35° | | 36° | | 37° | | 38° | | 39° | | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | |
| 0 | 57358 | 81915 | 58779 | 80902 | 60182 | 79864 | 61566 | 78801 | 62932 | 77715 | 60 |
| 1 | 57381 | 81899 | 58802 | 80885 | 60205 | 79846 | 61589 | 78783 | 62955 | 77696 | 59 |
| 2 | 57405 | 81882 | 58826 | 80867 | 60228 | 79829 | 61612 | 78765 | 62977 | 77678 | 58 |
| 3 | 57429 | 81865 | 58849 | 80850 | 60251 | 79811 | 61635 | 78747 | 63000 | 77660 | 57 |
| 4 | 57453 | 81848 | 58873 | 80833 | 60274 | 79793 | 61658 | 78729 | 63022 | 77641 | 56 |
| 5 | 57477 | 81832 | 58896 | 80816 | 60298 | 79776 | 61681 | 78711 | 63045 | 77623 | 55 |
| 6 | 57501 | 81815 | 58920 | 80799 | 60321 | 79758 | 61704 | 78694 | 63068 | 77605 | 54 |
| 7 | 57524 | 81798 | 58943 | 80782 | 60344 | 79741 | 61726 | 78676 | 63090 | 77586 | 53 |
| 8 | 57548 | 81782 | 58967 | 80765 | 60367 | 79723 | 61749 | 78658 | 63113 | 77568 | 52 |
| 9 | 57572 | 81765 | 58990 | 80748 | 60390 | 79706 | 61772 | 78640 | 63135 | 77550 | 51 |
| 10 | 57596 | 81748 | 59014 | 80730 | 60414 | 79688 | 61795 | 78622 | 63158 | 77531 | 50 |
| 11 | 57619 | 81731 | 59037 | 89713 | 60437 | 79671 | 61818 | 78604 | 63180 | 77513 | 49 |
| 12 | 57643 | 81714 | 59061 | 80696 | 60460 | 79653 | 61841 | 78586 | 63203 | 77494 | 48 |
| 13 | 57667 | 81698 | 59084 | 80679 | 60483 | 79635 | 61864 | 78568 | 63225 | 77476 | 47 |
| 14 | 57691 | 81681 | 59108 | 80662 | 60506 | 79618 | 61887 | 78550 | 63248 | 77458 | 46 |
| 15 | 57715 | 81664 | 59131 | 80644 | 60529 | 79600 | 61909 | 78532 | 63271 | 77439 | 45 |
| 16 | 57738 | 81647 | 59154 | 80627 | 60553 | 79583 | 61932 | 78514 | 63293 | 77421 | 44 |
| 17 | 57762 | 81631 | 59178 | 80610 | 60576 | 79565 | 61955 | 78496 | 63316 | 77402 | 43 |
| 18 | 57786 | 81614 | 59201 | 80593 | 60599 | 79547 | 61978 | 78478 | 63338 | 77384 | 42 |
| 19 | 57810 | 81597 | 59225 | 80576 | 60622 | 79530 | 62001 | 78460 | 63361 | 77366 | 41 |
| 20 | 57833 | 81580 | 59248 | 80558 | 60645 | 79512 | 62024 | 78442 | 63383 | 77347 | 40 |
| 21 | 57857 | 81563 | 59272 | 80541 | 60668 | 79494 | 62046 | 78424 | 63406 | 77329 | 39 |
| 22 | 57881 | 81546 | 59295 | 80524 | 60691 | 79477 | 62069 | 78405 | 63428 | 77310 | 38 |
| 23 | 57904 | 81530 | 59318 | 80507 | 60714 | 79459 | 62092 | 78387 | 63451 | 77292 | 37 |
| 24 | 57928 | 81513 | 59342 | 80489 | 60738 | 79441 | 62115 | 78369 | 63473 | 77273 | 36 |
| 25 | 57952 | 81496 | 59365 | 80472 | 60761 | 79424 | 62138 | 78351 | 63496 | 77255 | 35 |
| 26 | 57976 | 81479 | 59389 | 80455 | 60784 | 79406 | 62160 | 78333 | 63518 | 77236 | 34 |
| 27 | 57999 | 81462 | 59412 | 80438 | 60807 | 79388 | 62183 | 78315 | 63540 | 77218 | 33 |
| 28 | 58023 | 81445 | 59436 | 80420 | 60830 | 79371 | 62206 | 78297 | 63563 | 77199 | 32 |
| 29 | 58047 | 81428 | 59459 | 80403 | 60853 | 79353 | 62229 | 78279 | 63585 | 77181 | 31 |
| 30 | 58070 | 81412 | 59482 | 80386 | 60876 | 79335 | 62251 | 78261 | 63608 | 77162 | 30 |
| 31 | 58094 | 81395 | 59506 | 80368 | 60899 | 79318 | 62274 | 78243 | 63630 | 77144 | 29 |
| 32 | 58118 | 81378 | 59529 | 80351 | 60922 | 79300 | 62297 | 78225 | 63653 | 77125 | 28 |
| 33 | 58141 | 81361 | 59552 | 80334 | 60945 | 79282 | 62320 | 78206 | 63675 | 77107 | 27 |
| 34 | 58165 | 81344 | 59576 | 80316 | 60968 | 79264 | 62342 | 78188 | 63698 | 77088 | 26 |
| 35 | 58189 | 81327 | 59599 | 80299 | 60991 | 79247 | 62365 | 78170 | 63720 | 77070 | 25 |
| 36 | 58212 | 81310 | 59622 | 80282 | 61015 | 79229 | 62388 | 78152 | 63742 | 77051 | 24 |
| 37 | 58236 | 81293 | 59646 | 80264 | 61038 | 79211 | 62411 | 78134 | 63765 | 77033 | 23 |
| 38 | 58260 | 81276 | 59669 | 80247 | 61061 | 79193 | 62433 | 78116 | 63787 | 77014 | 22 |
| 39 | 58283 | 81259 | 59693 | 80230 | 61084 | 79176 | 62456 | 78098 | 63810 | 76996 | 21 |
| 40 | 58307 | 81242 | 59716 | 80212 | 61107 | 79158 | 62479 | 78079 | 63832 | 76977 | 20 |
| 41 | 58330 | 81225 | 59739 | 80195 | 61130 | 79140 | 62502 | 78061 | 63854 | 76959 | 19 |
| 42 | 58354 | 81208 | 59763 | 80178 | 61153 | 79122 | 62524 | 78043 | 63877 | 76940 | 18 |
| 43 | 58378 | 81191 | 59786 | 80160 | 61176 | 79105 | 62547 | 78025 | 63899 | 76921 | 17 |
| 44 | 58401 | 81174 | 59809 | 80143 | 61199 | 79087 | 62570 | 78007 | 63922 | 76903 | 16 |
| 45 | 58425 | 81157 | 59832 | 80125 | 61222 | 79069 | 62592 | 77988 | 63944 | 76884 | 15 |
| 46 | 58449 | 81140 | 59856 | 80108 | 61245 | 79051 | 62615 | 77970 | 63966 | 76866 | 14 |
| 47 | 58472 | 81123 | 59879 | 80091 | 61268 | 79033 | 62638 | 77952 | 63989 | 76847 | 13 |
| 48 | 58496 | 81106 | 59902 | 80073 | 61291 | 79016 | 62660 | 77934 | 64011 | 76828 | 12 |
| 49 | 58519 | 81089 | 59926 | 80056 | 61314 | 78998 | 62683 | 77916 | 64033 | 76810 | 11 |
| 50 | 58543 | 81072 | 59949 | 80038 | 61337 | 78980 | 62706 | 77897 | 64056 | 76791 | 10 |
| 51 | 58567 | 81055 | 59972 | 80021 | 61360 | 78962 | 62728 | 77879 | 64078 | 76772 | 9 |
| 52 | 58590 | 81038 | 59995 | 80003 | 61383 | 78944 | 62751 | 77861 | 64100 | 76754 | 8 |
| 53 | 58614 | 81021 | 60019 | 79986 | 61406 | 78926 | 62774 | 77843 | 64123 | 76735 | 7 |
| 54 | 58637 | 81004 | 60042 | 79968 | 61429 | 78908 | 62796 | 77824 | 64145 | 76717 | 6 |
| 55 | 58661 | 80987 | 60065 | 79951 | 61451 | 78891 | 62819 | 77806 | 64167 | 76698 | 5 |
| 56 | 58684 | 80970 | 60089 | 79934 | 61474 | 78873 | 62842 | 77788 | 64190 | 76679 | 4 |
| 57 | 58708 | 80953 | 60112 | 79916 | 61497 | 78855 | 62864 | 77769 | 64212 | 76661 | 3 |
| 58 | 58731 | 80936 | 60135 | 79899 | 61520 | 78837 | 62887 | 77751 | 64234 | 76642 | 2 |
| 59 | 58755 | 80919 | 60158 | 79881 | 61543 | 78819 | 62909 | 77733 | 64256 | 76623 | 1 |
| 60 | 58779 | 80902 | 60182 | 79864 | 61566 | 78801 | 62932 | 77715 | 64279 | 76604 | 0 |
| ′ | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | ′ |
| | 54° | | 53° | | 52° | | 51° | | 50° | | |

# NATURAL SINES AND COSINES.

| ′ | 40° | | 41° | | 42° | | 43° | | 44° | | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | |
| 0 | 64279 | 76604 | 65606 | 75471 | 66913 | 74314 | 68200 | 73135 | 69466 | 71934 | 60 |
| 1 | 64301 | 76586 | 65628 | 75452 | 66935 | 74295 | 68221 | 73116 | 69487 | 71914 | 59 |
| 2 | 64323 | 76567 | 65650 | 75433 | 66956 | 74276 | 68242 | 73096 | 69508 | 71894 | 58 |
| 3 | 64346 | 76548 | 65672 | 75414 | 66978 | 74256 | 68264 | 73076 | 69529 | 71873 | 57 |
| 4 | 64368 | 76530 | 65694 | 75395 | 66999 | 74237 | 68285 | 73056 | 69549 | 71853 | 56 |
| 5 | 64390 | 76511 | 65716 | 75375 | 67021 | 74217 | 68306 | 73036 | 69570 | 71833 | 55 |
| 6 | 64412 | 76492 | 65738 | 75356 | 67043 | 74198 | 68327 | 73016 | 69591 | 71813 | 54 |
| 7 | 64435 | 76473 | 65759 | 75337 | 67064 | 74178 | 68349 | 72996 | 69612 | 71792 | 53 |
| 8 | 64457 | 76455 | 65781 | 75318 | 67086 | 74159 | 68370 | 72976 | 69633 | 71772 | 52 |
| 9 | 64479 | 76436 | 65803 | 75299 | 67107 | 74139 | 68391 | 72957 | 69654 | 71752 | 51 |
| 10 | 64501 | 76417 | 65825 | 75280 | 67129 | 74120 | 68412 | 72937 | 69675 | 71732 | 50 |
| 11 | 64524 | 76398 | 65847 | 75261 | 67151 | 74100 | 68434 | 72917 | 69696 | 71711 | 49 |
| 12 | 64546 | 76380 | 65869 | 75241 | 67172 | 74080 | 68455 | 72897 | 69717 | 71691 | 48 |
| 13 | 64568 | 76361 | 65891 | 75222 | 67194 | 74061 | 68476 | 72877 | 69737 | 71671 | 47 |
| 14 | 64590 | 76342 | 65913 | 75203 | 67215 | 74041 | 68497 | 72857 | 69758 | 71650 | 46 |
| 15 | 64612 | 76323 | 65935 | 75184 | 67237 | 74022 | 68518 | 72837 | 69779 | 71630 | 45 |
| 16 | 64635 | 76304 | 65956 | 75165 | 67258 | 74002 | 68539 | 72817 | 69800 | 71610 | 44 |
| 17 | 64657 | 76286 | 65978 | 75146 | 67280 | 73983 | 68561 | 72797 | 69821 | 71590 | 43 |
| 18 | 64679 | 76267 | 66000 | 75126 | 67301 | 73963 | 68582 | 72777 | 69842 | 71569 | 42 |
| 19 | 64701 | 76248 | 66022 | 75107 | 67323 | 73944 | 68603 | 72757 | 69862 | 71549 | 41 |
| 20 | 64723 | 76229 | 66044 | 75088 | 67344 | 73924 | 68624 | 72737 | 69883 | 71529 | 40 |
| 21 | 64746 | 76210 | 66066 | 75069 | 67366 | 73904 | 68645 | 72717 | 69904 | 71508 | 39 |
| 22 | 64768 | 76192 | 66088 | 75050 | 67387 | 73885 | 68666 | 72697 | 69925 | 71488 | 38 |
| 23 | 64790 | 76173 | 66109 | 75030 | 67409 | 73865 | 68688 | 72677 | 69946 | 71468 | 37 |
| 24 | 64812 | 76154 | 66131 | 75011 | 67430 | 73846 | 68709 | 72657 | 69966 | 71447 | 36 |
| 25 | 64834 | 76135 | 66153 | 74992 | 67452 | 73826 | 68730 | 72637 | 69987 | 71427 | 35 |
| 26 | 64856 | 76116 | 66175 | 74973 | 67473 | 73806 | 68751 | 72617 | 70008 | 71407 | 34 |
| 27 | 64878 | 76097 | 66197 | 74953 | 67495 | 73787 | 68772 | 72597 | 70029 | 71386 | 33 |
| 28 | 64901 | 76078 | 66218 | 74934 | 67516 | 73767 | 68793 | 72577 | 70049 | 71366 | 32 |
| 29 | 64923 | 76059 | 66240 | 74915 | 67538 | 73747 | 68814 | 72557 | 70070 | 71345 | 31 |
| 30 | 64945 | 76041 | 66262 | 74896 | 67559 | 73728 | 68835 | 72537 | 70091 | 71325 | 30 |
| 31 | 64967 | 76022 | 66284 | 74876 | 67580 | 73708 | 68857 | 72517 | 70112 | 71305 | 29 |
| 32 | 64989 | 76003 | 66306 | 74857 | 67602 | 73688 | 68878 | 72497 | 70132 | 71284 | 28 |
| 33 | 65011 | 75984 | 66327 | 74838 | 67623 | 73669 | 68899 | 72477 | 70153 | 71264 | 27 |
| 34 | 65033 | 75965 | 66349 | 74818 | 67645 | 73649 | 68920 | 72457 | 70174 | 71243 | 26 |
| 35 | 65055 | 75946 | 66371 | 74799 | 67666 | 73629 | 68941 | 72437 | 70195 | 71223 | 25 |
| 36 | 65077 | 75927 | 66393 | 74780 | 67688 | 73610 | 68962 | 72417 | 70215 | 71203 | 24 |
| 37 | 65100 | 75908 | 66414 | 74760 | 67709 | 73590 | 68983 | 72397 | 70236 | 71182 | 23 |
| 38 | 65122 | 75889 | 66436 | 74741 | 67730 | 73570 | 69004 | 72377 | 70257 | 71162 | 22 |
| 39 | 65144 | 75870 | 66458 | 74722 | 67752 | 73551 | 69025 | 72357 | 70277 | 71141 | 21 |
| 40 | 65166 | 75851 | 66480 | 74703 | 67773 | 73531 | 69046 | 72337 | 70298 | 71121 | 20 |
| 41 | 65188 | 75832 | 66501 | 74683 | 67795 | 73511 | 69067 | 72317 | 70319 | 71100 | 19 |
| 42 | 65210 | 75813 | 66523 | 74664 | 67816 | 73491 | 69088 | 72297 | 70339 | 71080 | 18 |
| 43 | 65232 | 75794 | 66545 | 74644 | 67837 | 73472 | 69109 | 72277 | 70360 | 71059 | 17 |
| 44 | 65254 | 75775 | 66566 | 74625 | 67859 | 73452 | 69130 | 72257 | 70381 | 71039 | 16 |
| 45 | 65276 | 75756 | 66588 | 74606 | 67880 | 73432 | 69151 | 72236 | 70401 | 71019 | 15 |
| 46 | 65298 | 75738 | 66610 | 74586 | 67901 | 73413 | 69172 | 72216 | 70422 | 70998 | 14 |
| 47 | 65320 | 75719 | 66632 | 74567 | 67923 | 73393 | 69193 | 72196 | 70443 | 70978 | 13 |
| 48 | 65342 | 75700 | 66653 | 74548 | 67944 | 73373 | 69214 | 72176 | 70463 | 70957 | 12 |
| 49 | 65364 | 75680 | 66675 | 74528 | 67965 | 73353 | 69235 | 72156 | 70484 | 70937 | 11 |
| 50 | 65386 | 75661 | 66697 | 74509 | 67987 | 73333 | 69256 | 72136 | 70505 | 70916 | 10 |
| 51 | 65408 | 75642 | 66718 | 74489 | 68008 | 73314 | 69277 | 72116 | 70525 | 70896 | 9 |
| 52 | 65430 | 75623 | 66740 | 74470 | 68029 | 73294 | 69298 | 72095 | 70546 | 70875 | 8 |
| 53 | 65452 | 75604 | 66762 | 74451 | 68051 | 73274 | 69319 | 72075 | 70567 | 70855 | 7 |
| 54 | 65474 | 75585 | 66783 | 74431 | 68072 | 73254 | 69340 | 72055 | 70587 | 70834 | 6 |
| 55 | 65496 | 75566 | 66805 | 74412 | 68093 | 73234 | 69361 | 72035 | 70608 | 70813 | 5 |
| 56 | 65518 | 75547 | 66827 | 74392 | 68115 | 73215 | 69382 | 72015 | 70628 | 70793 | 4 |
| 57 | 65540 | 75528 | 66848 | 74373 | 68136 | 73195 | 69403 | 71995 | 70649 | 70772 | 3 |
| 58 | 65562 | 75509 | 66870 | 74353 | 68157 | 73175 | 69424 | 71974 | 70670 | 70752 | 2 |
| 59 | 65584 | 75490 | 66891 | 74334 | 68179 | 73155 | 69445 | 71954 | 70690 | 70731 | 1 |
| 60 | 65606 | 75471 | 66913 | 74314 | 68200 | 73135 | 69466 | 71934 | 70711 | 70711 | 0 |
| ′ | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | Cosine. | Sine. | ′ |
| | 49° | | 48° | | 47° | | 46° | | 45° | | |

N 648

Deacidified using the Bookkeeper process.
Neutralizing agent: Magnesium Oxide
Treatment Date: Jan. 2004

PreservationTechnologies
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